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Ontology-Based Representation and Reasoning about Precise and Imprecise Temporal Data: a Fuzzy-Based View

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ABSTRACT

Temporal representation and reasoning are important facets in the design of many Semantic Web applications. Several approaches exist to represent and reason about precise temporal data in ontology. However, most of them handle only time intervals and associated qualitative relations. Besides, to the best of our knowledge, there is no approach devoted to handle imprecise temporal data (e.g., "*late 1970s*"). In this paper, we propose an ontology-based approach for representing and reasoning about precise and imprecise temporal data. Quantitative temporal data (i.e., time intervals and points) and qualitative ones (i.e., relations between time intervals, relations between a time interval and a time point and relations between time points) are taken into consideration. Our approach is three folds: *(i)* extending the 4D-fluents approach with new crisp and fuzzy components, to represent precise and imprecise temporal data, (*ii*) extending the Allen's interval algebra to enable reasoning about precise and imprecise temporal data, and (*iii*) creating a Fuzzy-OWL 2 ontology TimeOnto that, based on the extended Allen's interval algebra, instantiates our 4D-fluents-based representation. The extension that we propose for the Allen's interval algebra handles precise and imprecise time intervals. Indeed, it enables expressing precise (e.g., "*before*") and imprecise (e.g., "*just before*") temporal relations. Compared to related work, our imprecise relations are personalized, in the sense that they are not limited to a defined set of interval relations and their meanings are determined by the domain expert. For instance, the classic Allen's relation "Before" may be generalized in 5 imprecise relations, where "$\text{Before}_{(1)}$" means "*just before*" and gradually the time gap between the two intervals increases until "$\text{Before}_{(5)}$" which means "*very long before*". To enable this representation, we propose an extension of the Vilain and Kautz's point algebra and redefined the Allen's relations by means of this extended algebra. We show in this paper that, unlike most related work, the resulting relations preserve many of the desirable properties of the Allen's interval algebra. The definitions of the resulting interval relations are adapted to allow relating a time interval and a time point, and two time points, where time intervals and points maybe both precise or both imprecise. These relations can be used for temporal reasoning by means of four transitivity tables. Finally, we describe a prototype based on "TimeOnto" that infers new relations using a set of SWRL and fuzzy IF-THEN rules. This prototype was integrated in an ontology-based memory prosthesis for Alzheimer's patients.

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1. Introduction

In the context of the VIVA[*] project ("*Vivre à Paris avec Alzheimer en 2030 grâce aux nouvelles technologies*"), we are proposing a memory prosthesis, called Captain Memo [1], to help Alzheimer's patients to palliate mnesic problems. This prosthesis supplies a set of services. Among these services, one is devoted to "remember thing(s) about people" i.e., it helps users to remember their convivial surroundings and relatives. Data in this service are structured semantically using an ontology called, PersonLink[†] [2]. It is a multicultural and multilingual OWL 2 ontology for storing, representing and reasoning about interpersonal relationships (e.g., husband, aunt and half-brother) and describing people (e.g., name, age, preferences and lived events). In this context, temporal inputs are especially numerous e.g., "John married to Maria *in 1982* and he remarried to Béatrice *in 2006*", "I traveled to India *in 1980*" and "My sister married *before* moving to Paris". This kind of data is not supported by PersonLink. Therefore, it is necessary to extend it to handle quantitative and qualitative temporal data.

Temporal data given by humans are often imprecise. This issue becomes worse in the context of Captain Memo. Indeed, imprecise temporal inputs are especially numerous when given by Alzheimer disease patients. For instance, if they give the information "John married to Béatrice from *early 2000s* to *by 2016*", two measures of imprecision are involved. On the one hand, the temporal data "early 2000s" is imprecise in the sense that it could mean, with a decreasingly possibility, from 2000 to 2004; on the other hand, the temporal data "*by 2016*" is imprecise in the sense that it could mean from 2014 to 2016. Moreover, they may express qualitative temporal relations in an imprecise way e.g., "John married to Maria *just after* he was graduated with a PhD". PersonLink needs to be extended to represent and reason about precise and imprecise temporal data.

In the Semantic Web field, several approaches have been proposed to represent and reason about precise temporal data. However, most of them handle only precise time intervals and associated qualitative relations i.e., they are not intended to handle time points and qualitative relations between a time interval and a time point or two time points. Besides, to the best of our knowledge, there is no approach devoted to handle imprecise temporal data.

In this paper, we propose an ontology-based approach to represent and reason about precise and imprecise temporal data. Quantitative temporal data (i.e., time intervals and points) and qualitative temporal data (i.e., relations between time intervals, relations between a time interval and a time point and relations between time points) are taken into consideration. We focus on handling temporal relations relating entities which both are precise or imprecise. Our approach is consistent with existing Semantic Web standards and tools. It consists of three parts:

- The first part focuses on representing precise and imprecise temporal data in ontology. Earlier work by [3] showed how precise time intervals and their evolution in time can be modeled in OWL using the so-called 4D-fluents approach. We extend this approach with new crisp and fuzzy ontological components in two ways: (*i*) representing quantitative temporal data i.e., imprecise time intervals and precise/imprecise time points, and (*ii*) representing precise and imprecise qualitative temporal relations between time intervals and points.
- The second part focuses on reasoning about temporal data by extending the Allen's interval algebra [4] that proposes 13 precise temporal relations between precise time intervals. This algebra is not designed to handle situations in which time intervals are imprecise and it is not suitable to express imprecise temporal relations such as "approximately at the same time", "middle during" and "just before". Besides, it is not intended to relate a time interval and a time point or even two time points. A number of approaches extend the Allen's interval algebra to propose temporal relations between precise and imprecise time intervals. Only very approaches propose imprecise temporal relations between time intervals that, unlike our approach, cannot be personalized. Besides, most of these approaches do not preserve the properties of the original Allen's interval algebra and do not study the compositions of the resulting relations. In this work, we propose an extension of the Allen's interval algebra that handles qualitative temporal relations between precise and imprecise time intervals and preserves important properties regarding reflexivity/irreflexivity, symmetry/asymmetry and transitivity. Our extension is not only suitable to model precise relations, but also imprecise ones. Compared to related work, our imprecise relations are personalized. Indeed, the classic Allen's relations {"Before", "After", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Ends" and "Ended-by"} maybe generalized in N imprecise relations. Their number N and their meanings are specified by the domain expert. For instance, the classic Allen's relation "After" may be generalized in 3 imprecise relations, where "$After_{(1)}$" means "approximately the same time"; "$After_{(2)}$" means "just after" and "$After_{(3)}$" means "long after". To enable this representation, we propose an extension of the Vilain and Kautz's point algebra [5] and redefined the Allen's relations by means of this extended algebra. We adapt the resulting interval relations to propose temporal relations between a time interval and a time point, and two time points. Furthermore, temporal relations that we propose in this work can be used for temporal reasoning by means of four transitivity tables.
- The third part consists of proposing a fuzzy ontology, called TimeOnto. This ontology allows representing and reasoning about precise and imprecise temporal data in OWL 2. It implements our extension of the 4D-fluents approach based on our extension of the Allen's interval algebra. Qualitative temporal relations are inferred via a set of SWRL and fuzzy IF-THEN rules.

The current paper is organized as follows. Section 2 is devoted to present some preliminary concepts. Related works are discussed in Section 3. Section 4 details our ontology-based approach for representing precise and imprecise temporal data. Section 5 introduces our approach for reasoning about the handled data. Section 6 presents the proposed TimeOnto ontology. Section 7 details the TimeOnto-based prototype and illustrates its usefulness within the context of the Captain Memo memory prosthesis. Finally, Section 8 draws conclusions and future research directions.

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[*] http://viva.cnam.fr/

[†] http://cedric.cnam.fr/isid/ontologies/files/PersonLink.html

2. Preliminaries

The purpose of this section is to recall some notions necessary to the reading of the paper.

2.1. Representing Temporal Data

Temporal data can be regarded as discrete or continuous, linear or cyclical and absolute or relative [6]. Temporal data are subject to imprecision and, thus, can be precise or imprecise. They are described using quantitative or qualitative terms. Precise quantitative data means precise time intervals and points. Imprecise quantitative data mean imprecise time intervals and points. Imprecise time intervals are classic (precise) time intervals characterized by gradual beginning and/or ending bounds (e.g., "the period from the *late 1960s* to the *early 1990s*"). Imprecise time points are defined as ill-defined classic time points (e.g., "*by 2000*"). Qualitative temporal relations may relate two time intervals (Interval-Interval), a time interval and a time point (Interval-Point and Point-Interval) or even two time points (Point-Point), where time intervals and points maybe precise or imprecise. These temporal relations maybe precise (e.g., "before") or imprecise (e.g., "just before"). Qualitative temporal data can be deduced from the quantitative ones.

2.1.1. Vilain and Kautz's Point Algebra

Vilain and Kautz have identified 3 qualitative comparators: "Precedes", "Same" and "Follows". They allow comparing two precise time points P and L. Their semantics are illustrated in Table 1.

Table 1 - Vilain and Kautz's temporal relations between two precise time points P (●) and L (○).

Comparator(P, L)	Definition	Illustration	Inverse(L, P)
Precedes(P, L)	P < L	● ○	Follows(L, P)
Same(P, L)	P = L	○ ●	Same(L, P)
Follows(P, L)	P > L	○ ●	Precedes(L, P)

This algebra does not handle imprecise time points and it does not propose imprecise qualitative relations. Indeed, in some situation, it cannot express the level of precision required. Taking the example of "John married *after* he was finished his PhD study" does not express the same precision if it is avowed as "John married *just after* he was finished his PhD study".

2.1.2. Allen's Interval Algebra

Allen [4] has proposed 13 precise qualitative relations that may hold between two precise time intervals. Their semantics are illustrated in Table 2. These relations are defined in terms of the ordering of the beginning and ending bounds of the corresponding intervals, which in turn are defined in terms of the ordering of the time points belonging to these intervals (thus, the Allen's relation maybe defined based on the Vilain and Kautz's point algebra).

Table 2 - Allen's temporal relations between two precise time intervals $I = [I^-, I^+]$ (▬) and $J = [J^-, J^+]$ (═).

Relation(I, J)	Symbol	Allen's Original Definition	Definition based on the Vilain and Kautz's point algebra	Illustration	Inverse(J, I)	Symbol
Before(I, J)	B	$I^+ < J^-$	Precedes(I^+, J^-)		After(J, I)	A
Meets(I, J)	M	$I^+ = J^-$	Same(I^+, J^-)		Met-by(J, I)	Mb
Overlaps(I, J)	O	$I^- < J^- \wedge J^- < I^+ \wedge I^+ < J^+$	Precedes(I^-, J^-) ∧ Precedes(J^-, I^+) ∧ Precedes(I^+, J^+)		Overlapped-by(J, I)	Ob
Starts(I, J)	S	$I^- = J^- \wedge I^+ < J^+$	Same(I^-, J^-) ∧ Precedes(I^+, J^+)		Started-by(J, I)	Sb
During(I, J)	D	$J^- < I^- \wedge I^+ < J^+$	Precedes(J^-, I^-) ∧ Precedes(I^+, J^+)		Contains(J, I)	C
Ends(I, J)	E	$J^- < I^- \wedge I^+ = J^+$	Precedes(J^-, I^-) ∧ Same(I^+, J^+)		Ended-by(J, I)	Eb
Equals(I, J)	Eq	$I^- = J^- \wedge I^+ = J^+$	Same(I^-, J^-) ∧ Same(I^+, J^+)		Equals(J, I)	Eq

In cases where the duration of time intervals are unknown (i.e., the beginning and/or ending bounds are not specified), the relations to other time intervals can still be asserted qualitatively by means of temporal relations (e.g., "a time interval I is *before* a time interval J" even in cases where the exact beginning and ending bounds of either I, J, or both are unknown). For nondegenerate time intervals (a time interval $I = [I^-, I^+]$ is nondegenerate if $I^- < I^+$), Allen's relations are mutually exclusive. This means that at most one relation holds between two given nondegenerate time intervals.

The crux of this algebra is the transitivity table. This table lets us deduce that $R_3(I, K)$ holds when $R_1(I, J)$ and $R_2(J, K)$; where $I = [I^-, I^+]$, $J = [J^-, J^+]$ and $K = [K^-, K^+]$ are precise time intervals and R_1, R_2 and R_3 are Allen's relations. For instance, we can deduce from "During(I, J)" and "Meet(J, K)" that "Before(I, K)" holds. Indeed by "During(I, J)", we have "Precedes(J^-, I^-) ∧ Precedes(I^+, J^+)", and by "Meet(J, K)", we have "Same(J^+, K^-)". From "Precedes(I^+, J^+)" and "Same(J^+, K^-)", we conclude that "Before(I, K)" holds. Not all compositions yield a unique relation as a result. For example, the composition of relations "Overlaps(I, J)" and "During(J, K)" yields three possible relations namely "Starts(I, K)", "Overlaps(I, K)" and "During(I, K)".

The Allen's interval algebra is not designed to handle situations in which time intervals are imprecise and it is not suitable to express imprecise qualitative temporal relations. Besides, it is not intended to relate a time interval and a time point or even two time points.

2.2. Fuzzy Ontology

An ontology is an "explicit specification of a conceptualization" [7]. It contains a set of classes, taxonomic and nontaxonomic relationships between them, assertions about them and instances [8]. A classic (crisp) ontology is confronted with natural language interpretation problems caused by imprecise terms. Indeed, it cannot represent and reason about imprecise data [14]. Based on that observation, fuzzy sets theory introduced by Zadeh [9] is integrated to ontology and fuzzy ontology has emerged. A number of formal definitions can be found for the fuzzy ontology. One of the most accepted definitions is an extension of crisp ontology that uses fuzzy logic to provide a natural representation of imprecise knowledge, and eases reasoning over it [10]. It has besides the crisp components, another fuzzy logic founded components. We define a fuzzy ontology as a 4-tuple (C, P, I, A) where:

- $C = (C_C, C_F)$ is a finite set of classes, where C_c is a finite set of crisp classes and C_F is a finite set of fuzzy classes. A fuzzy class is a class which possesses, at least, one fuzzy property;
- $P = (P_C, P_F, H)$ is a finite set of datatype and object properties, where P_C is a finite set of crisp properties, P_F is a finite set of fuzzy properties and H is a finite set of semantic relations. A fuzzy property is a property which can be represented in the form of a fuzzy linguistic variable. Thus, its instances have varying degrees of membership with a value in the interval [0, 1]. These degrees are calculated according to the membership function associated to the property. A semantic relation is a relation that holds between a crisp entity and its associated fuzzy entities;
- $I = (I_C, I_F)$ is a finite set of instances, where I_C is a finite set of instances corresponding to crisp classes and I_F is a finite set of instances corresponding to fuzzy classes;
- A is a finite set of axioms.

For all the examples given in the rest of this paper, we use the membership functions defined in [9] and shown in Fig. 1.

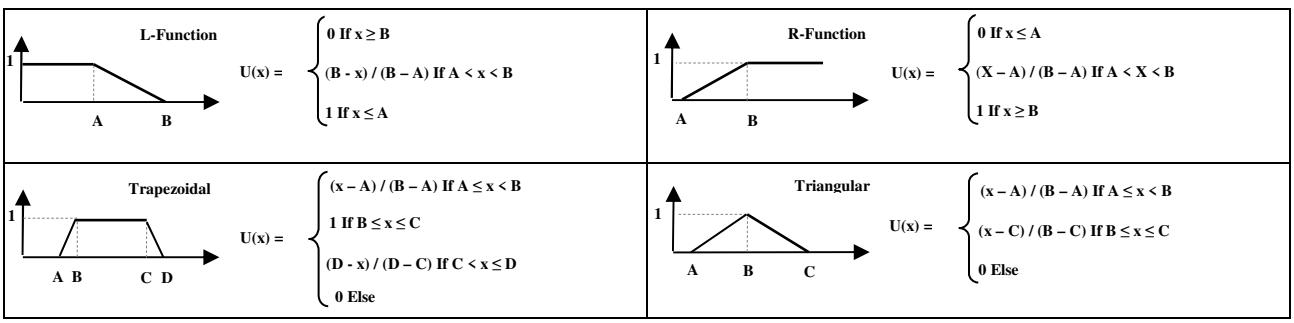


Fig. 1 - L-Function, R-Function, Trapezoidal and Triangular membership functions [9].

A number of extensions for OWL or RDF have been proposed to generate fuzzy ontology e.g., Fuzzy RDF [11], FOWL [12], f-OWL [13] and Fuzzy-OWL 2 [14]. However, these extensions are all based on binary relations that simply connect two instances without adding any temporal data. Several fuzzy reasoners have been proposed e.g., FuzzyJess [15], FiRE [16], FPLGERDS [17], YADLR [18], DeLorean [19], ONTOSEARCH2 [20], DLMedias [21], FuzzyDL [22], FuzzyRDF [23], GURDL [24], FRESG [25], SMT-based solver [26] and Lighweight Fuzzy Reasoner (LiFR) [27]. They are proposed basically for fuzzy static ontologies. However, we can use one of them if temporal data are structured using an existing fuzzy language syntax (without defining new operators).

Dynamic or temporal ontology enables representation and reasoning about data evolving in time. Adding the temporal dimension to fuzzy ontologies will enhance their ability to express and reason about temporal data.

3. State of the Art

In the present state of the art, we first address the issue of handling temporal data in the Semantic Web field. Then, we detail some extensions of the Allen's interval algebra.

3.1. Handling Temporal Data in the Semantic Web Field

Temporal concepts are represented by an ontology called OWL-Time[‡] [28]. It provides rich descriptions of temporal intervals, points, durations, and calendar terms. In October 2017, it becomes a W3C recommendation. However, it cannot specify how these concepts can be used to represent properties of objects changing in time and it does not propose inference rules to automatically infer new temporal data.

There is a need for representing the evolution of concepts in time in ontology. However, representation ontology languages such as OWL and RDF provide minimal support. They are all based on binary relations that simply connect two instances without adding any temporal data. Many approaches have been proposed to represent temporal data in ontology. We classify them into two categories. (*i*) The first category regroups approaches which extend the OWL or RDF(S) syntax to incorporate temporal data. These approaches involve defining new OWL or RDF operators and semantics. A variety of approaches have been proposed: Temporal Description Logics [29], Concrete Domains [30] and Temporal RDF [31]. However, there is still no agreement on a standard approach. (*ii*) The second category regroups approaches which are implemented directly using OWL or RDF(S) to represent temporal data. They do not modify OWL or RDF(S) syntax. They offer reasoning and querying support and they can be combined with the existing tools. A variety of approaches have been proposed: Reification [32], Versioning [33], N-Ary Relations [34], 4D-Fluents and Named Graphs [35].

Temporal Description Logics extend the standard description logics with additional temporal operators and semantics e.g., “until”, “sometime in the future” and “always in the past”. Contrary to other approaches, they do not suffer from data redundancy. Besides, they retain decidability (with an appropriate selection of allowable constructs). The main disadvantage of this approach is that it requires extending OWL or RDF(S). A number of researches are based on this approach. For instance, GFO-Time [36] offers a representation of temporal data using First Order Logic axioms that cover time and point algebra. See also [40].

Concrete Domains introduce datatypes and operators based on an underlying domain (such as decimal numbers). This approach requires extending OWL or RDF(S). TOWL [43] is an approach combining the concrete domains and 4D-fluents approaches. However, it does not support qualitative temporal relations and path consistency checking. Furthermore, it is not compatible with existing OWL editing, reasoning and querying tools.

Temporal RDF proposes extending RDF by the annotation of properties with the data about the time interval they hold on. It uses only RDF triples and requires extending the RDF syntax. Therefore, it does not have all the expressiveness of OWL. For instance, it is not possible to employ qualitative relations. A number of temporal representations are based on this approach. In [44], the authors introduce a constraint data model called stRDF. It extends RDF to represent temporal data. The authors extend SPARQL to propose a language called stSPARQL that query stRDF. StSPARQL does not express qualitative temporal data and does not have any reasoning support. In [45], Temporal RDF is enhanced with the support for undefined time intervals. Querying support for annotated properties is provided in [46].

Reification is “a general purpose technique for representing N-ary relations using a language such as OWL that permits only binary relations” [47]. For instance, if the relation R holds between two objects A and B at the time T, expressed as R(A, B, T), this is represented using reification as a new object R with three attributes A, B and T. Reification suffers from data redundancy because a new object is created whenever a temporal relation has to be represented. It offers limited OWL reasoning and inference capabilities [48]. Examples of temporal representation based on this approach are presented in [49], [50], [51], [52] and [53]. Connor and Das [51] propose a reification-based approach for representing and querying temporal data in OWL that uses the SWRL and SQWRL languages. In [52], temporal representation is combined with the application of specific SWRL rules for representing clinical narratives. In [53], a temporal representation is proposed, and an associated query language is introduced.

Versioning is described as the ability to handle changes in ontology by creating and managing different variants of it. When an ontology is modified, a new version is created to represent the temporal evolution of the ontology. This approach suffers from data redundancy as changes even on one single attribute require the creation of a new version of the ontology. All the created versions are independent from each other as there is no relation between evolving concepts. Searching requires exhaustive searches in all created versions [55]. Most implementations adopt a variety of optimization strategies to ensure that entire copies of the ontology are not generated for each new version. In [57], the authors introduce an RDF-centric versioning approach and an implementation named SemVersion. In [58], the authors propose an approach for schema versioning in OWL 2. T-SPARQL [59] is a query language that uses the versioning approach for the representation of temporal data.

The N-ary Relations[§] approach suggests representing an N-ary relation as two properties each related to a new object (rather than as the object of a property, as the reification does). It requires only one additional object for every time interval, and it maintains property semantics. However, it suffers from data redundancy in the case of inverse and symmetric properties. In [47], the N-ary relations approach is enhanced with qualitative temporal relations between time intervals. CHORONOS [60] is a Tab plug-in for the PROTÉGÉ editor that use N-ary relations approach to add temporal data to a static ontology. Another plug-in for the PROTÉGÉ editor supporting editing of temporal ontologies using N-ary is presented at [62]. A similar tool for both 4D-fluents and N-ary relations approaches is proposed in [61]. In [38], an extension of RDF and OWL that support the N-ary relations approach is proposed to represent temporal data. Querying support is also proposed and implemented. In [51], the N-ary relations approach is combined with SWRL rules to represent quantitatively temporal data. CNTRO [41] adopts a similar approach, combined with the concepts of time defined in OWL-Time.

The 4D-Fluents approach represents temporal data and its evolution in OWL. Concepts varying in time are represented as 4-dimensional objects with the 4th dimension being the temporal data. 4D-fluents minimizes the problem of data redundancy as the changes occur only on the temporal parts and keeping therefore the static part unchanged. It also maintains full OWL expressiveness and reasoning support [47]. According to [48], “It has a simple

[‡] https://www.w3.org/TR/owl-time/

[§] http://www.w3.org/TR/swbp-n-aryRelations

structure allowing to easily transform a static ontology into a dynamic one". The 4D-fluents approach has inspired several implementations e.g., [47], [48], [55], [64], [65], [66], [67] and [68]. SOWL [47] is a spatio-temporal ontology that uses the 4D-fluents approach. In SOWL, the 4D-fluents approach, is extended with qualitative temporal relations holding between time intervals whose beginning and ending bounds are not specified. The qualitative relations are supported via SWRL rules. In [67], the authors present a specialized temporal reasoner in conjunction with the SOWL ontology. In [47], a SPARQL-like query language supporting the SOWL ontology is introduced. In [56], the 4D-fluents approach is extended to represent precise time points. In [48] and [68], reasoning over temporal relations is achieved by means of SWRL rules embedded into an ontology based on the 4D-fluents approach. Temporal relation may be one of the Allen's relations. The MUSING system [39] uses both a 4D-fluents approach and an alternative approach based on extending RDF to represent temporal data. However, MUSING does not provide any qualitative reasoning support. PROTON [37] is an ontology that is based on the 4D-fluents approach. TOQL [69] is a SQL-like temporal query language for temporal ontologies using 4D-fluents for the representation of temporal data. It supports the Allen's relations, and the relations that allow comparisons between time points or time intervals.

The Named Graphs approach represents the temporal context of a property by the inclusion of a triple representing the property in a named graph (i.e., a subgraph into the RDF graph of the ontology specified by a distinct name). The main RDF graph contains definitions of interval beginning and ending bounds for each named graph, thus a property is stored in a named graph with beginning and ending bounds corresponding to the time interval that the property holds. Named graphs are not part of the OWL specification (i.e., there are not OWL constructs translated into named graphs) and they are not supported by OWL reasoners [47]. In [35] a SPARQL-based temporal query language, combined with an indexing mechanism is introduced applying only to the quantitatively defined time intervals.

Table 3 compares some approaches for representing temporal data in ontology. These approaches are mainly compared with respect to the compliance with the existing Semantic Web standards, the supported language (OWL, RDF(S)), the compatibility with querying and reasoning supports and the supported temporal data (precise/imprecise and quantitative/qualitative).

Table 3 - Comparison of some approaches for representing temporal data in ontology.

	W3C Compliance	Level of Operation	Interaction with a query language	Reasoning Support	Supported Temporal Data				Lowlights and Highlights
					Precise Data	Imprecise Data	Quantitative Data	Qualitative Data	
Approaches extending the OWL or RDF(S) syntax to represent temporal data									
Temporal Description Logics	*	OWL RDF(S)		*	*		*	*	(-) Require extending OWL or RDF(S) which is a tedious task. (+) Do not suffer from data redundancy. (+) Retain decidability.
Concrete Domains	*	OWL RDF(S)		*	*		*	*	(-) Require extending OWL or RDF(S) which is a tedious task.
Temporal RDF	*	RDF(S)	StSPARQL		*		*		(-) Require extending RDF(S) which is a tedious task. (-) Does not have all the expressiveness of OWL.
Approaches implemented directly using OWL or RDF(S) to represent temporal data									
Reification	*	OWL RDF(S)	SPARQL	*	*		*		(-) Suffers from data redundancy. (-) Offers limited OWL reasoning and inference capabilities.
Versioning	*	OWL RDF(S)	T-SPARQL		*		*	*	(-) Suffers from data redundancy. (-) Searching requires exhaustive searches in all created versions.
N-ary Relations	*	OWL RDF(S)	SPARQL	*	*		*	*	(-) Suffers from data redundancy.
4D-Fluents	*	OWL RDF(S)	TOQL SOWL QL	*	*		*	*	(-) Suffers from data redundancy. (+) 4D paradigm has solid background in Philosophy.
Named Graphs		RDF(S)	SPARQL τ-SPARQL		*		*		(-) Not W3C compliant solution. (-) Do not offer reasoning support.

All the mentioned approaches handle only precise temporal data and they do not allow the representation of imprecise ones. Most of them allow only representing time intervals and associated qualitative relations. In other words, they are not intended to handle time points or qualitative relations between a time interval and a time point or even two time points. Based on the present state of the art, we choose the 4D-fluents approach to be extended to represent precise and imprecise temporal data in ontology. Indeed, a basic design decision in our work is to choose an approach which relies on existing OWL constructs. Thus, we exclude the temporal description logic, concrete domain and temporal RDF approaches. We also exclude the named graphs

approach as it is not W3C compliant and does not support OWL. Compared to the reification, n-ary relations and versioning approaches, the 4D-fluents approach minimizes the problem of data redundancy as the changes occur only on the temporal parts and keeping therefore the static part unchanged.

3.2. Extending the Allen's Interval Algebra

In the literature, a number of approaches extend the Allen's interval algebra. Most of them are proposed in the context of the Artificial Intelligence and Databases fields.

A number of approaches extend the Allen's interval algebra to propose temporal relations between precise time intervals. Dubois and Prade [70] discuss the approximate reasoning on dates and time intervals. They represent a precise time interval as a pair of possibility distributions that define the possible values of the bounds of the interval. This approach proposes some imprecise relations such as "long before". Based on the possibility theory, Ryabov and Trudel [71] propose an approach to model uncertain interval relations by assigning a preference degree to every basic Allen's relation. This approach is proposed in the context of a probabilistic temporal interval network where the nodes are temporal intervals and the edges are uncertain interval relations. Badaloni and Giacomin [72] propose a fuzzy extension of the Allen's interval algebra, called IA^{fuz}. A degree of preference ($\in [0, 1]$) is associated to each temporal interval relation, e.g., the possibility that the relation "Meets" holds between two time intervals is 0.9. Imprecise temporal relations are not studied. Guesgen et al. [73] propose fuzzy temporal relations viewed as fuzzy sets of ordinary Allen's relations taking into account a neighborhood structure, a notion introduced in [74]. For instance, the temporal relation "Fuzz-Meets" covers the ordinary Allen's relation "Meets" as well as situations as "Slightly Before" and "Slight Overlap".

A number of approaches extend the Allen's interval algebra to propose temporal relations between imprecise time intervals. Nagypál and Motik [75] represent these intervals as a fuzzy set. They introduce a set of auxiliary operators on time intervals and define fuzzy counterparts of these operators. However, many of the properties of the original Allen's interval algebra are lost. For instance, the relation "Equals" is not reflexive. Thus, the compositions of the resulting relations cannot be studied by the authors. Imprecise temporal relations are not proposed. Ohlbach [63] extends the Allen's interval algebra based on fuzzy sets. This approach proposes some imprecise temporal relations as "more or less finishes". However, it cannot take into account imprecise temporal relations such as "Long before". It does not preserve many of the properties of the original Allen's interval algebra. Therefore, it is not suitable for temporal reasoning. Schockaert and Cock [42] propose a generalization of the Allen's interval algebra. This approach allows handling Allen's temporal relations, as well as some imprecise relations. However, a lot of imprecise temporal relations are not studied by the authors. Sadeghi and Goertzel [54] propose an approach allowing uncertain temporal inference. The Allen's interval algebra is extended to imprecise time intervals by representing them as trapeziums with distinct beginning, middle and end. An uncertain version of the transitivity table is developed. Gammoudi et al. [77] generalize the Allen's relations to make them applicable to imprecise time intervals in conjunctive and disjunctive ways. The compositions of the resulting relations are not studied by the authors. Imprecise temporal interval relations are not proposed by this approach.

4. Our Ontology-Based Approach for Representing Precise and Imprecise Temporal Data

Based on the state of the art, we choose the 4D-fluents approach to be extended to represent precise and imprecise temporal data.

4.1. The Classic 4D-Fluents Approach

The classic 4D-fluents approach introduces two crisp classes, "TimeSlice" and "TimeInterval", and four crisp properties, "TsTimeSliceOf", "TsTimeIntervalOf", "HasBeginning" and "HasEnd". "TimeSlice" is the domain class for entities representing temporal parts (i.e., "time slices"). The property "TsTimeSliceOf" connects an instance of "TimeSlice" with an entity. The property "TsTimeIntervalOf" connects an instance of "TimeSlice" with an instance of "TimeInterval". An instance of "TimeInterval" is related with two time points that specify the beginning and ending bounds of a precise time interval using, respectively, the "HasBeginning" and "HasEnd" properties. Properties having a time dimension are called "fluent properties". Fig. 2 illustrates the use of the 4D-fluents approach to represent this example: "John was enrolled in a doctorate from *1975* to *1978*".

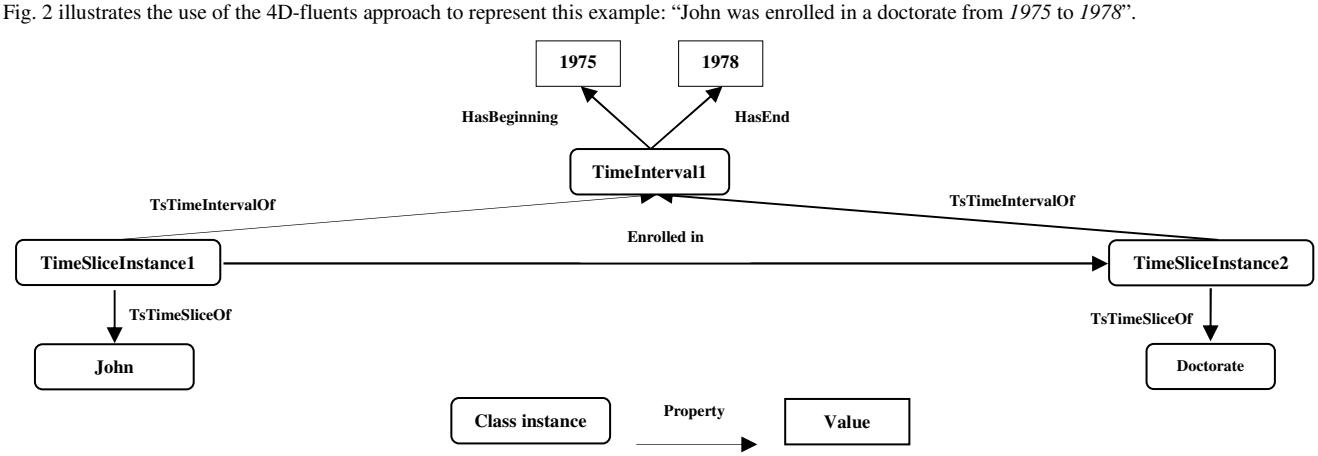


Fig. 2 - An instantiation of the classic 4D-fluents approach.

As already mentioned, in [47], the 4D-fluents approach is enhanced with precise qualitative temporal relations between precise time intervals. This is implemented by introducing a crisp object property between two instances of the "TimeInterval" class. This can be one of the Allen's relations. In [56], the approach is also extended to represent precise time points. The authors introduce a crisp class called "Instant" which has the "DateTime" datatype property representing the temporal data.

4.2. Extending the 4D-Fluents Approach

We extend the classic 4D-fluent approach with new crisp and fuzzy ontological components to represent not supported quantitative temporal data and qualitative temporal relations. The resulting ontology is fuzzy.

4.2.1. Representing Quantitative Temporal Data

The 4D-fluents approach is extended to represent imprecise time intervals and precise/imprecise time points.

To extend the 4D-fluents approach to represent an imprecise time interval, we introduce two fuzzy datatype properties "FuzzyHasBeginning" and "FuzzyHasEnd" associated to the class "TimeInterval". They represent, respectively, the imprecise beginning and ending bounds of the interval. The domain expert associates their corresponding membership functions. For instance, if we have the information "John started his PhD study in *1975* and was graduated in *late 1970s*", the beginning bound is precise. The ending bound is imprecise, in the sense that it could mean, with an increasingly possibility, 1976, 1977, 1978, 1979 or 1980. It is represented using the "FuzzyHasEnd" property which has the triangular membership function (A = 1976 and B = C = 1980).

To extend the 4D-fluents approach to represent precise and imprecise time points, we introduce a fuzzy class named "TimePoint". It has a crisp datatype property, named "CrispHasTimePoint", and a fuzzy datatype property, named "FuzzyHasTimePoint". "CrispHasTimePoint" and "FuzzyHasTimePoint" represent, respectively, the precise and imprecise temporal dimension. We also introduce a crisp object property, named "TsTimePointOf". It connects an instance of the class "TimeSlice" (domain) with an instance of the class "TimePoint" (range). The domain expert specifies the membership function associated to "FuzzyHasTimePoint". For instance, if we have the information "John started teaching at the Sorbonne University *by 1980*", "by 1980" is represented as an imprecise time point. It is represented using the "FuzzyHasTimePoint" property which has the triangular membership function (A = 1978, B = 1980 and C = 1982).

4.2.2. Representing Qualitative Temporal Data

The 4D-fluents approach is also enhanced to represent precise and imprecise qualitative temporal relations that may hold between time intervals and points (Interval-Interval, Interval-Point, Point-Interval and Point-Point relations). Fig. 3 represents our extended 4D-fluents approach.

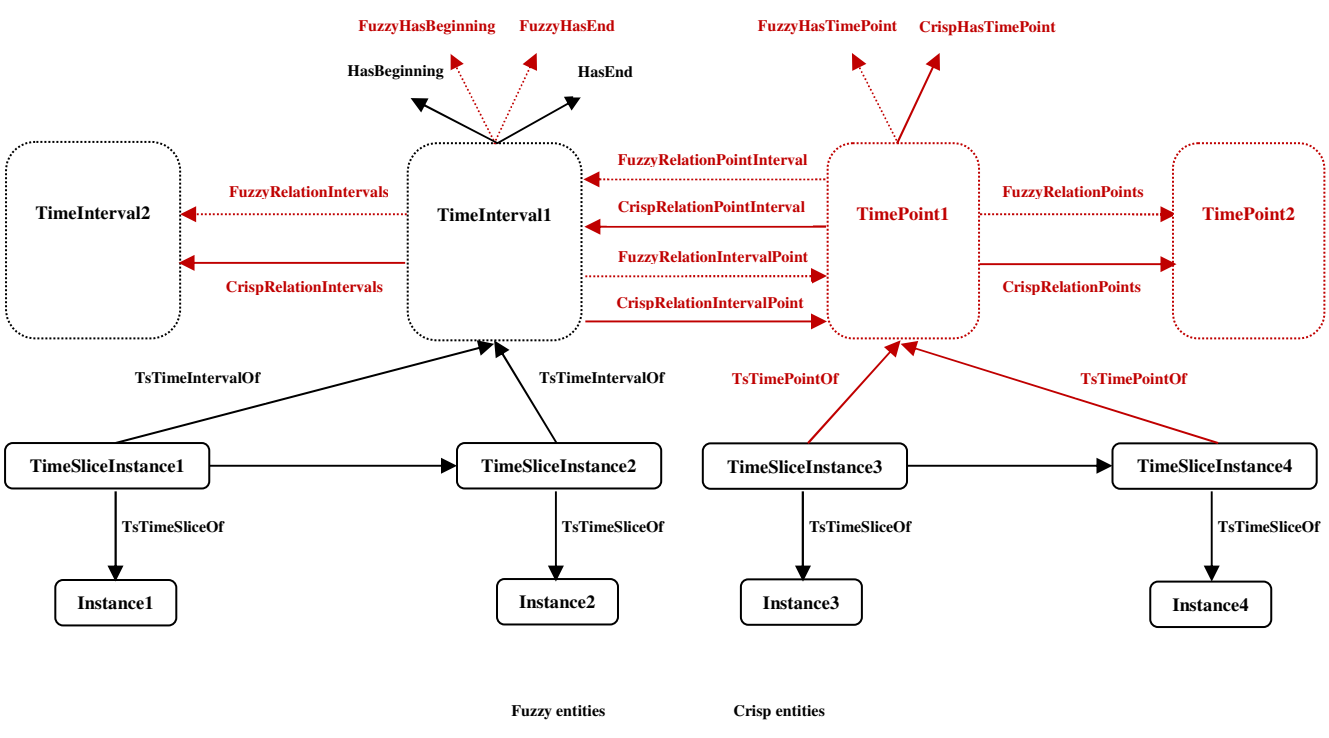


Fig. 3 - Our extended 4D-fluents approach.

We introduce four crisp object properties, named “CrispRelationIntervals”, “CrispRelationIntervalPoint”, “CrispRelationPointInterval” and “CrispRelationPoints”, and four fuzzy object properties, named “FuzzyRelationIntervals”, “FuzzyRelationIntervalPoint”, “FuzzyRelationPointInterval” and “FuzzyRelationPoints”. Fuzzy properties represent the imprecise temporal relations and crisp properties represent the precise ones. Fuzzy properties have the triangular membership function (A = 0 and B = C = 1). “FuzzyRelationIntervals” and “CrispRelationIntervals” connect two instances of the “TimeInterval” class to represent, respectively, precise and imprecise temporal relations between time intervals. “FuzzyRelationIntervalPoint” and “CrispRelationIntervalPoint” connect an instance of the “TimeInterval” class (domain) and an instance of the “TimePoint” class (range) to represent, respectively, precise and imprecise temporal relations between a time interval and a time point (Interval-Point relations). “FuzzyRelationPointInterval” and “CrispRelationPointInterval” connect an instance of the “TimePoint” class (domain) and an instance of the “TimeInterval” class (range) to represent, respectively, precise and imprecise temporal relations between a time point and a time interval (Interval-Point relations). “FuzzyRelationPoints” and “CrispRelationPoints” connect two instances of the “TimePoint” class to represent temporal relations between time points.

5. Our Approach for Reasoning about Precise and Imprecise Temporal Data

The aim of our approach is: (*i*) to reason about quantitative temporal data to infer qualitative temporal relations and (*ii*) to reason about the qualitative temporal relations to infer new ones. Precise and imprecise temporal data are taken into account. Our approach is mainly based on extending the Allen’s interval algebra. The temporal relations that we will define are based on orderings between the time points contained in the intervals. They may be expressed using time point comparators like those proposed in Vilain and Kautz’s Algebra. We chose this algebra because it allows a redefinition of a tractable fragment of the Allen’s interval algebra.

5.1. Extending the Vilain and Kautz’s Point Algebra

As already mentioned, the Vilain and Kautz’s point algebra proposes three precise comparators between precise time points. In our approach, we need to express in a gradual (imprecise) way the relations that may hold between two precise time points P and L e.g., P and L occur “*approximately at the same time*”. Based on fuzzy set theory, we extend this algebra. We generalize the two comparators “Precedes” and “Follows” to propose a set of imprecise personalized time point comparators. They are imprecise as we propose comparators such as “just precedes” and they are personalized as their number and meanings are set by the domain expert.

Let α and β be two parameters ∈ [0, +∞ [and let N be the number of the graduality. We propose the imprecise personalized comparators {“$Follows_{(1)}^{(\alpha, \beta)}$” … “$Follows_{(N)}^{(\alpha, \beta)}$”} to extend the precise comparator “Follows”. “$Follows_{(1)}^{(\alpha, \beta)}$(P, L)” means that P follows L w.r.t. (α, β) and P - L < α + β. Gradually the time gap between P and L increases until “$Follows_{(N)}^{(\alpha, \beta)}$(P, L)” which means that P follows L w.r.t. (α, β) and P - L > (N - 1) α + (N - 2) β. For instance, if we set N = 3, “$Follows_{(1)}^{(\alpha, \beta)}$” means “just follows”; “$Follows_{(2)}^{(\alpha, \beta)}$” means “middle follows” and “$Follows_{(3)}^{(\alpha, \beta)}$” means “long follows”.

We propose the imprecise personalized comparators {“$Precedes_{(1)}^{(\alpha, \beta)}$” … “$Precedes_{(N)}^{(\alpha, \beta)}$”} to extend the precise comparator “Precedes”. “$Precedes_{(1)}^{(\alpha, \beta)}$(P, L)” means that P precedes L w.r.t. (α, β) and P - L > - α - β. Gradually the time gap between P and L increases until “$Precedes_{(N)}^{(\alpha, \beta)}$(P, L)” which means that P precedes L w.r.t. (α, β) and P - L < - (N - 1) α - (N - 2) β. For instance, if we set N = 5, “$Precedes_{(1)}^{(\alpha, \beta)}$” means “approximately at the same time”; “$Precedes_{(2)}^{(\alpha, \beta)}$” means “just precedes”; “$Precedes_{(3)}^{(\alpha, \beta)}$” means “middle precedes”; “$Precedes_{(4)}^{(\alpha, \beta)}$” means “long precedes” and “$Precedes_{(5)}^{(\alpha, \beta)}$” means “too long precedes”. Table 4 summarizes the proposed imprecise personalized comparators.

Table 4 - Imprecise personalized time point comparators (P: ● and L:○).

Comparator(P, L)	Illustration	Inverse(L, P)
$Follows_{(1)}^{(\alpha, \beta)}$(P, L)	○● ⟶	$Precedes_{(1)}^{(\alpha, \beta)}$(L, P)
$Follows_{(2)}^{(\alpha, \beta)}$(P, L)	○ ● ⟶	$Precedes_{(2)}^{(\alpha, \beta)}$(L, P)
...		
$Follows_{(k)}^{(\alpha, \beta)}$(P, L)	○ ● ⟶	$Precedes_{(k)}^{(\alpha, \beta)}$(L, P)
...		
$Follows_{(N)}^{(\alpha, \beta)}$(P, L)	○ ● ⟶	$Precedes_{(N)}^{(\alpha, \beta)}$(L, P)

{“$Follows_{(1)}^{(\alpha, \beta)}$” … “$Follows_{(N)}^{(\alpha, \beta)}$”} and {“$Precedes_{(1)}^{(\alpha, \beta)}$”… “$Precedes_{(N)}^{(\alpha, \beta)}$”} are defined as fuzzy sets. “$Follows_{(1)}^{(\alpha, \beta)}$” has the R-Function membership function which has the parameters A = α and B = (α + β). All comparators {“$Follows_{(2)}^{(\alpha, \beta)}$”… “$Follows_{(N-1)}^{(\alpha, \beta)}$”} have the trapezoidal membership function which has the parameters A = ((k - 1) + (k - 2) α), B = ((k - 1) (α + β)), C = (k α + (k - 1) β) and D = (k (α + β)). “$Follows_{(N)}^{(\alpha, \beta)}$” has the L-Function membership function which has the parameters A = ((N - 1) α + (N - 2) β) and B = ((N - 1) (α + β)). “$Precedes_{(1)}^{(\alpha, \beta)}$” has the R-Function membership function which has the parameters A = (- α - β) et B = (-α). All comparators {“$Precedes_{(2)}^{(\alpha, \beta)}$”… “$Precedes_{(N-1)}^{(\alpha, \beta)}$”} have the trapezoidal membership function which has the parameters A = (- k (α + β)), B = (- k α - (k -1) β), C = (- (k - 1) (α + β)) and D = (- (k - 1) α - (k - 2) β). “$Precedes_{(N)}^{(\alpha, \beta)}$” has the L-Function membership function which has the parameters A = (- (N -1) (α + β)) and B = (- (N - 1) α - (N - 2) β). Fig. 4 shows the membership functions associated to the proposed time point comparators.

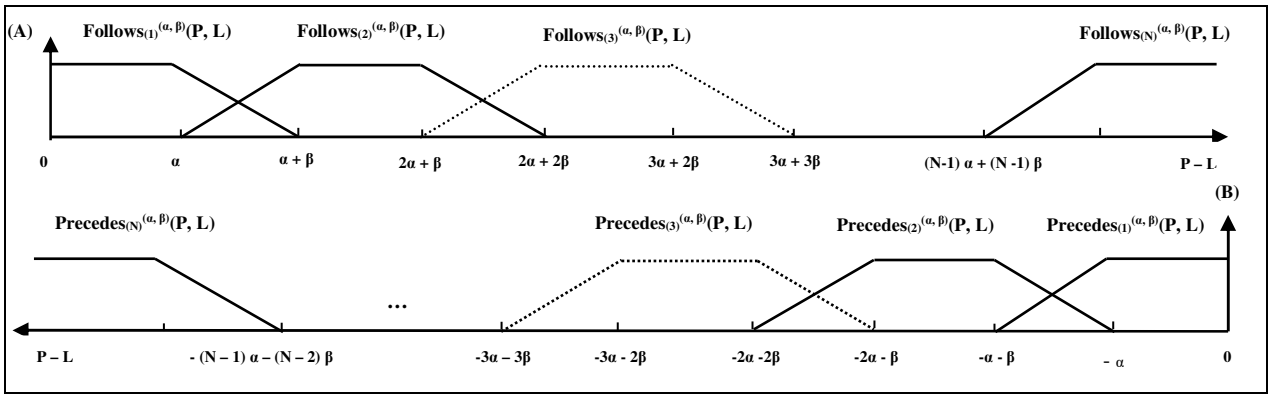


Fig. 4 - Membership functions associated to the imprecise personalized time point comparators.

We have the following formulas:

$$Follows_{(k)}^{(\alpha,\beta)}(P, L) = Precedes_{(k)}^{(\alpha,\beta)}(L, P) \quad (1)$$

$$Follows_{(k)}^{(\alpha,\beta)}(P, L) = 1 - Precedes_{(k)}^{(\alpha,\beta)}(P, L) \quad (2)$$

$$Follows(P, L) = \sum Follows_{(k)}^{(\alpha,\beta)}(P, L)\ (k \in [1, N]) \quad (3)$$

$$Precedes_{(k)}^{(\alpha,\beta)}(P, L) = Follows_{(k)}^{(\alpha,\beta)}(L, P) \quad (4)$$

$$Precedes_{(k)}^{(\alpha,\beta)}(P, L) = 1 - Follows_{(k)}^{(\alpha,\beta)}(P, L) \quad (5)$$

$$Precedes(P, L) = \sum Precedes_{(k)}^{(\alpha,\beta)}(P, L)\ (k \in [1, N]) \quad (6)$$

Example 1: Assume that P= 1974 and L = 2017. We use the following parameters: N = 3, α = 10 years and β = 20 years. "$Precedes_{(1)}^{(10,20)}$" means "just precedes"; "$Precedes_{(2)}^{(10,20)}$" means "middle precedes" and "$Precedes_{(3)}^{(10,20)}$" means "long precedes". Fig. 5 shows their membership functions.

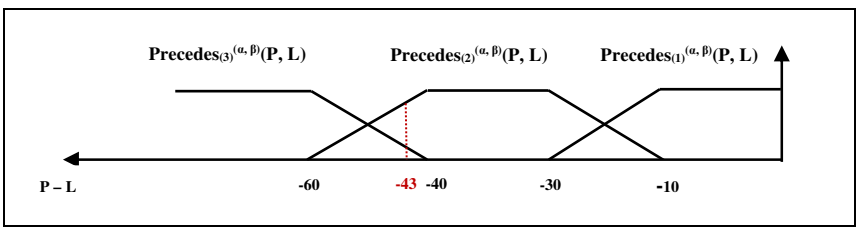


Fig. 5 - Membership functions associated to {$Precedes_{(1)}^{(10,20)}$, $Precedes_{(2)}^{(10,20)}$ and $Precedes_{(3)}^{(10,20)}$}.

We obtain:

Precedes(P, L) = 1

$Precedes_{(1)}^{(10,20)}(P, L) = 0$

$Precedes_{(2)}^{(10,20)}(P, L) = (-43 + 60) / (-40 + 60)) = 0{,}85$

$Precedes_{(3)}^{(10,20)}(P, L) = (-40 + 43) / (-40 + 60)) = 0{,}15$

Expressing that 1974 occurred "middle precedes" 2017 to a high degree and 1974 occurred "long precedes" 2017 to a low degree. On the other hand, using N = 5, α = 10 years and β = 5 years ("$Precedes_{(1)}^{(10,5)}$" means "approximately at the same time", "$Precedes_{(2)}^{(10,5)}$" means "just precedes", "$Precedes_{(3)}^{(10,5)}$" means "middle precedes", "$Precedes_{(4)}^{(10,5)}$" means "long precedes" and "$Precedes_{(5)}^{(10,5)}$" means "too long precedes"), we obtain:

Precedes(P, L) = 1

$Precedes_{(1)}^{(10,5)}(P, L) = 0$

$Precedes_{(2)}^{(10,5)}(P, L) = 0$

$Precedes_{(3)}^{(10,5)}(P, L) = 0{,}4$

$Precedes_{(4)}^{(10,5)}(P, L) = 0{,}6$

$Precedes_{(5)}^{(10,5)}(P, L) = 0$

5.2. Extending the Allen's Interval Algebra

First, we propose a representation of qualitative temporal relations between quantitative temporal data. Then, we introduce transitivity tables to derive new temporal knowledge from the resulted relations.

5.2.1. Qualitative Temporal Relations

We present in the present section our extension of the Allen's interval algebra to represent qualitative temporal relations between time intervals. We demonstrate that the resulting relations preserve many of the properties of the original algebra. Then, we adapt these relations to relate a time interval and a time point or two time points.

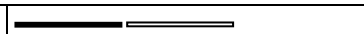
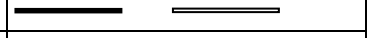
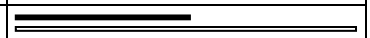
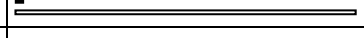
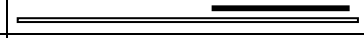
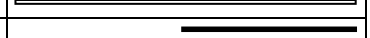
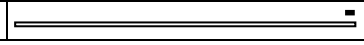
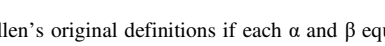
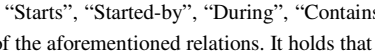
5.2.1.1. Qualitative Temporal Relations between Time Intervals

The extension that we propose enables representing qualitative temporal relations between precise and imprecise time intervals. It is not only suitable to express precise relations, but also imprecise personalized ones. Precise relations are based on the classic Vilain and Kautz's point algebra and imprecise personalized relations are based on our extension of the Vilain and Kautz's point algebra.

Qualitative Temporal Relations between Precise Time Intervals

Let $I = [I^-, I^+]$ and $J = [J^-, J^+]$ be precise time intervals. When considering precise temporal relations, our approach reduces to Allen's work. We also define imprecise personalized temporal relations between precise time intervals. The objective is to provide a way to model gradual, linguistic-like description of temporal interval relations. Compared to related work, our work is not limited to a given number of imprecise relations. It is possible to determinate the level of precision that it should be in a given context. The Allen's relations {"Before", "After", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Ends" and "Ended-by"} are generalized in N imprecise relations. Their number and meanings are set by the domain expert. For instance, the classic Allen's relation "Overlaps" may be generalized in N imprecise relations, where "$Overlaps_{(1)}^{(\alpha, \beta)}(I, J)$" means that I overlaps J and "$Precedes_{(1)}^{(\alpha, \beta)}(J^-, I^+) > 0$"; and gradually the time gap between the two bounds J^- and I^+ increase until "$Overlaps_{(N)}^{(\alpha, \beta)}(I, J)$" means that I overlaps J and "$Precedes_{(N)}^{(\alpha, \beta)}(J^-, I^+) > 0$". In a similar way, we define the other imprecise personalized relations between precise time intervals, as shown in Table 5.

Table 5 - Imprecise personalized temporal relations between the precise time intervals I (▬) and J (═).

Relation(I, J)	Definition	Illustration		Inverse(J, I)
$Before_{(k)}^{(\alpha, \beta)}(I, J)$	$Precedes_{(k)}^{(\alpha, \beta)}(I^+, J^-)$	$Before_{(1)}^{(\alpha, \beta)}$		$After_{(k)}^{(\alpha, \beta)}(J, I)$
		$Before_{(i)}^{(\alpha, \beta)}$		
		$Before_{(N)}^{(\alpha, \beta)}$		
$Overlaps_{(k)}^{(\alpha, \beta)}(I, J)$	$Precedes(I^-, J^-) \wedge Precedes_{(k)}^{(\alpha, \beta)}(J^-, I^+) \wedge Precedes(I^+, J^+)$	$Overlaps_{(1)}^{(\alpha, \beta)}$		$Overlapped\text{-}by_{(k)}^{(\alpha, \beta)}(J, I)$
		$Overlaps_{(i)}^{(\alpha, \beta)}$		
		$Overlaps_{(N)}^{(\alpha, \beta)}$		
$Starts_{(k)}^{(\alpha, \beta)}(I, J)$	$Same(I^-, J^-) \wedge Precedes_{(k)}^{(\alpha, \beta)}(I^+, J^+)$	$Starts_{(1)}^{(\alpha, \beta)}$		$Started\text{-}by_{(k)}^{(\alpha, \beta)}(J, I)$
		$Starts_{(i)}^{(\alpha, \beta)}$		
		$Starts_{(N)}^{(\alpha, \beta)}$		
$During_{(k)}^{(\alpha, \beta)}(I, J)$	$Precedes_{(k)}^{(\alpha, \beta)}(J^-, I^-) \wedge Precedes(I^+, J^+)$	$During_{(1)}^{(\alpha, \beta)}$		$Contains_{(k)}^{(\alpha, \beta)}(J, I)$
		$During_{(i)}^{(\alpha, \beta)}$		
		$During_{(N)}^{(\alpha, \beta)}$		
$Ends_{(k)}^{(\alpha, \beta)}(I, J)$	$Precedes_{(k)}^{(\alpha, \beta)}(J^-, I^-) \wedge Same(I^+, J^+)$	$Ends_{(1)}^{(\alpha, \beta)}$		$Ended\text{-}by_{(k)}^{(\alpha, \beta)}(J, I)$
		$Ends_{(i)}^{(\alpha, \beta)}$		
		$Ends_{(N)}^{(\alpha, \beta)}$		

Note that the definitions of the proposed temporal relations coincide with Allen's original definitions if each α and β equals zero and N equals one. The precise temporal relations {"Before", "After", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Ends" and "Ended-by"} may be calculated from the associated imprecise personalized ones. Let R be one of the aforementioned relations. It holds that

$$R(I, J) = \sum R_{(k)}^{(\alpha, \beta)(I}, J) \ (k \in [1, N]) \tag{7}$$

Qualitative Temporal Relations between Imprecise Time Intervals

Let I = [I^-, I^+] be an imprecise time interval. The imprecise beginning bound I^- could be one of the disjunctive ascending set {$I^{-(1)}$… $I^{-(B)}$} and the imprecise ending bound I^+ could be one of the disjunctive ascending set {$I^{+(1)}$… $I^{+(E)}$}. We take the example already mentioned: "John started his PhD study in *1975* and was graduated in *late 1970s*". The ending bound of the associated time interval is imprecise, in the sense that it could mean 1976, 1977, 1978, 1979 or 1980. It is represented as a disjunctive ascending set {1976…1980}.

Let J = [J^-, J^+] be imprecise time interval. We redefine the 13 Allen's relations to propose precise temporal relations between imprecise time intervals. For instance, the relation "Before(I, J)" is redefined as:

$$\forall I^{+(i)} \in I^+,\ \forall J^{-(j)} \in J^- \,/\, Precedes(I^{+(i)}, J^{-(j)}) \tag{8}$$

This means that the most recent time point of I^+($I^{+(E)}$) ought to precede the oldest time point of J^- ($J^{-(1)}$):

$$Precedes(I^{+(E)}, J^{-(1)}) \tag{9}$$

In a similar way, we redefine the other qualitative interval relations, as shown in Table 6.

Table 6 – Precise and imprecise personalized temporal relations between the imprecise time intervals I and J.

Relation(I, J)	Definition	Inverse(J, I)
Precise qualitative temporal relations		
Before(I, J)	Precedes($I^{+(E)}$, $J^{-(1)}$)	After(J, I)
Meets(I, J)	Same($I^{+(1)}$, $J^{-(1)}$) ∧ Same($I^{+(E)}$, $J^{-(B)}$)	Met-by(J, I)
Overlaps(I, J)	Precedes($I^{-(B)}$, $J^{-(1)}$) ∧ Precedes($J^{-(B)}$, $I^{+(1)}$) ∧ Precedes($I^{+(E)}$, $J^{+(1)}$)	Overlapped-by(J, I)
Starts(I, J)	Same($I^{-(1)}$, $J^{-(1)}$) ∧ Same($I^{-(B)}$, $J^{-(B)}$) ∧ Precedes($I^{+(E)}$, $J^{+(1)}$)	Started-by(J, I)
During(I, J)	Precedes($J^{-(B)}$, $I^{-(1)}$) ∧ Precedes($I^{+(E)}$, $J^{+(1)}$)	Contains(J, I)
Ends(I, J)	Precedes($J^{-(B)}$, $I^{-(1)}$) ∧ Same($I^{+(1)}$, $J^{+(1)}$) ∧ Same($I^{+(E)}$, $J^{+(E)}$)	Ended-by(J, I)
Equals(I, J)	Same($I^{-(1)}$, $J^{-(1)}$) ∧ Same ($I^{-(B)}$, $J^{-(B)}$) ∧ Same($I^{+(1)}$, $J^{+(1)}$) ∧ Same ($I^{+(E)}$, $J^{+(E)}$)	Equals(J, I)
Imprecise personalized qualitative temporal relations		
$Before_{(k)}^{(\alpha,\beta)}$(I, J)	$Precedes_{(k)}^{(\alpha,\beta)}$($I^{+(E)}$, $J^{-(1)}$)	$After_{(k)}^{(\alpha,\beta)}$(J, I)
$Overlaps_{(k)}^{(\alpha,\beta)}$(I, J)	Precedes ($I^{-(B)}$, $J^{-(1)}$) ∧ $Precedes_{(k)}^{(\alpha,\beta)}$($J^{-(B)}$, $I^{+(1)}$) ∧ $Precedes_{(k)}^{(\alpha,\beta)}$($I^{+(E)}$, $J^{+(1)}$)	$Overlapped\text{-}by_{(k)}^{(\alpha,\beta)}$(J, I)
$Starts_{(k)}^{(\alpha,\beta)}$(I, J)	Same($I^{-(1)}$, $J^{-(1)}$) ∧ Same($I^{-(B)}$, $J^{-(B)}$) ∧ $Precedes_{(k)}^{(\alpha,\beta)}$($I^{+(E)}$, $J^{+(1)}$)	$Started\text{-}by_{(k)}^{(\alpha,\beta)}$(J, I)
$During_{(k)}^{(\alpha,\beta)}$(I, J)	$Precedes_{(k)}^{(\alpha,\beta)}$($J^{-(B)}$, $I^{-(1)}$) ∧ Precedes($I^{+(E)}$, $J^{+(1)}$)	$Contains_{(k)}^{(\alpha,\beta)}$(J, I)
$Ends_{(k)}^{(\alpha,\beta)}$(I, J)	$Precedes_{(k)}^{(\alpha,\beta)}$($J^{-(B)}$, $I^{-(1)}$) ∧ Same($I^{+(1)}$, $J^{+(1)}$) ∧ Same($I^{+(E)}$, $J^{+(E)}$)	$Ended\text{-}by_{(k)}^{(\alpha,\beta)}$(J, I)

The precise temporal relations {"Before", "After", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Ends" and "Ended-by"} may be calculated from the associated imprecise personalized ones. Let R be one of the aforementioned relations. It holds that

$$R(I, J) = \sum R_{(k)}^{(\alpha,\beta)(I}, J)\ (k \in [1, N]) \tag{10}$$

Example 2: Let I_1= [I_1^-, I_1^+], I_2 = [I_2^-, I_2^+] and I_3 = [I_3^-, I_3^+] be imprecise time intervals; where I_1^- is represented using the disjunctive ascending set {1960 ... 1963}; I_1^+ is represented using the disjunctive ascending set {1971 ... 1974}; I_2^- is represented using the disjunctive ascending set {1940...1943}; I_2^+ is represented using the disjunctive ascending set {1971 ... 1974}; I_3^- is represented using the disjunctive ascending set {2017 ... 2020} and I_3^+ is represented using the disjunctive ascending set {2030 ... 2033}. We use N = 3, α = 10 years and β = 20 years. Based on Table 6, we obtain:

$Ends(I_1, I_2) = 1$

$End_{(1)}^{(10,20)}(I_1, I_2) = 0{,}65$

$End_{(2)}^{(10,20)}(I_1, I_2) = 0{,}35$

$End_{(3)}^{(10,20)}(I_1, I_2) = 0$

$Before(I_2, I_3) = 1$

$Before_{(1)}^{(10,20)}(I_2, I_3) = 0$

$Before_{(2)}^{(10,20)}(I_2, I_3) = 0{,}85$

$Before_{(3)}^{(\alpha,\beta)}(I_2, I_3) = 0{,}15$

Properties

The proposed interval relations preserve many properties of the Allen's algebra. Let I= [I^-, I^+], J = [J^-, J^+] and K = [K^-, K^+] be three time intervals. All demonstrations are available online**.

Reflexivity/Irreflexivity: The temporal relations {"Before", "After", "Meets", "Met-by", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Ends", "Ended-by", "$Before_{(k)}^{(\alpha, \beta)}$", "$After_{(k)}^{(\alpha, \beta)}$", "$Overlaps_{(k)}^{(\alpha, \beta)}$", "$Overlapped\text{-}by_{(k)}^{(\alpha, \beta)}$", "$Starts_{(k)}^{(\alpha, \beta)}$", "$Started\text{-}by_{(k)}^{(\alpha, \beta)}$", "$During_{(k)}^{(\alpha, \beta)}$", "$Contains_{(k)}^{(\alpha, \beta)}$", "$Ends_{(k)}^{(\alpha, \beta)}$" and "$Ended\text{-}by_{(k)}^{(\alpha, \beta)}$"} are irreflexive, i.e., let R be one of the aforementioned relations. It holds that

$$R(I, I) = 0 \quad (For\ instance,\ Before_{(k)}^{(\alpha, \beta)}(I, I) = Precedes_{(k)}^{(\alpha, \beta)}(I^{+(E)}, I^{-(1)}) = 0\ as\ I^{+(E)} - I^{-(1)} > 0) \tag{11}$$

Furthermore, the temporal relation "Equals" is reflexive. It holds that

$$Equals(I, I) = Same(I^{-(1)}, I^{-(1)}) \wedge Same(I^{-(B)}, I^{-(B)}) \wedge Same(I^{+(1)}, I^{+(1)}) \wedge Same(I^{+(E)}, I^{+(E)}) = 1 \tag{12}$$

Symmetry/Asymmetry: The temporal relations {"Before", "After", "Meets", "Met-by", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Ends", "Ended-by", "$Before_{(k)}^{(\alpha, \beta)}$", "$After_{(k)}^{(\alpha, \beta)}$", "$Overlaps_{(k)}^{(\alpha, \beta)}$", "$Overlapped\text{-}by_{(k)}^{(\alpha, \beta)}$", "$Starts_{(k)}^{(\alpha, \beta)}$", "$Started\text{-}by_{(k)}^{(\alpha, \beta)}$", "$During_{(k)}^{(\alpha, \beta)}$", "$Contains_{(k)}^{(\alpha, \beta)}$", "$Ends_{(k)}^{(\alpha, \beta)}$" and "$Ended\text{-}by_{(k)}^{(\alpha, \beta)}$"} are asymmetric, i.e., let R be one of the aforementioned relations. It holds that

$$R(I, J)\ and\ R(J, I) \Rightarrow I = J \tag{13}$$

Furthermore, the temporal relation "Equals" is symmetric. It holds that

$$Equals(I, J) = Equals(J, I) \tag{14}$$

Transitivity: The temporal relations {"Before", "After", "Overlaps", "Overlapped-by", "Starts", "Started-by", "During", "Contains", "Equals", "$Before_{(k)}^{(\alpha, \beta)}$", "$After_{(k)}^{(\alpha, \beta)}$", "$Overlaps_{(k)}^{(\alpha, \beta)}$", "$Overlapped\text{-}by_{(k)}^{(\alpha, \beta)}$", "$Starts_{(k)}^{(\alpha, \beta)}$", "$Started\text{-}by_{(k)}^{(\alpha, \beta)}$", "$During_{(k)}^{(\alpha, \beta)}$" and "$Contains_{(k)}^{(\alpha, \beta)}$"} are transitive, i.e., let R be one of the aforementioned relations. It holds that

$$R(I, J)\ and\ R(J, K) \Rightarrow R(I, K) \tag{15}$$

For instance, we can deduce from "$Before_{(k)}^{(\alpha, \beta)}(I, J)$" and "$Before_{(k)}^{(\alpha, \beta)}(J, K)$" that "$Before_{(k)}^{(\alpha, \beta)}(I, K)$" holds. Indeed by "$Before_{(k)}^{(\alpha, \beta)}(I, J)$", we have "$Precedes_{(k)}^{(\alpha, \beta)}(I^{+(E)}, J^{-(1)})$", and by "$Before_{(k)}^{(\alpha, \beta)}(J, K)$", we have "$Precedes_{(k)}^{(\alpha, \beta)}(J^{+(E)}, K^{-(1)})$". From "$Precedes_{(k)}^{(\alpha, \beta)}(I^{+(E)}, J^{-(1)})$" and "$Precedes_{(k)}^{(\alpha, \beta)}(J^{+(E)}, K^{-(1)})$", we conclude "$Precedes_{(k)}^{(\alpha, \beta)}(I^{+(E)}, K^{-(1)})$", or in other words, "$Before_{(k)}^{(\alpha, \beta)}(I, K)$".

5.2.1.2. Qualitative Temporal Relations between a Time Interval and a Time Point

We adapt the resulting qualitative temporal relations between time intervals to propose Interval-Point and Point-Interval relations.

Qualitative Temporal Relations between a Precise Time Interval and a Precise Time Point

We adapt the resulting temporal relations between precise time intervals to propose precise and imprecise qualitative temporal relations between a precise time interval and a precise time point, as shown in Table 7. Let I = [I^-, I^+] be a precise time interval and P be a precise time point.

Table 7 - Precise and imprecise personalized temporal relations between a precise time interval I and a precise time point P.

Relation(P, I)	Definition	Illustration	Inverse(I, P)
Precise qualitative temporal relations			
Before(P, I)	Precedes(P, I^-)		After(I, P)
After(P, I)	Precedes(I^+, P)		Before(I, P)
Starts(P, I)	Same(P, I^-)		Started-by(I, P)
During(P, I)	Precedes(I^-, P) ∧ Precedes(P, I^+)		Contains(I, P)

** https://cedric.cnam.fr/~hamdif/upload/DKE19/Proofs.pdf

Ends(P, I)	Same(P, I^+)		Ended-by(I, P)
Imprecise personalized qualitative temporal relations			
$Before_{(k)}^{(\alpha,\beta)}$(P, I)	$Precedes_{(k)}^{(\alpha,\beta)}$(P, I^-)		$After_{(k)}^{(\alpha,\beta)}$(I, P)
$Overlaps_{(k)}^{(\alpha,\beta)}$(P, I)	Precedes(P, I^-) ∧ $Precedes_{(k)}^{(\alpha,\beta)}$($I^-$, P) ∧ Precedes(P , I^+)		$Before_{(k)}^{(\alpha,\beta)}$(I, P)
$Starts_{(k)}^{(\alpha,\beta)}$(P, I)	Same(P , I^-) ∧ $Precedes_{(k)}^{(\alpha,\beta)}$(P , I^+)		$Started\text{-}by_{(k)}^{(\alpha,\beta)}$(I, P)
$After_{(k)}^{(\alpha,\beta)}$(P, I)	$Precedes_{(k)}^{(\alpha,\beta)}$($I^+$, P)		$Contains_{(k)}^{(\alpha,\beta)}$(I, P)
$During_{(k)}^{(\alpha,\beta)}$(P, I)	$Precedes_{(k)}^{(\alpha,\beta)}$($I^-$, P) ∧ Precedes(P , I^+)		$Ended\text{-}by_{(k)}^{(\alpha,\beta)}$(I, P)

The precise temporal Interval-Point relations {"After", "Before", "Started-by", "Contains" and "Ended-by"} may be calculated from the associated imprecise personalized ones. Let R be one of the aforementioned relations. It holds that

$$R(I, P) = \sum R_{(k)}^{(\alpha,\beta)}(I, P)\ (k \in [1, N]) \tag{16}$$

The precise temporal Point-Interval relations {"Before", "After", "Starts", "During" and "Ends"} may be calculated from the associated imprecise personalized ones. Let R be one of the aforementioned relations. It holds that

$$R(P, I) = \sum R_{(k)}^{(\alpha,\beta)}(P, I)\ (k \in [1, N]) \tag{17}$$

Qualitative Temporal Relations between an Imprecise Time Interval and an Imprecise Time Point

Let P be an imprecise time point. We represent P as a disjunctive ascending set $\{P^{(1)}\ldots P^{(P)}\}$. For instance, we take the following example: "John travelled to Paris in *late 1960s*". The imprecise temporal data "late 1960s" could mean 1966, 1967, 1968, 1969 or 1970. It is represented as a disjunctive ascending set {1966…1970}.

We adapt the resulting precise and imprecise personalized qualitative temporal relations between imprecise time intervals to propose precise and imprecise qualitative temporal relations between an imprecise time interval and an imprecise time point, as shown in Table 8. Let I = [I^-, I^+] be an imprecise time interval.

Table 8 - Precise and imprecise personalized temporal relations between an imprecise time interval I and an imprecise time point P.

Relation(P, I)	**Definition**	**Inverse(I, P)**
Precise qualitative temporal relations		
Before(P, I)	Precedes($P^{(P)}$, $I^{-(1)}$)	After(I, P)
After(P, I)	Precedes($I^{+(E)}$, $P^{(1)}$)	Before(I, P)
Starts(P, I)	Same($P^{(1)}$, $I^{-(1)}$) ∧ Same($P^{(P)}$, $I^{-(B)}$)	Started-by(I, P)
During(P, I)	Precedes($I^{-(E)}$, $P^{(1)}$) ∧ Precedes($P^{(P)}$, $I^{+(1)}$)	Contains(I, P)
Ends(P, I)	Same($P^{(1)}$, $I^{+(1)}$) ∧ Same($P^{(P)}$, $I^{+(E)}$)	Ended-by(I, P)
Imprecise personalized qualitative temporal relations		
$Before_{(k)}^{(\alpha,\beta)}$(P, I)	$Precedes_{(k)}^{(\alpha,\beta)}$($P^{(P)}$, $I^{-(1)}$)	$After_{(k)}^{(\alpha,\beta)}$(I, P)
$After_{(k)}^{(\alpha,\beta)}$(P, I)	$Precedes_{(k)}^{(\alpha,\beta)}$($I^{+(E)}$, $P^{(1)}$)	$Before_{(k)}^{(\alpha,\beta)}$(I, P)
$During_{(k)}^{(\alpha,\beta)}$(P, I)	$Precedes_{(k)}^{(\alpha,\beta)}$($I^{-(E)}$, $P^{(1)}$) ∧ Precedes($P^{(P)}$, $I^{+(1)}$)	$Contains_{(k)}^{(\alpha,\beta)}$(I, P)

The precise temporal Interval-Point relations {"Before", "After" and "Contains"} may be calculated from the associated imprecise personalized ones. Let R be one of the aforementioned relations. It holds that

$$R(I, P) = \sum R_{(k)}^{(\alpha,\beta)}(I, P)\ (k \in [1, N]) \tag{18}$$

The precise temporal Point-Interval relations {"Before", "After" and "During"} may be calculated from the associated imprecise personalized ones. Let R be one of the aforementioned relations. It holds that

$$R(P, I) = \sum R_{(k)}^{(\alpha,\beta)}(P, I)\ (k \in [1, N]) \tag{19}$$

Example 3: Let P_1 and P_2 be two imprecise time points and $I_1 = [I_1^-, I_1^+]$ be the imprecise time intervals mentioned in Example 2. P_1 is represented using the disjunctive ascending set {1960 … 1963}. P_2 is represented using the disjunctive ascending set {1963 … 1965}. Using N = 3, α = 10 years and β = 20 years, we obtain:

$$\text{Starts}(P_1, I_1) = 1$$

$$\text{During}(P_2, I_1) = 1 \qquad \text{During}_{(1)}^{(\alpha,\beta)}(P_2, I_1) = 1$$

$$\text{During}_{(2)}^{(\alpha,\beta)}(P_2, I_1) = 0 \qquad \text{During}_{(3)}^{(\alpha,\beta)}(P_2, I_1) = 0$$

5.2.1.3. Qualitative Temporal Relations between Time Points

We adapt the qualitative temporal relations between time intervals to propose qualitative temporal relations between time points.

Qualitative Temporal Relations between Precise Time Points

We adapt the resulting precise and imprecise personalized qualitative temporal relations between precise time intervals to propose precise and imprecise personalized qualitative temporal relations between precise time points, as shown in Table 9. When considering precise temporal relations, our approach reduces to Vilain and Kautz's work. When considering imprecise personalized temporal relations, our approach reduces to our extension of the Vilain and Kautz's algebra. Let P and L be precise time points.

Table 9 - Precise and imprecise personalized temporal relations between precise time points P and L.

Relation(P, L)	Definition	Inverse(L, P)
Precise qualitative temporal relations		
Before(P, L)	Precedes(P, L)	After(L, P)
Equals(P, L)	Same(P, L)	Equals(L, P)
Imprecise personalized qualitative temporal relations		
$\text{Before}_{(k)}^{(\alpha,\beta)}(P, L)$	$\text{Precedes}_{(k)}^{(\alpha,\beta)}(P, L)$	$\text{After}_{(k)}^{(\alpha,\beta)}(L, P)$

The precise temporal relations {"Before" and "After"} may be calculated from the associated imprecise personalized ones. Let R be one of the aforementioned relations. It holds that

$$R(P, L) = \sum R_{(k)}^{(\alpha,\beta)}(P, L) \; (k \in [1, N]) \tag{20}$$

Qualitative Temporal Relations between Imprecise Time Points

We adapt the resulting precise and imprecise qualitative temporal relations between imprecise time intervals to propose precise and imprecise qualitative temporal relations between imprecise time points, as shown in Table 10. Let P and L be imprecise time points.

Table 10 - Precise and imprecise personalized temporal relations between imprecise time point P and L.

Relation(P, L)	Definition	Inverse(L, P)
Precise qualitative temporal relations		
Before(P, L)	$\text{Precedes}(P^{(P)}, L^{(1)})$	After(L, P)
Equals(P, L)	$\text{Same}(P^{(1)}, L^{(1)}) \wedge \text{Same}(P^{(P)}, L^{(L)})$	Equals(L, P)
Imprecise personalized qualitative temporal relations		
$\text{Before}_{(k)}^{(\alpha,\beta)}(P, L)$	$\text{Precedes}_{(k)}^{(\alpha,\beta)}(P^{(P)}, L^{(1)})$	$\text{After}_{(k)}^{(\alpha,\beta)}(L, P)$

The precise temporal relations {"Before" and "After"} may be calculated from the associated imprecise personalized ones. Let R be one of the aforementioned relations. It holds that

$$R(P, L) = \sum R_{(k)}^{(\alpha,\beta)}(P, L) \; (k \in [1, N]) \tag{21}$$

Example 4: Let P_1, P_2 and P_3 be three imprecise time points; where P_1 is represented using the disjunctive ascending set {1960 ... 1963}; P_2 is represented using the disjunctive ascending set {1971 ... 1974} and P_3 is represented using the disjunctive ascending set {1940 ... 1943}. Using N = 3, α = 10 years and β = 20 years, we obtain:

$Before(P_1, P_2) = 1$ $\quad$ $After(P_1, P_3) = 1$

$Before_{(1)}^{(10,20)}(P_1, P_2) = 1$ $\quad$ $After_{(1)}^{(10, 20)}(P_1, P_3) = 0{,}65$

$Before_{(2)}^{(10,20)}(P_1, P_2) = 0$ $\quad$ $After_{(2)}^{(10, 20)}(P_1, P_3) = 0{,}35$

$Before_{(3)}^{(10,20)}(P_1, P_2) = 0$ $\quad$ $After_{(3)}^{(10, 20)}(P_1, P_3) = 0$

5.2.2. Reasoning about Qualitative Temporal Data: Transitivity Tables

We introduce four transitivity tables to derive new temporal knowledge from the resulted qualitative temporal relations between time intervals and points.

5.2.2.1. Qualitative Temporal Relations between Time Intervals

The Allen's transitivity table lets us obtain from $R_1(I, J)$ and $R_2(J, K)$ that $R_3(I, K)$ holds, where $I = [I^-, I^+]$, $J = [J^-, J^+]$ and $K = [K^-, K^+]$ are precise time intervals and R_1, R_2 and R_3 are Allen's relations. As already mentioned, we can deduce from "During(I, J)" and "Meet(J, K)" that "Before(I, K)" holds. We generalize such deductions using the three imprecise time intervals $I = [I^-, I^+]$, $J = [J^-, J^+]$ and $K = [K^-, K^+]$. Based on Table 6, we can deduce from "During(I, J)" and "Meet(J, K)" that "Before(I, K)" holds. Indeed by "During(I, J)", we have "$Precedes(J^{-(B)}, I^{-(1)}) \wedge Precedes(I^{+(E)}, J^{+(1)})$", and by "Meet(J, K)", we have "$Same(J^{+(1)}, K^{-(1)}) \wedge Same(J^{+(E)}, K^{-(B)})$". From "$Precedes(I^{+(E)}, J^{+(1)})$" and "$Same(J^{+(1)}, K^{-(1)})$", we conclude that "Before(I, K)". When considering only precise relations, our transitivity table coincides with the Allen's one.

We can also make other deductions when considering imprecise personalized relations. Let I, J and K be precise or imprecise time intervals; N be the number of graduality and k be an integer ($\in [2, N - 1]$) For instance, we can deduce from "Ends(I, J)" and "$Before_{(k)}^{(\alpha, \beta)}(J, K)$" which holds with a degree $D_{(k)}$ that "$Before_{(k)}^{(\alpha, \beta)}(I, K)$" holds with a degree $D_{(k)}$ (min (1, $D_{(k)}$)) and "Before(I, K)". Table 11 shows a part of the transitivity table.

Table 11 – A part of the transitivity table that allows reasoning $R_3(I, K)$ from $R_1(I, J)$ and $R_2(J, K)$.

$R_1(I, J)$ \ $R_2(J, K)$	**B**	**$B_{(1)}$**	**$B_{(k)}$**	**$B_{(N)}$**	**M**	**S**	**$S_{(1)}$**	**$S_{(k)}$**	**$S_{(N)}$**	**Eq**	**E**	**$E_{(1)}$**	**$E_{(k)}$**	**$E_{(N)}$**	**D**	**$D_{(1)}$**	**$D_{(k)}$**	**$D_{(N)}$**
B	B	B	B	$B \wedge B_{(N)}$	B	B	B	B	B	B	B	B	B	B	B	B	B	B
$B_{(1)}$	B	B	B	$B \wedge B_{(N)}$	B	B	B	B	B	$B_{(1)}$	$B \wedge B_{(1)}$	$B \wedge B_{(1)}$	$B \wedge B_{(1)}$	$B \wedge B_{(1)}$	B	B	B	B
$B_{(k)}$	B	B	B	$B \wedge B_{(N)}$	B	B	B	B	B	$B_{(k)}$	$B \wedge B_{(k)}$	$B \wedge B_{(k)}$	$B \wedge B_{(k)}$	$B \wedge B_{(k)}$	B	B	B	B
$B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$
M	B	B	B	$B \wedge B_{(N)}$	B	B	$B \wedge B_{(1)}$	$B \wedge B_{(K)}$	$B \wedge B_{(N)}$	M	M	M	M	M	B	B	B	B
S	B	$B \wedge B_{(1)}$	$B \wedge B_{(k)}$	$B \wedge B_{(N)}$	M	S	S	S	S	S	D	D	D	D	D	$D \wedge D_{(1)}$	$D \wedge D_{(k)}$	$D \wedge D_{(N)}$
$S_{(1)}$	B	$B \wedge B_{(1)}$	$B \wedge B_{(k)}$	$B \wedge B_{(N)}$	M	S	S	S	S	$S_{(1)}$	D	D	D	D	D	$D \wedge D_{(1)}$	$D \wedge D_{(k)}$	$D \wedge D_{(N)}$
$S_{(k)}$	B	$B \wedge B_{(1)}$	$B \wedge B_{(k)}$	$B \wedge B_{(N)}$	M	S	S	S	S	$S_{(k)}$	D	D	D	D	D	$D \wedge D_{(1)}$	$D \wedge D_{(k)}$	$D \wedge D_{(N)}$
$S_{(N)}$	B	$B \wedge B_{(1)}$	$B \wedge B_{(k)}$	$B \wedge B_{(N)}$	M	S	S	S	S	$S_{(N)}$	D	D	D	D	D	$D \wedge D_{(1)}$	$D \wedge D_{(k)}$	$D \wedge D_{(N)}$
Eq	B	$B_{(1)}$	$B_{(k)}$	$B_{(N)}$	M	S	$S_{(1)}$	$S_{(k)}$	$S_{(N)}$	Eq	E	$E_{(1)}$	$E_{(k)}$	$E_{(N)}$	D	$D_{(1)}$	$D_{(k)}$	$D_{(N)}$

Example 5: Let $I_1 = [I_1^-, I_1^+]$, $I_2 = [I_2^-, I_2^+]$ and $I_3 = [I_3^-, I_3^+]$ be the three imprecise time intervals mentioned in Example 2. We use N = 3, α = 10 years and β = 20 years. Based on the transitivity relation that allows inferring "$Before_{(k)}^{(\alpha, \beta)}(I_1, I_3)$" and "$Before(I_1, I_3)$" from "$Ends(I_1, I_2)$" and "$Before_{(k)}^{(\alpha, \beta)}(I_2, I_3)$", we automatically infer from the results shown in Example 2 the following relations:

$Before_{(2)}^{(\alpha, \beta)}(I_1, I_3) = 0{,}85$ $\quad$ $Before_{(3)}^{(\alpha, \beta)}(I_1, I_3) = 0{,}15$ $\quad$ $Before(I_1, I_3) = 1$

Note that we have the same results if we calculate "$Before(I_1, I_3)$", "$Before_{(2)}^{(\alpha, \beta)}(I_1, I_3)$" and "$Before_{(3)}^{(\alpha, \beta)}(I_1, I_3)$" based on Table 6.

5.2.2.2. Qualitative Temporal Relations between a Time Interval and a Time Point

We introduce two transitivity tables to reason about the qualitative temporal relations between a time interval and a time point. Let I and J be time intervals, P and L be time points, N be the number of graduality and k be an integer (∈ [2, N – 1]). Temporal data may be precise or imprecise. Table 12 shows a part of the transitivity table that allows to reason R_3(P, L) from R_1(P, I) and R_2(I, L). R_1 is a Point-Interval relation; R_1 is an Interval-Point relation and R_1 is a Point-Point relation. There are some composition relations that are undecidable. For instance, the relations "After$_{(k)}^{(\alpha, \beta)}$(P, I)" and "Before$_{(1)}^{(\alpha, \beta)}$(I, L)" yields three possible relations namely "Before(P, L)", "Equals(P, L)" and "After(P, L)". One of them is right. However, if the degree associated to "After$_{(k)}^{(\alpha, \beta)}$(P, I)" is 1 and the degree associated to "Before$_{(1)}^{(\alpha, \beta)}$(I, L)" is 1, we infer that the right one is "After(P, L)". In the composition table, such relations are marked with the symbol (*).

Table 12 – A part of the transitivity table that allows reasoning R_3(P, L) from R_1(P, I) and R_2(I, L).

R_1(P, I) / R_2(I, L)	**B**	**B$_{(1)}$**	**B$_{(k)}$**	**B$_{(N)}$**	**A**	**A$_{(1)}$**	**A$_{(k)}$**	**A$_{(N)}$**	**S**	**D**	**D$_{(1)}$**	**D$_{(K)}$**	**D$_{(N)}$**	**E**
B	B	B	B	B ∧ B$_{(N)}$	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B	B	B	B	B	B
B$_{(1)}$	B	B	B	B ∧ B$_{(N)}$	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A* ∨ Eq	B ∨ A* ∨ Eq	B	B	B	B	B	B ∧ B$_{(1)}$
B$_{(k)}$	B	B	B	B ∧ B$_{(N)}$	B ∨ A ∨ Eq	B* ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A* ∨ Eq	B	B	B	B	B	B ∧ B$_{(k)}$
B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∨ A ∨ Eq	B* ∨ A ∨ Eq	B* ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$
A	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A ∨ Eq	A	A	A	A ∧ A$_{(N)}$	A	A	A	A	A ∧ A$_{(N)}$	A
A$_{(1)}$	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B* ∨ A ∨ Eq	B* ∨ A ∨ Eq	A	A	A	A ∧ A$_{(N)}$	A ∧ A$_{(1)}$	A	A	A	A ∧ A$_{(N)}$	A
A$_{(k)}$	B ∨ A ∨ Eq	B ∨ A* ∨ Eq	B ∨ A ∨ Eq	B* ∨ A ∨ Eq	A	A	A	A ∧ A$_{(N)}$	A ∧ A$_{(k)}$	A	A	A	A ∧ A$_{(N)}$	A
A$_{(N)}$	B ∨ A ∨ Eq	B ∨ A* ∨ Eq	B ∨ A* ∨ Eq	B ∨ A ∨ Eq	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$	A ∧ A$_{(N)}$
Sb	B	B ∧ B$_{(1)}$	B ∧ B$_{(k)}$	B ∧ B$_{(N)}$	A	A	A	A ∧ A$_{(N)}$	Eq	A	A ∧ A$_{(1)}$	A ∧ A$_{(K)}$	A ∧ A$_{(N)}$	A
C	B	B	B	B ∧ B$_{(N)}$	A	A	A	A ∧ A$_{(N)}$	B	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A ∨ Eq	A
C$_{(1)}$	B	B	B	B ∧ B$_{(N)}$	A	A	A	A ∧ A$_{(N)}$	B ∧ B$_{(1)}$	B ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A* ∨ Eq	B ∨ A* ∨ Eq	A
C$_{(k)}$	B	B	B	B ∧ B$_{(N)}$	A	A	A	A ∧ A$_{(N)}$	B ∧ B$_{(k)}$	B ∨ A ∨ Eq	B* ∨ A ∨ Eq	B ∨ A ∨ Eq	B ∨ A* ∨ Eq	A
C$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	B ∧ B$_{(N)}$	A	A	A	A ∧ A$_{(N)}$	B ∧ B$_{(N)}$	B ∨ A ∨ Eq	B* ∨ A ∨ Eq	B* ∨ A ∨ Eq	B ∨ A ∨ Eq	A
Eb	B	B	B	B ∧ B$_{(N)}$	A	A ∧ A$_{(1)}$	A ∧ A$_{(k)}$	A ∧ A$_{(N)}$	B	B	B	B	B	Eq

Example 6: Let I_1 be the imprecise time interval mentioned in Example 2 and P_1 and P_2 are the time points mentioned in Example 3. We use N = 3, α = 10 years and β = 20 years. Based on the transitivity relation that allows inferring "Before$_{(k)}^{(\alpha, \beta)}$(P_1, P_2)" from "Starts(P_1, I_1)" and "Contains(I_1, P_2)" (as we have "During(P_2, I_1) =1"), we automatically infer from the results shown in Example 3 the following relation:

$$\text{Before}\ (P_1, P_2) = 1$$

Table 13 allows to reason R_3(I, J) from R_1(I, P) and R_2(P, J). R1 is an Interval-Point relation; R_2 is a Point- Interval relation and R_3 is an Interval-Interval relation.

Table 13 – A part of the transitivity table that allows reasoning $R_3(I, J)$ from $R_1(I, P)$ and $R_2(P, J)$.

$R_2(P, J)$ \ $R_1(I, P)$	**B**	$\mathbf{B_{(1)}}$	$\mathbf{B_{(k)}}$	$\mathbf{B_{(N)}}$	**A**	$\mathbf{A_{(1)}}$	$\mathbf{A_{(k)}}$	$\mathbf{A_{(N)}}$
B	B	B	B	$B \wedge B_{(N)}$	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations
$\mathbf{B_{(1)}}$	B	B	B	$B \wedge B_{(N)}$	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations
$\mathbf{B_{(k)}}$	B	B	B	$B \wedge B_{(N)}$	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations
$\mathbf{B_{(N)}}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	$B \wedge B_{(N)}$	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations
A	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	A	A	A	$A \wedge A_{(N)}$
$\mathbf{A_{(1)}}$	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	A	A	A	$A \wedge A_{(N)}$
$\mathbf{A_{(k)}}$	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	A	A	A	$A \wedge A_{(N)}$
$\mathbf{A_{(N)}}$	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	One of the precise interval relations	$A \wedge A_{(N)}$	$A \wedge A_{(N)}$	$A \wedge A_{(N)}$	$A \wedge A_{(N)}$

5.2.2.3. Qualitative Temporal Relations between Time Points

We introduce a transitivity table to reason about the resulting qualitative temporal relations between time points. It allows to reason $R_3(P, M)$ from $R_1(P, L)$ and $R_2(L, M)$, where P, L and M are precise or imprecise time points. R_1, R_2 and R_3 are Point-Point relations. Let N be the number of graduality and k be an integer ($\in [2, N - 1]$).

Table 14 – A part of the transitivity table that allows reasoning $R_3(P, M)$ from $R_1(P, L)$ and $R_2(L, M)$.

$R_2(L, M)$ \ $R_1(P, L)$	**B**	$\mathbf{B_{(1)}^{(\alpha, \beta)}}$	$\mathbf{B_{(k)}^{(\alpha, \beta)}}$	$\mathbf{B_{(N)}^{(\alpha, \beta)}}$	**Eq**	**A**	$\mathbf{A_{(1)}^{(\alpha, \beta)}}$	$\mathbf{A_{(k)}^{(\alpha, \beta)}}$	$\mathbf{A_{(N)}^{(\alpha, \beta)}}$
B	B	B	B	$B \wedge B_{(N)}^{(\alpha, \beta)}$	B	$B \vee A \vee Eq$	$B \vee A \vee Eq$	$B \vee A \vee Eq$	$B \vee A \vee Eq$
$\mathbf{B_{(1)}^{(\alpha, \beta)}}$	B	B	B	$B \wedge B_{(N)}^{(\alpha, \beta)}$	$B \wedge B_{(1)}^{(\alpha, \beta)}$	$B \vee A \vee Eq$	$B \vee A \vee Eq$	$B \vee A^* \vee Eq$	$B \vee A^* \vee Eq$
$\mathbf{B_{(k)}^{(\alpha, \beta)}}$	B	B	B	$B \wedge B_{(N)}^{(\alpha, \beta)}$	$B \wedge B_{(k)}^{(\alpha, \beta)}$	$B \vee A \vee Eq$	$B^* \vee A \vee Eq$	$B \vee A \vee Eq$	$B \vee A^* \vee Eq$
$\mathbf{B_{(N)}^{(\alpha, \beta)}}$	$B \wedge B_{(N)}^{(\alpha, \beta)}$	$B \wedge B_{(N)}^{(\alpha, \beta)}$	$B \wedge B_{(N)}^{(\alpha, \beta)}$	$B \wedge B_{(N)}^{(\alpha, \beta)}$	$B \wedge B_{(N)}^{(\alpha, \beta)}$	$B \vee A \vee Eq$	$B^* \vee A \vee Eq$	$B^* \vee A \vee Eq$	$B \vee A \vee Eq$
Eq	B	$B \wedge B_{(1)}^{(\alpha, \beta)}$	$B \wedge B_{(k)}^{(\alpha, \beta)}$	$B \wedge B_{(N)}^{(\alpha, \beta)}$	Eq	A	$A \wedge A_{(1)}^{(\alpha, \beta)}$	$A \wedge A_{(k)}^{(\alpha, \beta)}$	$A \wedge A_{(N)}^{(\alpha, \beta)}$
A	$B \vee A \vee Eq$	$B \vee A \vee Eq$	$B \vee A \vee Eq$	$B \vee A \vee Eq$	A	A	A	A	$A \wedge A_{(N)}^{(\alpha, \beta)}$
$\mathbf{A_{(1)}^{(\alpha, \beta)}}$	$B \vee A \vee Eq$	$B \vee A \vee Eq$	$B^* \vee A \vee Eq$	$B^* \vee A \vee Eq$	$A \wedge A_{(1)}^{(\alpha, \beta)}$	A	A	A	$A \wedge A_{(N)}^{(\alpha, \beta)}$
$\mathbf{A_{(k)}^{(\alpha, \beta)}}$	$B \vee A \vee Eq$	$B \vee A^* \vee Eq$	$B \vee A \vee Eq$	$B^* \vee A \vee Eq$	$A \wedge A_{(k)}^{(\alpha, \beta)}$	A	A	A	$A \wedge A_{(N)}^{(\alpha, \beta)}$
$\mathbf{A_{(N)}^{(\alpha, \beta)}}$	$B \vee A \vee Eq$	$B \vee A^* \vee Eq$	$B \vee A^* \vee Eq$	$B \vee A \vee Eq$	$A \wedge A_{(N)}^{(\alpha, \beta)}$	$A, A_{(N)}^{(\alpha, \beta)}$	$A \wedge A_{(N)}^{(\alpha, \beta)}$	$A \wedge A_{(N)}^{(\alpha, \beta)}$	$A \wedge A_{(N)}^{(\alpha, \beta)}$

Example 7: Let P_1, P_2 and P_3 be the three imprecise time points mentioned in Example 4. We use N = 3, α = 10 years and β = 20 years. From "Before(P_1, P_2) = 1", we deduce "After(P_2, P_1) = 1". Based on the transitivity relation that allows to infer "After(P_2, P_3)" from "After(P_2, P_1)" and "After(P_1, P_3)", we obtain:

$$After(P_2, P_3) = 1$$

6. Our TimeOnto Ontology for Representing and Reasoning about Precise and Imprecise Temporal Data in OWL 2

In this section, we introduce the proposed temporal ontology called TimeOnto. This ontology implements our extension of the 4D-fluents approach which is instantiated based on our extension of the Allen's interval algebra. It is implemented using Fuzzy-OWL 2.

We use the ontology editor PROTÉGÉ version 4.3 and the Fuzzy OWL 2 PROTÉGÉ's plug-in[††] version 2.21. This plug-in allows representing fuzzy classes and relations as annotations. For reasoning, we use the Pellet [76] precise reasoner and the FuzzyDL fuzzy reasoner.

We instantiate the crisp object properties {"CrispRelationIntervals", "CrispRelationIntervalPoint", "CrispRelationPointInterval" and "CrispRelationPoints"} based on our extension of the Allen's interval algebra. For instance, "CrispRelationIntervals" may be one of the Allen's relations. In other words, 13 object properties are associated: {"BeforeIntervals", "MeetsIntervals", "OverlapsIntervals", "StartsIntervals", "DuringIntervals", "EndsIntervals", "AfterIntervals", "Met-byIntervals", "Overlapped-byIntervals", "Started-byIntervals", "ContainsIntervals", "Ended-byIntervals" and "EqualsIntervals"}.

To represent the imprecise personalized interval relations, we fuzzify the crisp object properties {"BeforeIntervals", "OverlapsIntervals", "StartsIntervals", "DuringIntervals", "EndsIntervals", "AfterIntervals", "Overlapped-byIntervals", "Started-byIntervals", "ContainsIntervals" and "Ended-byIntervals"} into N fuzzy object properties. These properties are associated to the fuzzy object property "CrispRelationIntervals". For instance, using N = 3, the crisp object property "BeforeIntervals" is fuzzified into 3 fuzzy object properties {"BeforeIntervals_1", "BeforeIntervals_2" and "BeforeIntervals_3"} (see Fig. 6). In the same way, we implement the other imprecise properties relating a time interval and a time point or time points.

Fig. 6 - Fuzzification of the object property "BeforeIntervals" (N = 3).

TimeOnto proposes a set of SWRL and fuzzy IF-THEN rules to infer missing qualitative temporal relations. Crisp rules are passed to Pellet and fuzzy rules are passed to FuzzyDL. SWRL rules are associated to the crisp part of TimeOnto. They deduce precise temporal relations. We associate a SWRL rule for each precise temporal relation to deduce it from the precise quantitative temporal data given by the user. Based on transitivity tables, we associate a SWRL rule for each transitivity relation that implies only precise temporal relations. Let P, L and M be precise time points. For instance, based on Table 9, we associate two SWRL rules to infer the object properties "BeforePoints(P, L)" and "EqualsPoints(L, M)". Based on the transitivity table associated to time points, "BeforePoints(P, M)" is inferred as the following:

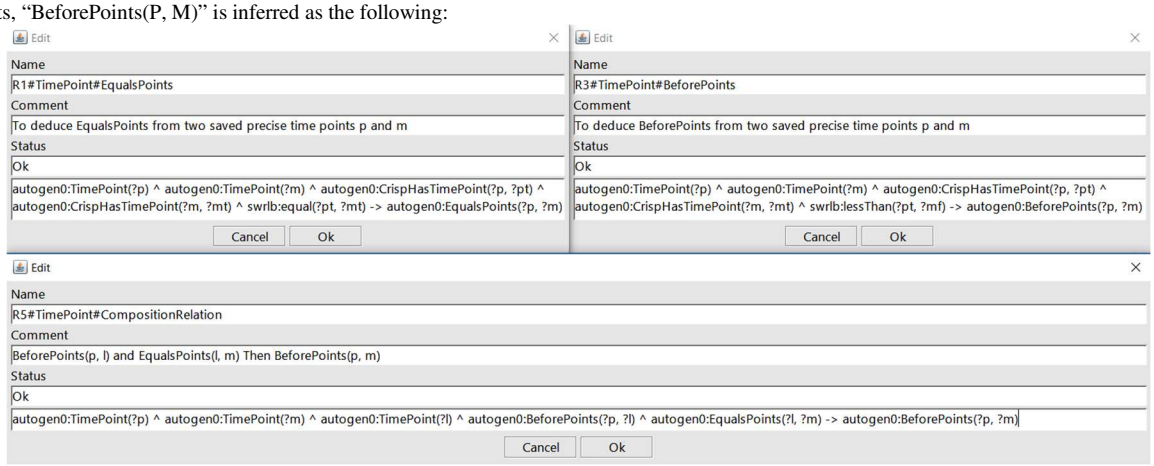


Fig. 7 - Examples of SWRL rules.

[††] http://www.umbertostraccia.it/cs/software/FuzzyOWL/index.html

Fuzzy IF-THEN rules are associated to the fuzzy part of TimeOnto. We associate a fuzzy IF-THEN rule for each imprecise personalized temporal relation deduced from quantitative temporal data entered by the user or each transitivity relation that implies imprecise personalized temporal relations. Rules are expressed with the Mamdani structure. Fig. 8 shows the fuzzy IF-THEN rule that allows to infer the "$Before_{(1)}^{(\alpha,\beta)}$" relation. In our experiment, we use $\alpha = 10$ and $\beta = 20$.

```
TimeOnto.fdl
26 (define-fuzzy-concept Fulfilled Right-shoulder(0,-30,-10,0))
27 (define-fuzzy-concept True Right-shoulder(0,0,1,1))
28 (define-concept Rule0 (g-and (some VarPrecedes_1 Fulfilled) (some BeforeIntervals_1 True)))
```

Fig. 8 - Example of fuzzy IF-THEN rule.

TimeOnto is serialized in the OWL 2 format. The syntax used is OWL 2 functional-style syntax[‡‡]. It contains 1 crisp class, 2 fuzzy classes, 6 crisp datatype properties, 3 fuzzy datatype properties, 26 crisp object properties and (22 * N) fuzzy object properties. Fig. 9 shows the TimeOnto ontology structure.

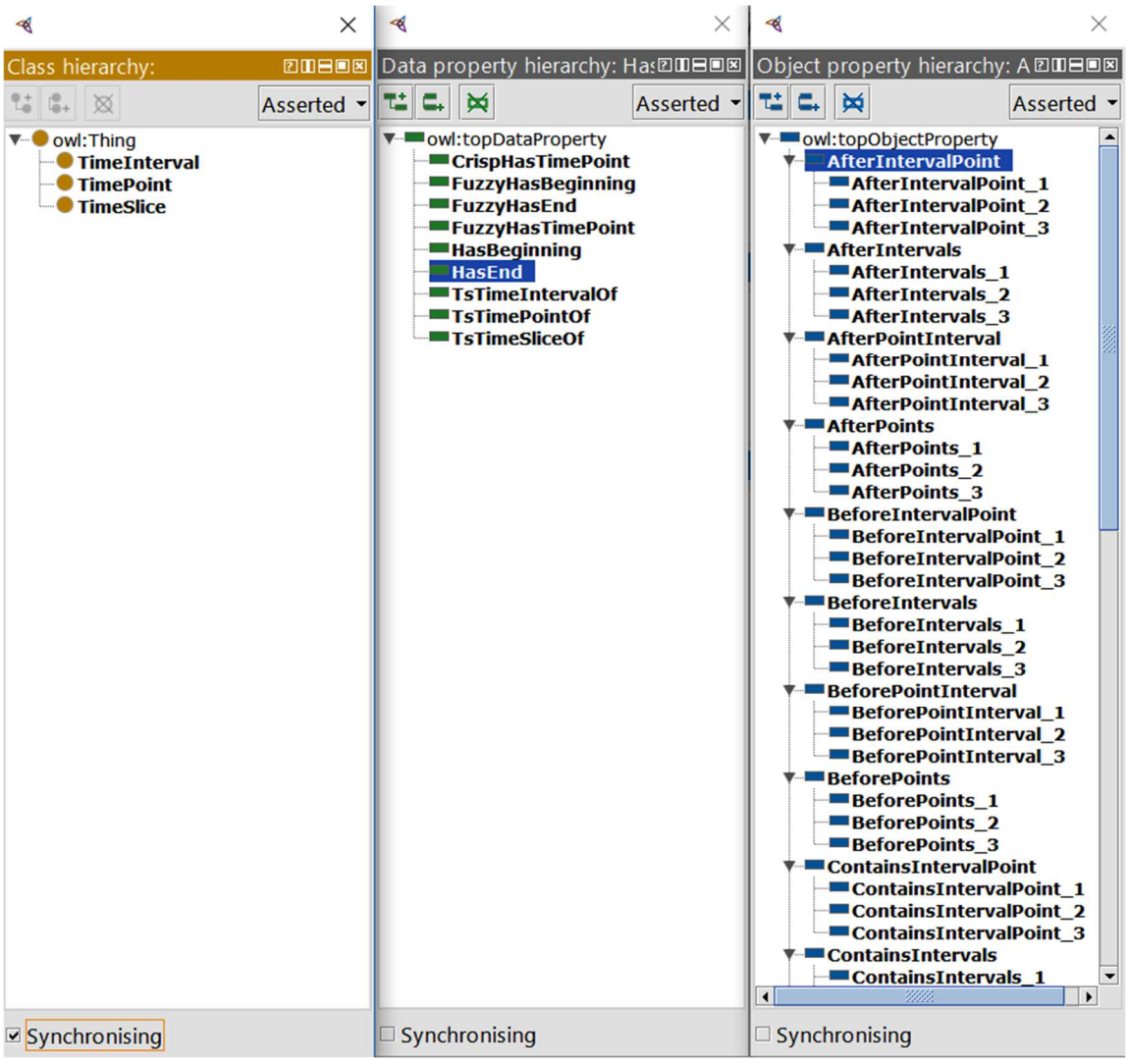


Fig. 9 - Excerpt of classes, datatype and object properties in the TimeOnto ontology (N = 3).

[‡‡] 6 https://www.w3.org/TR/owl2-syntax/

7. Validation

We developed a prototype based on TimeOnto. Then, we integrated it in the Captain Memo memory prosthesis to manage the temporal dimension in the context of the PersonLink ontology. Finally, we show how we use TimeOnto to manage temporal data in the context of the Travel[§§] ontology [78].

7.1. TimeOnto-Based Prototype

We propose a prototype that implements the proposed approach. This prototype, implemented in JAVA, is based on the TimeOnto ontology and allows users to interact through user interfaces. It uses Fuzzy Owl API (for managing fuzzy ontologies), FuzzyDL API (for managing the FuzzyDL fuzzy reasoner) and SPARQL-DL API[***] (for querying crisp ontology).

The architecture of the prototype includes four main components: “TimeOnto Population”, “Qualitative Temporal Data Inference”, “Configuration” and “Querying”. First, the user instantiates the TimeOnto ontology via the user interface shown in Fig. 10. This interface enables them to enter imprecise temporal data using the R-Function, L-Function, Trapezoidal, Triangular and Linear membership functions. We are limited to these functions as they are the only membership functions supported by the Fuzzy OWL 2 PROTÉGÉ’s plug-in. We propose a set of search criteria to show selected parts of the saved temporal data. For each search criterion, a pre-established query is associated. For instance, in Fig. 10, we show all saved time intervals using the following fuzzy query: (all instances? TimeInterval).

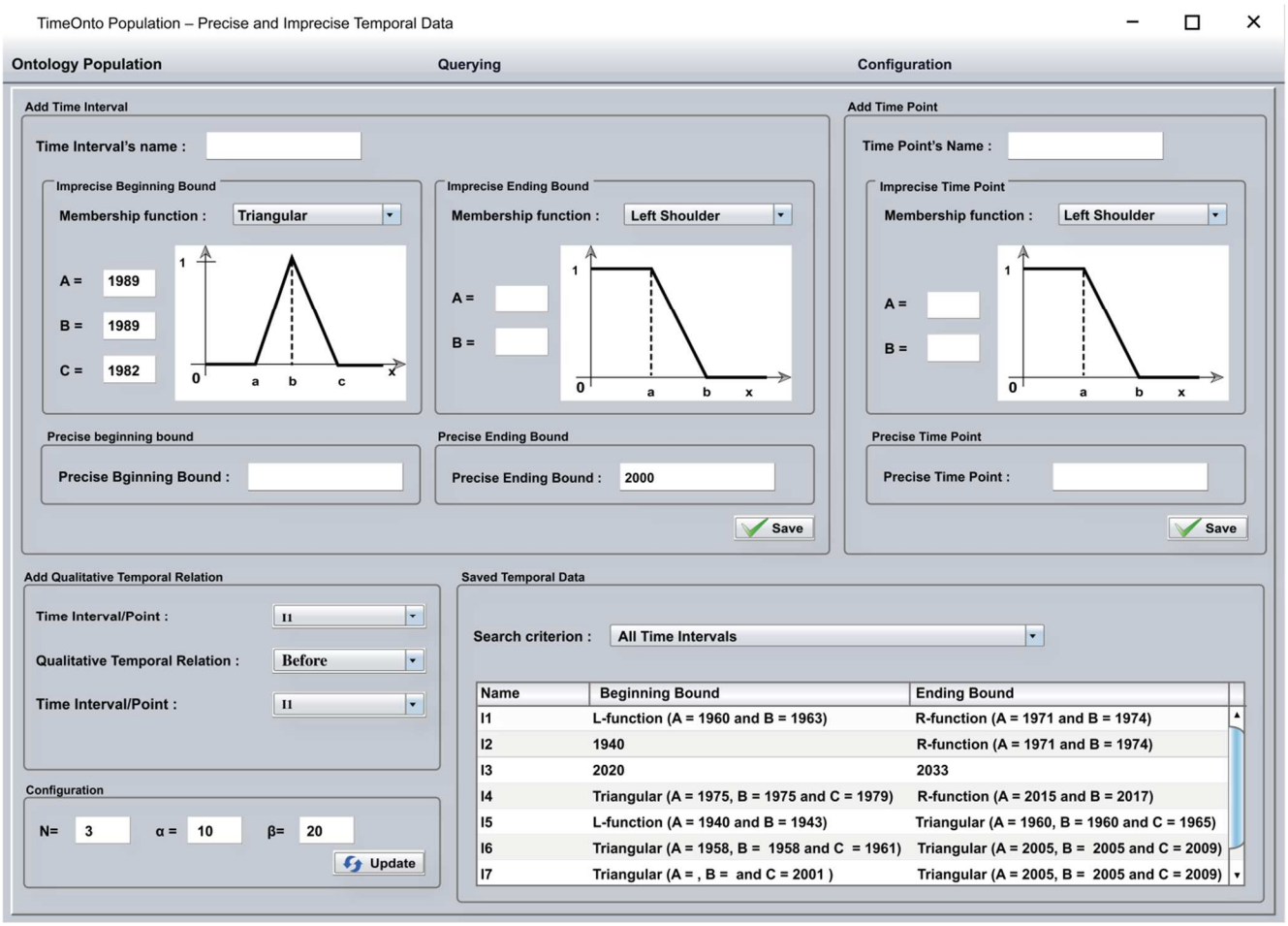


Fig. 10 - “TimeOnto Population” user interface.

[§§] https://protege.stanford.edu/ontologies/travel.owl

[***] http://www.derivo.de/en/resources/sparql-dl-api.html

After each user's new temporal data input, the "Qualitative Temporal Data Inference" component is automatically executed to infer missing data. This component is based on the proposed SWRL and fuzzy IF-THEN rules embedded into TimeOnto. The third component allows configuring the prototype. By default, we use the following parameters: N = 1, α = 0, β = 0. It also allows the domain expert to associate the meaning of each imprecise temporal relation. The fourth component allows users to query TimeOnto. However, querying this ontology is a tedious task. It requires using constructs specific to the extended 4D-fluent approach (e.g., "TsTimeSliceOf" and "TsTimeInterval"). This leads to complicated expressions and the user has to be familiar with the extended representation model. To address the issue, we introduce two temporal languages to query TimeOnto. The first one extends SPARQL to query the crisp part of the TimeOnto. It uses the same clauses as SPARQL and it supports all SPARQL features. The second language extends the syntax related to the FuzzyDL reasoner to query the fuzzy part of TimeOnto. For both languages, we propose "simple" temporal operators e.g., "BEFORE", "DURING" and "EQUALS". For each operator, we associate the corresponding SPARQL/syntax related to the FuzzyDL translation. For instance, in the context of PersonLink, the following query will retrieve all the countries visited by a person from 2005 to 2018.

A Query Based on the Proposed Crisp Temporal Language	Translation into SPARQL
SELECT ?Country **WHERE** ?Person plink:TravelTo ?Country **DURING("2005", "2018")**	**SELECT** ?Country **WHERE** ?Person plink:TravelTo ?Country. ?Person TsTimeSliceOf ?PersonTS. ?Country TsTimeSliceOf ?CountryTS. ?PersonTS TsTimePoint ?TravelTimePoint. ?CountryTS TsTimePoint ?TravelTimePoint. ?TimeInterval HasBeginning "2005". ?TimeInterval HasEnd "2018". ?TravelTimePoint DuringPointInterval ?TimeInterval.

7.2. Application to the Captain Memo Memory Prosthesis

We integrate the TimeOnto-based prototype in the Captain Memo memory prosthesis to manage the temporal dimension in the context of the PersonLink ontology (we merge TimeOnto and PersonLink ontologies).

Let's take the following example: "John married to Maria after one year he was graduated with a PhD. John was graduated with a PhD in *early 1980s*. Their marriage lasts *15 years*. John remarried to Béatrice and their marriage lasts *about 10 years*. They divorced in *2016*". Let P be an imprecise time point which represents the year of the award of a PhD degree. P has the Triangular membership function (A = B = 1980 and B = 1984). Let I = [I^-, I^+] and J = [J^-, J^+] be two imprecise time intervals representing, respectively, the duration of the marriage of John with Maria and the one with Béatrice. I^- is represented with the fuzzy datatype property "FuzzyHasBeginning" which has the Triangular membership function (A = B = 1981 and B = 1985). I^+ is represented with the fuzzy datatype property "FuzzyHasEnd" which has the Triangular membership function (A = B = 1996 and B = 2000). J^- is represented with the fuzzy datatype property "FuzzyHasBeginning" which has the Triangular membership function (A = 2004, B = 2006 and C = 2008). J^+ is represented with the crisp datatype property "HasEnd" which has the value "2016". We use the following parameters: α = 5 years, β = 3 years and N = 5. Fig. 11 shows a part of PersonLink that represents the imprecise time intervals I and J. The associated qualitative temporal relations are inferred from I and J based on the "Qualitative Temporal Data Inference" component.

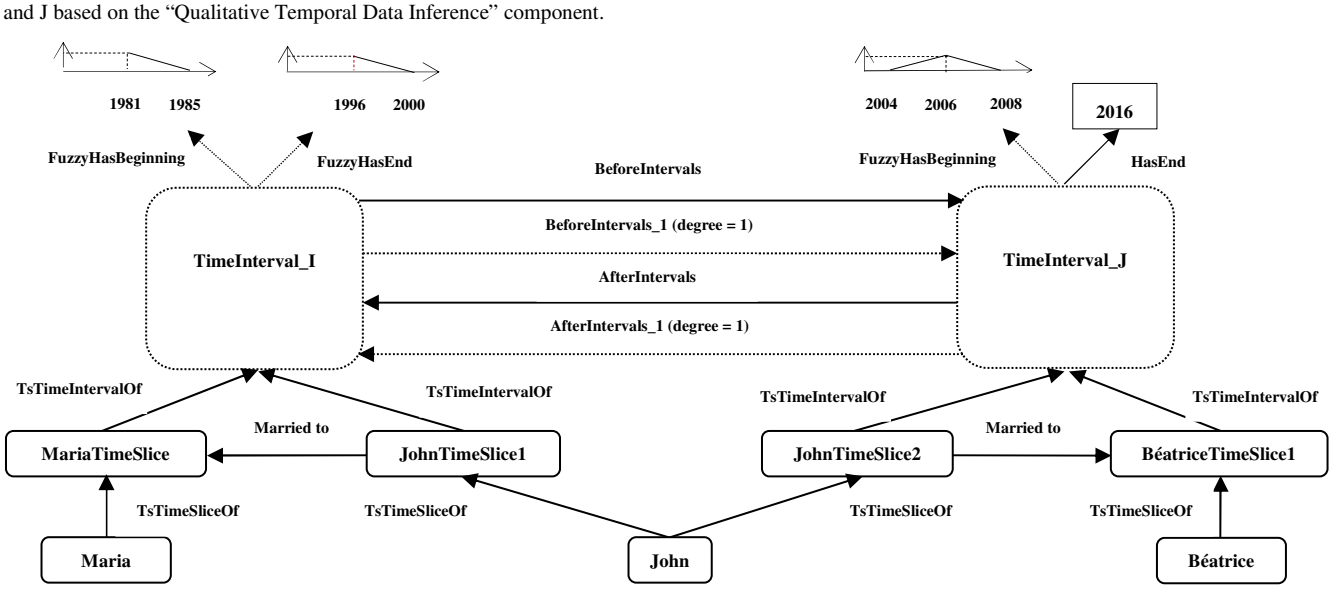


Fig. 11 - A part of PersonLink representing two imprecise time intervals.

Fig. 12 shows a part of PersonLink that represents the imprecise time intervals I and the imprecise time point P. However, in this case, a limitation of the proposed approach should be discussed. Indeed, none of the proposed SWRL and fuzzy IF-THEN rules is activated. This is explained by the fact that the ascendant set {1981 … 1985} associated to I overlaps with the ascendant set {1980 … 1984} associated to P (we have Precedes(1984, 1981) = 0). The property "AfterIntervalPoint" is given by the user and the property "BeforePointInterval" is automatically inferred as it is defined as "Inverse Of" the property "AferIntervalPoint" in PROTÉGÉ.

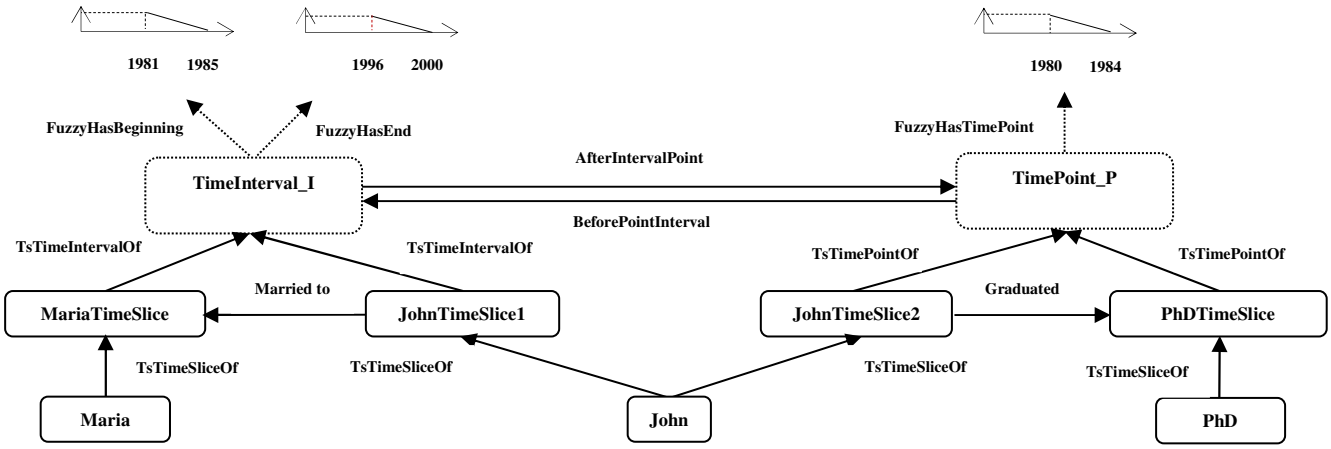


Fig. 12 - A part of PersonLink representing an imprecise time interval and an imprecise time point.

Alzheimer's patients present own characteristics that differ from other user groups, parts of them are related to Alzheimer's disease and the other parts are related to the normal effects of aging. We note that they give dates in reference to other dates or events e.g., "My daughter was born when I was in Paris. However, my son was born when I was in Nantes", "I bought a house when I was a teacher at the Sorbonne University" and "I was two teacher contracts when I was a PhD student". An interesting point in this work is to deal with a personalized slicing of the person's life in order to sort the different events. For each user, we define their slices of life. They serve as reference intervals. For each slice of life, we define its own parameters α, β and N. For instance, for the reference interval which represents the period of living in Paris ([1979, Now]), we use the parameters α = 5, β = 3 and N = 5. However, for the period which represents the period of living in Nantes ([1971, 1979]), we use the parameters α = 2, β = 1 and N = 3.

For instance, an Alzheimer's disease patient enters the following information: "I taught at the Sorbonne University from *by 1980* to *late 1990s*". In response to the following question: "When did I start teaching at the Sorbonne University?", our prototype compares the temporal data already entered "by 1980" to all entered temporal data and all references intervals. For instance, using the parameters already mentioned, we obtain that the patient started teaching at this university approximately at the same time when he moved to Paris and married to Maria and just after he moved from Nantes, as shown in Fig. 13.

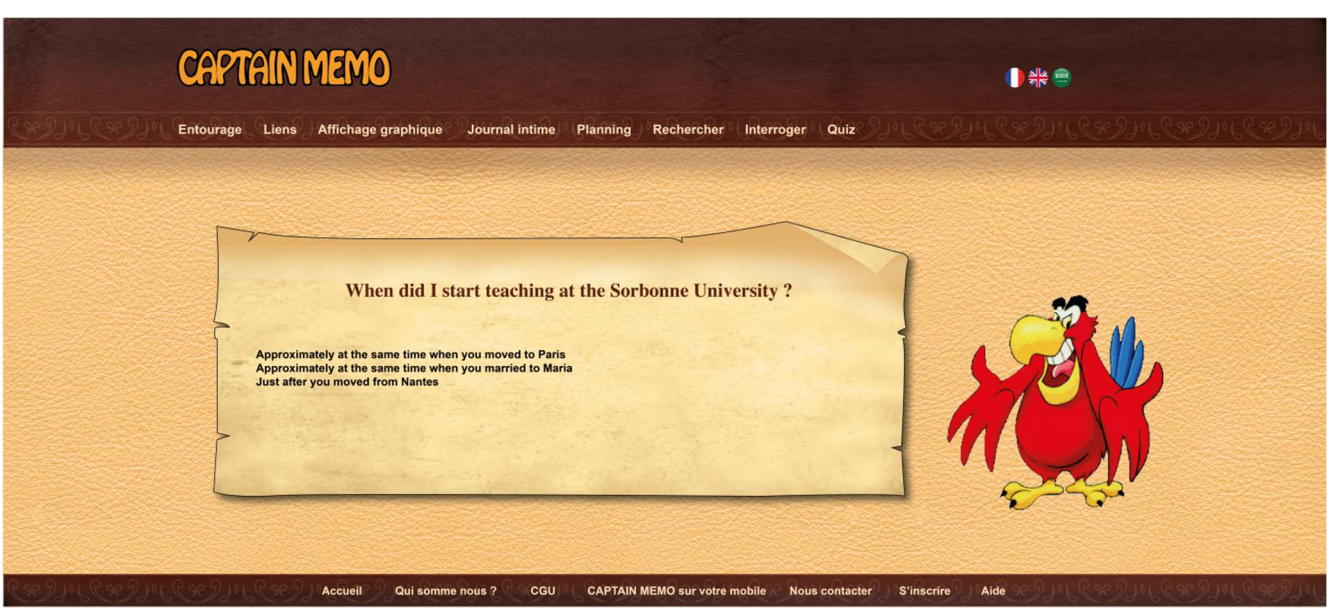


Fig. 13 - Screenshot of Captain Memo which shows the response returned by the "Querying" component.

7.3. Application to the Travel Ontology

The proposed approach is also applied to the Travel ontology. It allows representing and reasoning about data related to the tourism domain. In this context, temporal data are especially numerous e.g., "The journey starts *by June 5th* and finishes *by the end of July*", "The accommodation in the hotel starts in *June 05th* and ends in *June 12th*" and "We should leave the hotel *after* lunch time". Managing temporal data is necessary. For this reason, we merge it with the TimeOnto ontology. An interesting point in this work is to deal with temporal entities "with finer granularity" e.g., time clocks, date and vernacular names for months as well as days. To handles such data, we extend TimeOnto. We present two examples. Further details are presented in [79].

The first example concerns representing dates. Indeed, we propose a class called "Date" that is a subclass of "TimePoint". For the dates, let D, Mo and Y be, respectively, precise day, month and year. We introduce three datatype properties named "HasDay", "HasMonth" and "HasYear" to relate, respectively, the "Date" class and D, Mo and Y. For instance, if we have "The journey begins in *June 05th 2019*", "June 05th 2019" is represented as a precise date. "HasDay" has the range "05th", "HasMonth" has the range "June" and "HasYear" has the range "2019".

The second example concerns representing time clocks. We introduce a subclass of the class "TimePoint" called "Clock". Similarly, for the time clocks, let S, Mi and H be, respectively, precise seconds, minutes and hours. We introduce three datatype properties, named "HasSeconds", "HasMinutes" and "HasHours", to connect, respectively, the "Clock" class with S, Mi and H. For instance, if we have "The breakfast in the hotel starts *at 07:30:00*", "07:30:00" is represented as a precise time clock. "HasSeconds" has the range "00", "HasMinutes" has the range "30" and "HasHours" has the range "07".

8. Conclusion

In this paper, we proposed an approach for representing and reasoning about precise and imprecise temporal data in ontology. This approach takes into account quantitative and qualitative temporal data. It is compliant with existing Semantic Web standards and W3C recommendations. The first contribution of our work focuses on representing precise and imprecise temporal data in ontology. We extended the 4D-fluents approach which allows representing precise time intervals in OWL. It is extended to represent imprecise time intervals and precise/imprecise time points. It also deals with precise and imprecise qualitative relations between time intervals and points. The second contribution focuses on reasoning about precise and imprecise temporal data. We extended the Allen's interval algebra and proposed precise and imprecise temporal relations between precise and imprecise time interval. Compared to related work, our imprecise relations are personalized as their number and their meanings are set by the domain expert. This extension has advantages in terms of preserving many desirable properties possessed by the original Allen's algebra. We adapted the resulting relations to relate a time interval and a time point or two time points, where time intervals and points may be precise or imprecise. The proposed imprecise temporal relations are based on our extension, with an imprecise personalized view, of the Vilain and Kautz's point algebra. We introduced transitivity tables to reason about the resulting temporal relations. The third contribution consists of proposing an ontology called TimeOnto that implements our 4D-fluents approach which is instantiated based on our extension of the Allen's interval algebra. Inferences are done using a set of SWRL and fuzzy IF-THEN rules. As a fuzzy ontology is an extension of crisp ontology, TimeOnto may be integrated in crisp or fuzzy ontologies to represent and reason about temporal data. Finally, we proposed a TimeOnto-based prototype that was integrated in the prototype of the Captain Memo memory prosthesis to handle temporal inputs in the context of the PersonLink ontology.

The extension of the Allen's interval algebra that we proposed in this work in not limited to the semantic web field but may be applied to other research fields. For instance, it may be applied to the databases field. In this case, inferences may be made using database querying languages such SQL.

Furthermore, users/researchers who do not want to learn technologies related to fuzzy ontology may benefit from our approach. Indeed, they can use only the crisp part of TimeOnto to represent and reason about precise temporal data.

Future works are categorized in four main axes:

- We plan to test our approach in the ANR DAPHNE project that aims to allow Middle Ages specialized historians to deal with prosopographical databases storing Middle age academic's career histories.
- We plan to allow the TimeOnto ontology to be referenced by the Linked Open Vocabularies (LOV)[†††] platform.
- We plan to semi-automate the choosing values process of the setting parameters (N, α and β). The idea is to propose a fuzzy inference system that takes as input a set of criteria and returns the parameters values. These criteria are set by the domain expert. They differ from one application to another. For instance, in the context of Captain Memo, we plan to take into consideration the rate of progression of the Alzheimer's disease when setting the value of the number of graduality N.
- We plan to extend the proposed approach to handle another type of imperfection: uncertainty.

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