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# NONCIRCULAR WAVEFORMS EXPLOITATION FOR RADAR SIGNAL PROCESSING : SURVEY AND STUDY FOR AGILE RADAR WAVEFORM

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**Abstract**—With new generation of Active Digital Radar Antenna, there is a renewal of waveform generation and processing approaches, and new strategies can be explored to optimize waveform design and waveform analysis and to benefit of all potential waveform diversity. Among these strategies, building and exploitation of the Noncircularity of waveforms is a promising issue. Up to the middle of the nineties, most of the signals encountered in practice are assumed to be second order (SO) circular (or proper), with a zero second correlation function. However, in numerous operational contexts such as in radio communications, the observed signals are either SO noncircular (or improper) or jointly SO noncircular with a particular signal to estimate, to detect or to demodulate, with some information contained in the second correlation function of the signals. Exploitation of this information in the processing of SO noncircular signals may generate dramatic gain in performance with respect to conventional processing and opens new perspective in signal processing. The purpose of this paper is to present a short overview of the interest of taking into account the potential SO noncircularity of the signals in signal processing and to describe the potential interest of SO noncircular waveforms for radar applications.

*Keywords*- Noncircular, improper, widely linear, Single Antenna Interference Cancellation (SAIC), Binary signals, AM, ASK, BPSK, MSK, GMSK, demodulation, equalization, detection, beamforming, direction finding, MIMO, radar, waveform diversity.

## I. INTRODUCTION

Up to the middle of the nineties, most of the signals encountered in practice are assumed to be SO circular [1],

which means that their second correlation function is zero. Despite of the fact that many signals used in practical situations are indeed SO circular, some of them, mainly borrowed from the radio communications context, are SO noncircular [1] and contain information in their second correlation function. It is in particular the case for rectilinear signals such as Amplitude Modulated (AM), Amplitude Shift Keying (ASK) or Binary Phase Shift Keying (BPSK) signals [2]. It is also the case for signals which become quasi-rectilinear (i.e. a complex filtered version of a rectilinear signal) after a simple derotation operation such as Offset Quadrature Amplitude Modulations (OQAM), Minimum Shift Keying (MSK) or Gaussian MSK (GMSK) modulations [2]. Moreover, in the context of MIMO wireless systems [3] and whatever the chosen constellation, some Spatio-Temporal Block Coding (STBC) schemes, such as the Alamouti's scheme [4], generate observations which become jointly SO noncircular with the transmitted symbols. For observations which are either SO noncircular or jointly SO noncircular with a given input signal, exploitation of the information contained in the second correlation function of the signals may generate dramatic gain in performance with respect to conventional processing. This may even open new perspectives in signal processing.

The purpose of this paper is to present a brief overview of the interest of taking into account the potential SO noncircularity of the signals in signal processing and to describe the potential interest of noncircular waveforms in radar applications.

## II. SECOND ORDER NONCIRCULAR OBSERVATIONS

The SO statistics of a  $(N \times 1)$  complex random vector process  $\mathbf{x}(t)$  are completely defined by the two correlation matrices  $R_{\mathbf{x}}(t, \tau) = \overset{\Delta}{E}[\mathbf{x}(t) \mathbf{x}(t - \tau)^\dagger]$  and  $C_{\mathbf{x}}(t, \tau) = \overset{\Delta}{E}[\mathbf{x}(t) \mathbf{x}(t - \tau)^\top]$ , for all the values of  $(t, \tau)$ , where  $\dagger$  and  $\top$  are the transposition conjugation and the transposition operation respectively. The vector  $\mathbf{x}(t)$  is said to be SO noncircular [1] if the pseudo-correlation matrix [5],  $C_{\mathbf{x}}(t, \tau)$ , is not zero for at least one couple  $(t, \tau)$ . Such situations may for instance occur in the presence of rectilinear signals such as BPSK signals used in secondary radar or quasi-rectilinear signals such as GMSK signals used in the GSM network. Extension of the SO noncircularity concept to higher order may be found in [1] [6].

### III. SO NONCIRCULARITY EXPLOITATION THROUGH OPTIMAL WIDELY LINEAR FILTERING

#### A. Widely linear filtering

It is shown in [7] that for stationary observations, whose complex envelope is necessarily SO circular [1], SO optimal complex filters are linear and Time Invariant (TI), i.e. of the form  $y(t) = \Delta \mathbf{w}^\dagger \mathbf{x}(t)$ , where  $\mathbf{w}$  is a TI complex filter. However, for a nonstationary observation, whose complex envelope may be SO noncircular, SO optimal complex filters are Time Variant (TV) and, under some conditions of SO noncircularity presented in [7] [8], Widely Linear (WL) [8], i.e. of the form  $y(t) = \Delta \mathbf{w}_1(t)^\dagger \mathbf{x}(t) + \mathbf{w}_2(t)^\dagger \mathbf{x}(t)^*$ , where  $*$  means complex conjugate and where  $\mathbf{w}_1(t)$  and  $\mathbf{w}_2(t)$  are TV complex filters. For this reason, since the first use of WL filters in signal processing [9] and the first works about their SO optimality in noncircular contexts [7] [8], there has been an increasing interest for such filters in signal processing. The main applications of WL filtering currently concern Co-Channel Interference (CCI) cancellation of noncircular interferences, equalization of frequency selective channels for noncircular transmissions and optimization of MIMO systems using either SO noncircular constellations or some particular “noncircular” STBC schemes.

#### B. Cochannel Interference Rejection

##### 1) Cellular F/TDMA radio communications systems

SO optimal WL filters have been used for CCI rejection in F/TDMA radio communications cellular networks using either rectilinear modulations such as BPSK signals [10] or quasi-rectilinear modulations such as MSK [10] or GMSK [10-11] signals. In such contexts, it has been shown in [10] that SO optimal WL filters allow the processing of up to  $P = 2N - 1$  rectilinear or quasi-rectilinear interferences from an array of  $N$  sensors. This proves in particular the capability of these filters to do Single Antenna Interference Cancellation (SAIC) ( $P = 1$  for  $N = 1$ ). For this reason, the 3G Americas [12] presents the SAIC technology as a great improvement for GSM mobile station receivers, already standardized and allowing significant network’s capacity gains for the GSM system.

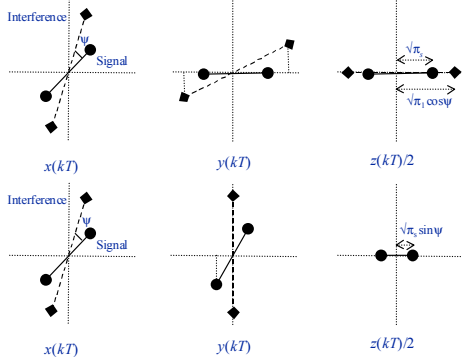


Figure 1. SAIC concept for BPSK signal and interference

Figure 1 illustrates the behavior of both the conventional receiver and the optimal receiver, implementing the SAIC concept, when a BPSK useful signal is corrupted by a BPSK interference. Contrary to the conventional receiver, which compensates the phase of the useful signal before taking the real part of the observation, the SAIC receiver puts the interference on the y-axis before taking the real part of the observation, which eliminates the latter.

##### 2) CDMA radiocommunications systems

SO optimal WL filters have been proposed to mitigate Multiple Access Interference (MAI) in CDMA radio communications networks using either BPSK modulation [13-16] or OQPSK modulation [17], through the implementation of WL Multi-user Detection (MUD) schemes. For Direct Sequence Spread Spectrum (DSSS) BPSK signals, it is shown in [15-16] that the use of optimal WL filters for MUD allows to double the capacity of the system in terms of number of users for a given spreading factor. For OQPSK signals, which concern IS’95 and CDMA 2000 standard, the use of optimal WL filters strongly improves the performance of the receivers.

##### 3) Detection systems

Very recently, optimal WL filters have been applied to the problem of detection of either a random signal [18] or a known deterministic signal with unknown parameters [19] corrupted by SO noncircular interferences. Several new GLRT-based detectors associated to several choices of unknown desired signal parameters have been developed in [19] for binary coded signals and arbitrary SO noncircular interferences. One of this new detector is also well-suited for synchronization purposes in radio communications [20]. All the proposed detectors allow to perform SAIC in the presence of a rectilinear interference as it is illustrated by figure 2.

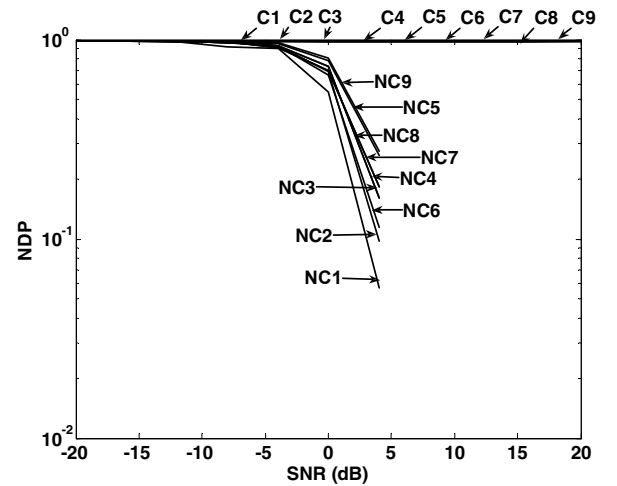


Figure 2. Non detection probability as a function of SNR

The latter shows the variations of the Non Detection Probability of the desired BPSK signal at the output of several conventional (Ci) and optimal detectors (Nci) as a function of the input SNR in the presence of a BPSK interference such that  $\text{INR} = \text{SNR} + 20$  dB, for a false alarm rate equal to 0.001, for a signal containing 64 symbols and for  $N = 1$  sensor. Phase difference between the sources is equal to  $45^\circ$ . Note the non detection of the signal by the conventional receivers and the detection of the latter by the optimal receivers.

#### 4) MVDR Beamforming for passive listening

For passive listening applications, two WL MVDR beamformers have been proposed recently [21] to improve the performance of the well-known Capon's beamformer [22-23] in the presence of SO noncircular interferences. The first one is TI whereas the second one, TV, allows to process SO noncircular interferences having a nonzero frequency offset or carrier residue. The proposed TI MVDR beamformer allow the processing of up to  $2(N - 1)$  rectilinear interferences from an array of  $N$  sensors.

### C. Egalisation of Linear Frequency Selective channels

#### 1) Single carrier links

In [24] several MMSE equalization schemes based on WL processing are proposed for rectilinear and quasi-rectilinear single carrier signals propagating through frequency selective channels, generating SO noncircular Inter-Symbol Interference (ISI) at the reception. It is shown in particular that WL equalizers outperforms Linear ones in such contexts.

#### 2) DMT or OFDM links

Optimal Narrow-Band Interference (NBI) resistant WL receivers/equalizers have also been proposed in [25] and [26] for Discrete Multitone (DMT) and Orthogonal Frequency Division Multiplex (OFDM) systems using SO noncircular constellations. Reference [26] considers both Cyclic-Prefix (CP) and zero-padded based multicarrier (MC) systems whereas [25] limits to CP systems.

#### 3) UWB links

In [27] and [28], WL equalizers are proposed to mitigate ISI in the context of direct-sequence based Ultra-Wideband (DS-UWB) systems using either BPSK [27] or 4BOK [28] modulations. The use of WL receivers allows very powerful high rate and short range transmission from the standardized UWB wave-forms under investigation for wireless personal area networks.

### D. MIMO systems optimization

In [29], [30] and [31], WL equalizers are proposed to mitigate ISI in the context of MIMO systems using either BPSK constellation [29], [30] or STBC schemes, such as the

Alamouti's scheme, which generate observation jointly SO noncircular with the transmitted symbols [31]. Strong gain in performance with respect to conventional linear equalizers are proved. In [32] and [33], WL receivers are proposed to mitigate interferences generated by some spatial multiplexing schemes involving either SO noncircular constellations [32] or transmitted symbols and their complex conjugates [33]. Finally reference [34] proposes optimized WL precoder schemes from rectilinear constellations and Channel state information knowledge at the transmitter.

### IV. SO NONCIRCULARITY EXPLOITATION FOR PARAMETERS ESTIMATION

The SO noncircularity of the signals may also be exploited for some parameter estimation problems without the implementation of WL filters. Such a property has been exploited for direction finding of rectilinear [35] [36] or SO noncircular signals [37], where up to  $2N - 1$  or  $2(N - 1)$  rectilinear signals may be processed from an array of  $N$  sensors. In [38] it is shown that for SO noncircular signals, blind channel identification is possible from SO statistics only. Finally several algorithms for blind frequency offset estimation of SO noncircular single carrier [39] [40] and OFDM [41] links through frequency selective channels have been proposed recently.

### V. NONCIRCULAR WAVEFORM EXPLOITATION FOR RADAR SYSTEMS

Radar waveforms have been classically optimized relatively to ambiguity function for target detection (good range resolution; sufficient energy on target, non sensitivity to target Doppler offset, robustness to clutter and jamming, ...). For the time being, Frequency Modulation waveforms are widely used and preferred to Phase Coding ones for Radar Applications. But noncircularity properties of phase coded radar waveforms have not yet been exploited, and this breakthrough coming from radiocommunication could modify balance between frequency & phase coding choice.

From this matter of fact, we have to explore classical phase coded waveform : BPSK, Pseudo-Noise coded (PN) sequences, Multi-Phase codes, ... .

Waveforms based on Binary phase-coded sequences, like BPSK, are used in Radar but their noncircularity properties are not exploited. In radio communications, this Binary phase-coded sequences are exploited to transmit information in a multi-user environment or for discretion purposes (e.g. for spread-spectrum communication system using binary PSK modulation, M-sequences are applied as sub-baud codes for bandwidth expansion), but for radar applications, these codes are used in an other framework for "pulse compression" to increase range resolution. Furthermore in radar, target motion gives rise to a Doppler frequency offset on the received signals. It is thus necessary not only to analyse the coded waveforms auto-correlation functions, but also to study the ambiguity function  $X(t, f)$  in time-frequency (t, f) plan, introduced by P. M. Woodward in 1953, to evaluate the

performance under various Doppler frequency returns :

$$X(t, f) = \int_{-\infty}^{+\infty} s(\tau) s^*(\tau + t) e^{-i2\pi f t} d\tau \quad (1)$$

Radar Engineering will select  $s(t)$  that will give an ambiguity function as close as possible to the ideal “thumbtack” response, with most of the weight around the origin in the  $(t, f)$  plane :

$$|X(t, f)| \leq |X(0, 0)| \quad \text{with} \quad \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |X(t, f)|^2 dt df = |X(0, 0)|^2 = 1 \quad (2)$$

The ambiguity should still be small in some strip of the  $(t, f)$  plane containing the delay axis and because of the sidelobe on the Doppler axis, the strip must be narrower than  $1/(MT)$  in the Doppler direction.

Considering binary phase-coded sequences, like BPSK (with 0 and  $\pi$  phase coding of the carrier waveform), Barker sequences have been introduced [42], as their aperiodic autocorrelations are all  $1/M$  or 0, for  $k = 1, 2, \dots, M - 1$ . However, while the ambiguity function is small on the delay axis, for all known Barker sequences there are substantial ridges away from the zero Doppler axis. Poor performance of Barker sequences is mainly due because they are all short sequences (there are no Barker sequences of length greater than 13, some elements of this proof in [43]), but exploitation of their non-circularity in anti-jamming or DOA could improve their advantages and other performances that could be attractive.

Pseudo-Noise coded (PN) sequences have been also studied as alternative of Barker sequences. The fact, that we have longer sequence could be also an advantage for non-circularity exploitation. They are generated by maximum length shift-register sequences, and have the property that the cyclic autocorrelation sidelobe levels are very small, equal to  $-1/M$  for  $1 \leq k \leq M - 1$ . If we consider their aperiodic autocorrelations, their sidelobes are at most  $\log(M)/\sqrt{M}$  (see [44]). The low level of cyclic autocorrelation Ambiguity function sidelobes ( $-1/M$ ) only occurs for the zero Doppler case, while the remainder of the plot is similar to the aperiodic performance. The idea is that subsequences of long M-sequences might be sufficiently random to have good autocorrelation properties, as studied in [46] where some properties of such subsequences (e.g. moments, mean weights) are calculated.

In practice, the ambiguity performance of binary phase coded sequences is lower than linear Frequency Modulation (FM) chirp signals. This is why Shift-register sequences have been deeply analyzed algebraically and investigated experimentally by simulation. For example, Bartee and Wood [49] have studied interlacing several short M-sequences having relatively prime lengths to obtain long codes having good short-term correlation properties to promote rapid acquisition in a radar ranging application. The general problem area involves the pseudo-randomness properties and/or weight distributions of segments from M-sequences. Most of the well-known good pseudo-random properties of M-

sequences pertain to averages over an entire period of the M-sequence. The detailed weight distribution of segments from M-sequences is not well known. Bartee and Wood [49] have considered an exhaustive comparison of all M-sequences of lengths up to and including  $M = 2^{14} - 1 = 16\,383$  for a wide range of subsequence lengths to select “optimum” M-sequences. For a given subsequence length,  $M$ , and fixed-length M-sequences the “optimum” M-sequence, in the sense of Bartee and Wood [49], is essentially that one whose minimum weight M-tuple has greatest weight. In one of their examples, a particular-system performance parameter varies over more than five orders of magnitude from worst-case M-sequence to best-case M-sequence. Unfortunately, Bartee and Wood found no systematic relationship between their set of optimum M-sequences and the corresponding generating functions.

Alternatively, Lindholm has considered the moments of the distribution of weights of M-tuples from any given M-sequence, and develop a systematic relationship between the moments of the distribution and the characteristic polynomial via the notion of coset functions. In addition, the coset function approach yields a systematic relationship between the distributions of the entire family of M-sequences of any given length. The first few moments of a distribution are sufficient to describe the main geometrical features of the distribution.

As in communication, noncircularity could be exploited on Multi-Phase Codes. In Radar, the design of Multi-Phase codes has focused on small aperiodic autocorrelation, rather than maximum sidelobes minimization of the whole ambiguity function. For instance, Golomb [48] has developed a 6-phase analogues of Barker codes. But, as with Barker codes, the maximum Ambiguity function sidelobes of these codes, away from the delay axis, are high.

Performance of Phase coded sequences should be now considered in the new framework of noncircularity, as it is done in radio communications. As we have seen in the first part of the paper, we could accept degraded performance of ambiguity function if we exploit noncircularity to improve Radar detection, anti-jamming (twice more number of interferences sources that could be rejected, hostile interference or interferences from other co-localized sensors that share same frequency plan) and DOA (better resolution than MUSIC with circularity assumption) performances by new processing chains exploiting statistics contained in the complex symmetric unconjugated spatial covariance matrix, and not only the classical covariance matrix. A potential improvement could be obtained if the two covariance matrices are used simultaneously. Phase coded waveforms can be used also for data transmission (plots & tracks) in netted radar system (in a dense network of co-localized radars, Phase coded waveforms could be used for communication modes between radars).

## VI. SIGNAL RADAR MODEL OF NON-CIRCULAR WAVEFORMS

Classically, Radar Signal can be modeled by a Normal Complex Random variable  $Z$  of zero mean. As we have previously seen the covariance matrix  $\Gamma = E[ZZ^+]$ , that is

Hermitian  $\Gamma^+ = \Gamma$  and definite positive does not completely describe the second order statistical properties for non-circular waveforms. A second matrix, called relation matrix  $C = E[ZZ^t]$  is needed. The matrix  $C$  is complex and symmetric  $C^* = C^+$ . B. Picinbono has proved that assuming  $\Gamma$  is complex and positive definite and  $C$  is complex and symmetric, this matrix  $C$  is a relation matrix of a random vector  $Z$  if and only if the matrix  $\Gamma^* - C^+\Gamma^{-1}C$  is non-negative definite.

Normal Random Vector is classically assumed to be circular. Without this assumption, consider random vector that is defined to be normal if its real and imaginary parts  $X$  and  $Y$  are jointly normal, with the PDF for a vector of zero mean :

$$p(X, Y) = (2\pi)^{-n} [\det \Gamma_{2r}]^{-1} e^{-\frac{1}{2} V^T \Gamma_{2r}^{-1} V} \quad (3)$$

$$\text{with } V^T = [X^T, Y^T] \text{ and } \Gamma_{2r} = \begin{bmatrix} \Gamma_{XX} & \Gamma_{XY} \\ \Gamma_{YX} & \Gamma_{YY} \end{bmatrix} = \begin{bmatrix} E[XX^T] & E[XY^T] \\ E[YX^T] & E[YY^T] \end{bmatrix}$$

$$\text{or more precisely : } \Gamma_{2r} = \frac{1}{2} \begin{bmatrix} \text{Re}(\Gamma + C) & \text{Im}(-\Gamma + C) \\ \text{Im}(\Gamma + C) & \text{Re}(\Gamma - C) \end{bmatrix} \quad (4)$$

We can built complex variable  $Z$ , given by  $Z = X + iY$ , and the vector  $W$  defined by  $W = [Z^T, Z^+]^T$  with  $W = M^{-1}V$  and  $V = M^{-1}W$  where  $M = \frac{1}{2} \begin{bmatrix} I & I \\ -iI & iI \end{bmatrix}$ .

As a consequence,  $v^T \Gamma_{2r}^{-1} V = W^+ \Gamma_w^{-1} W$  with  $\Gamma_w^{-1} = M^+ \Gamma_{2r}^{-1} M$  and  $[\det(\Gamma_{2r})]^{-1/2} = [|\det M|]^{-1} [\det(\Gamma_w)]^{-1/2} = 2^n [\det(\Gamma_w)]^{-1/2}$  (5)

where  $\det(M) = i^n 2^{-n}$

The covariance matrix  $\Gamma_w$  is related to initial covariance matrix  $\Gamma = E[ZZ^+]$  and relation matrix  $C = E[ZZ^t]$  :

$$\Gamma_w = \begin{bmatrix} \Gamma & C \\ C^* & \Gamma^* \end{bmatrix} = \begin{bmatrix} \Gamma & C \\ C^+ & \Gamma^* \end{bmatrix} = \begin{bmatrix} I & 0 \\ R & I \end{bmatrix} \begin{bmatrix} \Gamma & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} I & R^+ \\ 0 & I \end{bmatrix} \quad (6)$$

$$\text{where } \begin{cases} \Gamma = \Gamma_{XX} + \Gamma_{YY} + i(\Gamma_{YX} - \Gamma_{XY}) \\ C = \Gamma_{XX} + \Gamma_{YY} + i(\Gamma_{YX} - \Gamma_{XY}) \end{cases}$$

Cholesky block factorization are given by :

$$R = C^+ \Gamma^{-1} \text{ and } P = \Gamma^* - C^+ \Gamma^{-1} C$$

At this step of development, we can then give initial PDF  $p(X, Y)$  only according to variable  $Z$  and matrix  $C$  and  $\Gamma$  :

$$p(X, Y) = p(Z, Z^*) = (\pi)^{-n} [\det(\Gamma) \det(\Gamma^* - C^+ \Gamma^{-1} C)]^{-1/2} e^{-q(Z, Z^*)} \quad (7)$$

$$\text{with } q(Z, Z^*) = \begin{bmatrix} Z \\ Z^* \end{bmatrix}^+ \begin{bmatrix} \Gamma & C \\ C^+ & \Gamma^* \end{bmatrix}^{-1} \begin{bmatrix} Z \\ Z^* \end{bmatrix}$$

A normal random vector  $Z$  is said to be circular when  $C = 0$ , and the PDF becomes the classical PDF used by signal processing :  $p(X, Y) = p_C(Z) = (\pi)^{-n} [\det(\Gamma)]^{-1} e^{-Z^T \Gamma^{-1} Z}$  (8)

This PDF  $p_C(Z)$  could be factorized in general expression :

$$p(X, Y) = p_C(Z) [\det(I - R^+ R^t)]^{-1/2} e^{-r(Z, Z^*)} \quad (9)$$

$$\text{with } r(Z, Z^*) = Z^+ R^+ P^{-1} R Z - \text{Re}(Z^T R^t (P^{-1})^+ Z)$$

These developments done for non circular PDF can be also used for characteristic function :

$$\Phi(\alpha, \beta) = E[e^{i(\alpha^T X + \beta^T Y)}] = e^{-\frac{1}{2} \gamma^T \Gamma_{2r} \gamma} \text{ with } \gamma = [\alpha^T \quad \beta^T]^T \quad (10)$$

By using vector  $\Theta = \alpha + i\beta$ , characteristic function is given by:

$$\Phi(\alpha, \beta) = \Phi(\Psi, \Psi^*) = e^{-\frac{1}{4} [\Theta^+ \Gamma_{\Theta} + \text{Re}(\Theta^+ C \Theta^*)]} \quad (11)$$

When we apply non circular model in place of circular model for Radar Processing, we use, in place of  $n \times n$  matrix  $\Gamma = E[ZZ^+]$ , a  $2n \times 2n$  covariance matrix  $\Gamma_w$  with  $\Gamma_w^{-1} = M^+ \Gamma_{2r}^{-1} M$  where and the vector  $W$  is defined by  $W = [Z^T, Z^+]^T$  and complex variable  $Z$ , given by  $Z = X + iY$  with  $X$  and  $Y$  (In-Phase & quadrature IQ Radar signal). On this basis, we can define "Non-Circular" Anti-jamming and "Non-circular" MUSIC DOA Algorithm that out-perform classical algorithm based on circular waveforms.

## VII. CONCLUSION

The area of waveform design enjoys a new attention due to advances in "software radar" concept (Digital Array, use of software in Front-End,...). Classically, radar waveforms have been designed to optimize ambiguity function for target detection properties (good range resolution; sufficient energy on target, non sensitivity to target Doppler offset, robustness to clutter and jamming,...). In practice, Frequency Modulation have been preferred to Phase Coding for Radar Applications. But for the time being, noncircularity properties of phase coded radar waveforms have not been exploited as it has been done in radio communications. We propose to study new phase coded waveforms and exploit statistics contained in the complex symmetric unconjugated spatial covariance matrix (pseudocovariance matrix), to improve radar performances (detection, anti-jamming, DOA,...) and to allow communication, with the same phase coded waveform, between co-localized radar sensors (information on targets plots/tracks are exchanged between radar sensors in a collaborative way).

In that respect, it highlights the importance of designing waveforms with diverse objectives in terms of global radar performances and not with only classical ambiguity function constraints.

Exploitation of radar waveforms noncircularity property could be also extended to be applied for multi-static, MIMO and Ultra-Wideband Radars applications.

Concept of Waveform Diversity is a new challenging project that will promote new solutions and paradigms for advanced agile waveform resources management as explained in [50].

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