Acceleration Conservation Principle and Relativity

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Abstract

Discrete mechanics postulates the conservation of acceleration; it establishes a law of conservation where the proper acceleration of the material medium or of the particle is equal to the sum of the accelerations which are applied to it. The abandonment of the mass of Newton's second law is not in contradiction with the principle of equivalence of Galileo; it leads to acceleration being considered as an absolute quantity, independent of the local reference frame chosen. The immediate consequence is that the fundamental quantities necessary for the description of any mechanical problem are limited to two and only two, a length and a time. These two intrinsic quantities are independent of all the others which result from it.

The theory of relativity, like classical mechanics, gives mass a primordial role by setting as a principle the conservation of momentum; in special relativity, mass is a function of velocity through a Lorentz transformation. The discrete mechanics formulated without mass within the equation of motion is directly confronted with the indisputable results of the theory of relativity verified by experience. The solutions of two cases of relativity are found: the movement of a uniformly accelerated isolated particle and the deflection of light by the sun. An application to the interference produced by two sources of coherent light in direct simulation makes it possible to show the robustness of the discrete equation at very small time scales.

Keywords

Discrete Mechanics; Hodge-Helmholtz Decomposition; Acceleration Conservation Principle; Special Relativity; General relativity

1 Introduction

The subject developed here concerns the potential duality between two concepts, the continuous medium and the discrete medium. The concept of continuous medium is based on the reduction, at a point, in all the quantities associated with a volume losing the passage orientation vectors attached to it, e.g., normal to its surface. The concept of discrete framework [1] is the development of a physical model on a geometric topology with a characteristic length of d and the reduction of this length in a homothetic manner while keeping directions and angles. The geometric model can be observed at all spatial scales. Like distance, time, or the elapsed time dt between two observations of the geometric model, is considered a discrete value which is as small as necessary to understand the true nature of a phenomenon. Length and time are thus the only two independent intrinsic quantities.

There is a pitfall in the transition from discrete to continuous during the reduction to zero of the characteristic length d: the disappearance of the mass. It is therefore useful to question the need to conserve mass in the modeling of physical phenomena. The equivalence principle of Galileo, considering that gravity and inertia have the same effect, clears the way in the fundamental law of dynamics to derive a conservation law of acceleration. The mass can of course vary, but only under the influence of a specific conservation law. There is a difficulty in relativity [3] where the mass varies with the rest mass and velocity through the Lorentz factor; the problem due to abandonment of this approach is solved very differently in discrete mechanics.

The left aside of the concept of mass leads to a revisiting of the meaning of the classical quantities of force, energy, momentum, etc., with which it is associated. All these amounts will be divided by the mass, for example energy becomes e/m which has the dimension of a velocity squared. Additionally, all quantities of physics that refer to the mass involve this mass to order one, so it is possible to define these quantities per unit mass. The constants of physics, those of Planck and Boltzmann for example, can be defined per unit of mass without calling into question the underlying concepts attached to massive particles. This change in the status of mass then opens up the possibility of considering material media or particles with or without mass, without distinction. Particle/wave and discrete/continuous dualities open up perspectives for modeling the movement of particles and/or material media.

Discrete mechanics was established as a physical model from the principles of Galileo, namely the equivalence between gravitational and inertial effects and the principle of relativity of velocities. The Galilean or inertial reference frame is replaced by a local reference frame where the remote effects are considered causal. The mechanical and electromagnetic effects may be described by only two actions, the first concept relating to compression/expansion and the second associated with the effects of rotation/shear. The compression effects correspond to a directional movement and the rotation to an action by planar facet formalized by a Hodge-Helmholtz decomposition into a curl-free part and another divergence-free part. The general formulation obtained results in a law of conservation of acceleration, i.e. the acceleration of the particle or of a material medium is equal to the sum of the accelerations which are applied to it, associated with specific scalar and vector potentials.

The discrete formalism already applied to the movements of fluids and solids [1] shows that the equation of motion replaces the Navier-Stokes and Navier-Lamé equations by unifying these domains of mechanics. The extension of this approach to particle physics is presented using this same modeling. The set of variables in these areas of physics is reduced to a small number, acceleration and its scalar and vector potentials; similarly, the physical parameters are only the longitudinal and transverse celerities. All these quantities are expressed with only the two fundamental units, i.e. length and time.

2 Discrete formulation

The geometric topology created in discrete mechanics corresponds to the diagram in figure (1). A straight oriented edge Γ of length d and of unit vector $\mathbf{t}$ is limited by two points a and b; three of these edges define the primal topology, a contour Γ^* and a planar facet $\mathcal{S}$ of oriented unitary normal $\mathbf{n}$. The dual planar surface Δ joins the barycentres of the cells formed by the set of facets $\mathcal{S}$ having the edge Γ in common.

The unit vectors are thus orthogonal by construction, $\mathbf{t} \cdot \mathbf{n} = 0$; a third unit vector $\mathbf{m}$ orthogonal to the other two makes it possible to construct a local reference frame $(\mathbf{m}, \mathbf{n}, \mathbf{t})$, for each of the primal facets. A particle with or without mass is associated with the stencil, which can be considered as fixed in a Eulerian vision or attached to the particle in a Lagrangian approach. Acceleration and velocity, or rather their components, are defined on each edge where they are considered to be constant. The scalar quantities such as scalar potential ϕ or mass are attached to the vertices a or b of the primal geometry and the same is true for the divergence of the velocity $\nabla \cdot \mathbf{V}$. The primal curl $\nabla \times \mathbf{V}$, calculated as the circulation of the velocity vector along the contour Γ^*, is carried by the unit normal $\mathbf{n}$. The gradient operator $\nabla \phi$ is simply defined on the edge Γ as a difference in potential at the extremities, and the dual curl $\nabla \times \psi$ projects the result of the circulation along the contour of the Δ facet on the segment Γ.

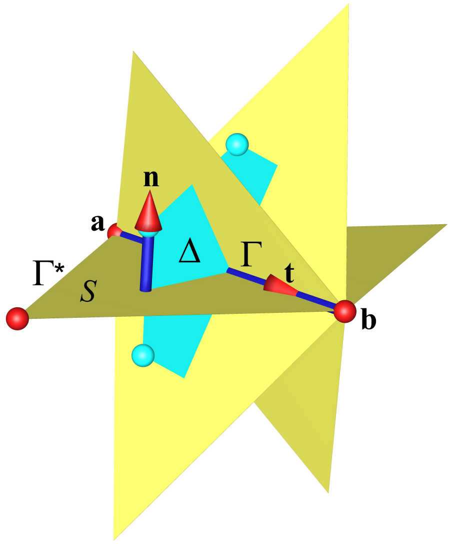

Figure 1. *Discrete geometric topology from the direct local reference frame* $(\mathbf{m}, \mathbf{n}, \mathbf{t})$*; a set of primitive planar facets* $\mathcal{S}$ *are associated with the edge* Γ *of unit vector* $\mathbf{t}$ *whose ends a and b are distant by a length d. Each facet is defined by a contour* Γ^* *is oriented according to the normal* $\mathbf{n}$ *such that* $\mathbf{n} \cdot \mathbf{t} = 0$*; the dual surface* Δ *connecting the centroids of the cells is also flat.*

The conservation law of the acceleration $\boldsymbol{\gamma} = \boldsymbol{h}$ of the particle or the material medium is written in the form of a Hodge-Helmholtz decomposition:

$$\boldsymbol{\gamma} = -\nabla \phi + \nabla \times \boldsymbol{\psi} + \boldsymbol{h}_s \tag{1}$$

where $\boldsymbol{\gamma}$ is the proper acceleration and where the second member $\boldsymbol{h} = -\nabla \phi + \nabla \times \boldsymbol{\psi} + \boldsymbol{h}_s$ represents the sum of the accelerations imposed by the outside neighborhood. The modeling of each of the phenomena in each of the fields of physics [1] leads to a unique system of equations:

$$\begin{cases} \boldsymbol{\gamma} = -\nabla \left(\phi^o - dt\, c_l^2\, \nabla \cdot \mathbf{V} \right) + \nabla \times \left(\boldsymbol{\psi}^o - dt\, c_t^2\, \nabla \times \mathbf{V} \right) + \boldsymbol{h}_s \\ \phi^o - dt\, c_l^2\, \nabla \cdot \mathbf{V} \longmapsto \phi^o \\ \boldsymbol{\psi}^o - dt\, c_t^2\, \nabla \times \mathbf{V} \longmapsto \boldsymbol{\psi}^o \\ \mathbf{V}^o + dt\, \boldsymbol{\gamma} \longmapsto \mathbf{V}^o \end{cases} \tag{2}$$

where c_l and c_t are the longitudinal and transverse velocities of the media concerned, whether fluid, solid or vacuum. The symbol $\longmapsto$ corresponds to an explicit upgrade of the quantity concerned after solving the vector equation of the system (2). The factors α_l and α_t between 0 and 1 correspond to the attenuation of the longitudinal and transverse waves, respectively. The source term $\boldsymbol{h}_s$, for example gravitational, capillary effects, etc., will be written in the same way, in the form of a Hodge-Helmholtz decomposition. All the variables and the physical parameters of this system are expressed by the two fundamental units, length and time.

The autonomous nature of this formulation should be emphasized here: the solution is obtained without any constitutive law or additional conservation law. In particular, the conservation of the mass is not necessary to obtain a solution, as is the case for the Navier-Stokes equation to which it is adjoined. This formulation in $(\boldsymbol{\gamma}, \phi^o, \boldsymbol{\psi}^o)$ serves to evaluate *a posteriori* the velocity, the displacement, etc. It is presented as a law of conservation of energy per unit of mass, where

ϕ^o is the compression energy and $\boldsymbol{\psi}^o$ the shear-rotation energy. The energy-mass duality would possibly favor this latter energy, but it would unnecessarily increase the number of fundamental quantities.

Inertial potential curvature: The material derivative $\boldsymbol{\gamma} = d\mathbf{V}/dt$ can bring out the partial derivative in time and another term, which will be named inertia κ_i, in the form $\boldsymbol{\gamma} = \partial \mathbf{V}/\partial t + \boldsymbol{\kappa}_i$. This last term represents the advection of the medium by itself. In a Lagrangian description, by following the particle during its movement, the material derivative is equal to the derivative in time. Inertia has a specific form which cannot be deduced from one of the ones associated with the continuous medium; it is written:

$$\boldsymbol{\kappa}_i = -\nabla \left(\frac{1}{2} \, \|\mathbf{V}\|^2 \right) + \nabla \times \left(\frac{1}{2} \, \|\mathbf{V}\|^2 \, \mathbf{n} \right) \tag{3}$$

Inertia $\boldsymbol{\kappa}_i$ is presented as the mean curvature of the inertial potential $\phi_i = \|\mathbf{V}\|^2/2$ defined both on the vertices and on the barycenters of the facets of the geometric topology of the figure (1). Vector $\boldsymbol{\kappa}_i$ is thus expressed on the edges Γ. In an accelerated movement following a rectilinear trajectory, only the first contribution in gradient of the inertial potential remains and it is not zero.

3 Application to the Light

The movement of light, propagation of the wave and advection of the photon provide an opportunity to test discrete mechanics on the results of the theories of special or general relativity, confirmed by experiments for over a century. Different cases can be envisaged depending on whether the particle is emitted by a source at a well-defined velocity or whether it is subjected to uniform acceleration from a state of rest; the latter acceleration is that of particles charged in an accelerator. An emblematic case of general relativity is that of light deviated from its rectilinear trajectory by a gravitational effect; it will be treated by comparing the inertial and gravitational potentials. A final case of interference fringes produced by two point monochromatic sources allows us to show that the system of equations (2) is representative of phenomena on all scales of time and space.

3.1 Inflow of particles

The problem treated here relates to the velocity of a particle and the propagation of the associated wave in equilibrium. It can be a flow of water in an already filled tube or the movement of electrons in an electrical conductor, for example a copper wire. If the tube or the conductor has a length L, the water or the electrons will exit after a period of time equal to $\tau = L/c$, where c is the velocity of the water ($c \approx 1500 \, m \, s^{-1}$) or that of electromagnetic waves, close to the velocity of light. However, the velocity of the water in the tube or of the electrons in the conductor is much lower, of the order of a centimeter per hour for the latter. A photon or a stream of photons ejected from a source at a velocity equal to the celerity of light is also part of this category. The problem posed schematizes the experiment of Michelson and Morlay [5], carried out in 1887. To demonstrate the existence of the luminiferous ether, these authors emitted light in different directions but no difference was recorded on what was known as the "‘speed of light". This experiment is reproduced very schematically from the system (2). Before going into the details of the calculation of the evolution of velocity over time for an accelerated rectilinear movement, let us consider some orders of magnitude of the main parameters of the vector equation:

$$\frac{d\mathbf{V}}{dt} = -\nabla \left(\phi^o - dt\, c^2\, \nabla \cdot \mathbf{V} \right) + \boldsymbol{h}_s \tag{4}$$

The terms of rotation have been removed from the equation (2) in order to consider only longitudinal actions. The analysis in order of magnitude leads to $\mathbf{V} \approx u$ where u is the norm of $\mathbf{V}$, the time of propagation of the wave along the edge Γ is equal to $dt \approx d/c$, where d is its length, and, like $\nabla \cdot \mathbf{V} \approx u/d$, the compression energy per unit of mass ϕ^o must be of the same order of magnitude as $dt\, c^2\, \nabla \cdot \mathbf{V}$ or $\phi^o \approx u\, c$. In fact, solving this equation for a velocity $u \leq c$ shows that this result is exact:

$$\phi^o = u\, c \tag{5}$$

We find an energy equal to $e = m\, c^2$ for a finite mass m when the velocity is equal to the celerity. Figure (2) shows the numerical solution obtained, with $\boldsymbol{h}_s = 0$, using the model (2) for a domain with one dimension of space, where in $x = 0$ a constant velocity equal to $u < c$ is imposed. The numerical resolution can be carried out directly with the real values, $c = c_0$ for the velocity and times chosen according to dimensionless quantities, as here.

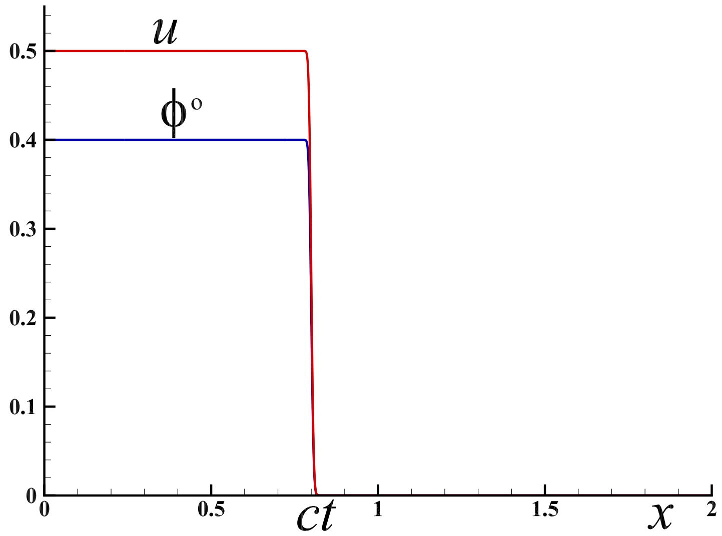

Figure 2. *Numerical solution obtained from the system (2) for a domain with a dimension of space with $c = 0.8$, $u = 0.5$ for a time $t = 1$. The wavefront is located at $x = c\, t$ and the energy upstream of it is equal to $\phi^o = u\, c$.*

The propagation of the wave in the domain leads to a front located at a distance equal to $x = x_o + c\, t$ for a total time t. Upstream of the wave front, the velocity is equal to u everywhere and the energy per unit of mass is equal to $\phi^o = u\, c$. Apart from the inevitable difficulties of numerical diffusion errors, the solution corresponds to the theoretical value (5). We can reproduce the experiment with different velocities u, positive or negative, and the distance traveled by the wave front is always equal to $x - x_0 = c\, t$ whatever the velocity u. If a perturbation is emitted at a point in the domain, the propagation extends in the two directions of the domain at a distance equal to $x = x_0 \pm c\, t$. The superposition of a constant velocity $\mathbf{V}_0$ simulating the potential presence of an ether changes nothing; indeed the system (2) is invariant in this transformation. The conclusion is that the "speed of light" is invariant. The interpretation given by A. Einstein, in his theory of special relativity, of the experiment of Michelson and Morlay is that the "speed of light" in the vacuum is equal to c_0 in all inertial frames of reference and does not depend on the movement of the source.

The interpretation which one can advance in discrete mechanics is certainly not in contradiction with this statement, but it provides a slightly more obvious physical sense to this result. The light sources used in these real or virtual experiments emit photons directly at a velocity equal to the celerity in the vacuum c_0; these continue their rectilinear trajectory, thus satisfying the principle of inertia in a movement that can be described as incompressible. The physical meaning of $\nabla \cdot \mathbf{V}$ and, more generally, of the term $dt\, c^2\, \nabla \cdot \mathbf{V}$ is linked to the compressibility of the longitudinal propagation. Indeed, the volume character which is too often attached to the divergence in continuum mechanics has no reason to be, as a unidirectional movement can be compressible. This notion of compressibility or incompressibility is not intrinsic, it depends on the period of time dt between two states of mechanical equilibrium in which the phenomenon is observed. The propagation of electromagnetic waves is a compressible phenomenon; there is no reason to differentiate acoustic waves in a material medium from light waves in a vacuum. The phenomenon of sonic blocking in a convergent orifice or nozzle, which is well known to fluid mechanics, is equivalent in nature to limiting the velocity of charged particles to the celerity c_0 of the wave in vacuum. The following example shows that, like a massive particle, the photon has its own velocity which can differ from the celerity of the wave and, in some cases, exceed it.

3.2 Uniform acceleration of an isolated particle

The case of a single particle with or without mass subjected to an external acceleration $\boldsymbol{h}_s = g\, \mathbf{e}_x = \nabla \phi_s$ from a state of rest is of a different nature, as the movement is not in mechanical equilibrium. Consider a particle at rest at the initial time, set in motion by means of an acceleration imposed from the outside, a gravitational field or a magnetic field, to the charged particles. In a Lagrangian description movement is described by the time-dependent position of the particle $x(t)$ and velocity $u(t) = dx/dt$. The equation of the discrete motion (4) to the rectilinear movement of the particle from its origin located at $x = 0$ to $t = 0$ is rewritten:

$$\frac{du}{dt} = -\frac{d}{dx}\left(\phi^o - dt\, u^2\, \frac{du}{dx}\right) + g \tag{6}$$

The difference with equation (4) lies within the compression term; in fact, nothing suggests that the wave upstream and downstream is already in a state of equilibrium. The particle whose instantaneous velocity is equal to $u(t)$ compresses the downstream wave in order to reduce the divergence of the velocity until it is zero. The compression energy is equal to $\phi^o(t)$. The velocity of the particle tends towards the celerity of the medium $u \rightarrow c$ and the second member then becomes zero. The physical interpretation described by the equation of motion (4) becomes clearer: the source term $\boldsymbol{h}_s$, which is constant, induces an acceleration of the particle or of the material medium in the direction of this vector; the velocity increases and the energy accumulates over time in the equilibrium potential ϕ. Then the divergence of the velocity tends towards zero and the mechanical equilibrium is reached when the acceleration becomes zero; we thus have:

$$-\nabla \phi + \boldsymbol{h}_s = 0 \tag{7}$$

The particle continues its rectilinear movement at a constant velocity equal to its celerity. The principle of relativity intrinsic to the equation of discrete motion allows the particle to keep the velocity acquired. In order to quantify the movement of the particle in terms of velocity and position in a more general vision, let us pose the following quantities to resize distance, time, velocity and energy per unit of mass:

$$x^* = \frac{x\, g}{c^2}; \quad t^* = \frac{t\, g}{c}; \quad u^* = \frac{u}{c}; \quad \phi^{*o} = \frac{\phi^o}{c^2} \tag{8}$$

where the starred quantities are the dimensionless variables. To lighten the writing, the same symbols are used to express the dimensionless quantities. The equation thus becomes:

$$\frac{du}{dt} = -\frac{d}{dx}\left(\phi^o - dt\, u^2\, \frac{du}{dx}\right) + 1 \tag{9}$$

The compression energy ϕ^o is that accumulated at time t from the initial time:

$$\phi^o(t) = -\int_0^t u^2\, \frac{du}{dx} d\tau = -\int_0^t \frac{dt}{dx}\, u^2\, du = -\frac{u^2}{2} \tag{10}$$

Thus, at each instant, the compression energy of the wave is exactly equal to the kinetic energy acquired by the particle. As $g\, \mathbf{e}_x = \nabla(g\, x)$ in real variables, the scalar potential of the acceleration becomes equal to:

$$\phi(t) = x - \phi^o(t) = x - \frac{u^2}{2} \tag{11}$$

Equation (9) becomes:

$$\frac{du}{dt} = \frac{d}{dx}\left(\frac{u^2}{2}\right) \tag{12}$$

The position of the particle x and time t are not independent and the relation (12) is not an equation but an identity; each member expresses the material derivative in a Lagrangian description, because dx/dt is none other than the velocity u. In fact the equation of discrete motion is not a simple differential equation. It involves a temporal process called "accumulation" where the description of the mechanical equilibrium at the instant t is deduced from that at the previous equilibrium instant t^o by a Lagrangian formulation composed of an equation on acceleration and its upgrades on velocity, energy and spatial coordinates:

$$\begin{cases} \boldsymbol{\gamma} = 1 - \dfrac{1}{u}\, \dfrac{d}{dt}\left(\phi^o - u\, \dfrac{du}{dt}\right) \\[2ex] \phi^o - u\, \dfrac{du}{dt} \longmapsto \phi^o \\[2ex] u^o + dt\, \boldsymbol{\gamma} \longmapsto u^o \\[2ex] x^o + dt\, u \longmapsto x^o \end{cases} \tag{13}$$

This system is autonomous, it does not require *a priori* recourse to knowledge of the position of the particle as a function of time, nor to one of the Lorentz transformations. This problem of the evolution of the uniformly accelerated particle requires only knowledge of the initial conditions (u^o, x^o, ϕ^o), zero for a particle at rest at the initial time. While the resolution of the Eulerian version given by the system (2) is very robust, the Lagrangian version (2) has a term in $1/u$ in the equation which makes it substantially more complicated to obtain the solution $(u(t), \phi(t), x(t))$. The implication of the variable velocity achievable for the system (2) is more difficult for the system (2). These numerical difficulties in Lagrange variables lead, however, to a behavior of the expected solution, with velocity naturally tending towards a constant value over time. It is possible to find a more robust alternative by considering the energy necessary for the particle to overcome inertia and a velocity which tends towards celerity. This energy, by definition equal to $c^2/2$, is expressed as a function of the integral of the acceleration from zero to infinity, i.e. in dimensionless form:

$$\phi_c = \int_0^\infty \boldsymbol{\gamma}(x)\, dx = \frac{1}{2} \tag{14}$$

Acceleration is thus a decreasing function of x in $\mathcal{O}(x^{-n})$, whereby $n = 3$ in order to satisfy the condition (14). The initial condition corresponding to an acceleration equal to unity for $x = 0$, satisfied by the system equation (13), leads to the following form:

$$\gamma = \frac{1}{(1+x)^3} \tag{15}$$

The imposed acceleration $\boldsymbol{h}_s = 1$ is transformed partly into compression energy equal to ϕ^o. This compression energy is transferred to the particle to increase its own kinetic energy ϕ_c and therefore its own acceleration $\boldsymbol{\gamma}$. At each instant we have $\phi_c = \phi^o$ and the vector potential of the acceleration is equal to $\phi = x - \phi^o$; equilibrium is reached when $-\nabla \phi + 1 = 0$. The imposed acceleration $\boldsymbol{h}_s$ then no longer acts on the particle, the movement is incompressible and the particle moves at a velocity equal to the celerity $u = c$.

The solution to the problem in dimensionless form obtained by the incremental system (13) corresponds very precisely to the solution obtained in the context of special relativity.

$$\begin{cases} \gamma(t) = \dfrac{1}{(1+x)^3} = \dfrac{t}{(1+t^2)^{3/2}} \\[2ex] u(t) = \dfrac{\sqrt{x(2-x)}}{(1+x)} = \dfrac{t}{\sqrt{1+t^2}} \\[2ex] \phi(t) = x - \dfrac{u^2}{2} \end{cases} \tag{16}$$

Figure (3) shows the evolutions obtained numerically by the system (13) and the acceleration (15) over time: (i) acceleration $\boldsymbol{\gamma}$, (ii) velocity $u(t)$, (iii) scalar potential $\phi(t)$ and (iv) the coordinate $x(t)$. The divergence (v) $\delta = \nabla \cdot \mathbf{V}$ shows that the movement becomes incompressible, explaining that the velocity of the particle tends towards celerity.

With short times, the acceleration is equal to unity, the velocity increases in the form $u(t) \approx t^2/2$ and the divergence of the velocity is then in $\mathcal{O}(t^{-1})$; at this point the accumulated energy is very close to zero. When the time increases beyond $t \approx 0.5$ the potential $\phi = x - \phi^o$ increases, the natural acceleration $\boldsymbol{\gamma}$ decreases and the growth in velocity slows down. The acceleration then tends towards zero and the velocity towards the celerity of the medium. The root cause of this limitation is the divergence of the motion of the particle, which goes from $\mathcal{O}(t^{-1})$ for Newtonian mechanics to $\mathcal{O}(t^{-3})$ in relativity and in discrete mechanics when time increases.

It is possible to compare the approach of special relativity to that of discrete mechanics. They can be synthesized in the formal writing of the equations by returning to the real variables:

$$\begin{cases} S.R.: \quad \dfrac{d(m\, u)}{dt} = m_0\, g \\[2ex] D.M.: \quad \dfrac{du}{dt} = -\nabla \phi + g \end{cases} \tag{17}$$

where the mass in motion is equal to $m = m_0\, \gamma$ with $\gamma = 1/\sqrt{1 - u^2/c^2}$ the Lorentz factor and m_0 the mass at rest.

While the result is identical, the physical modeling of the phenomenon is not at all the same. First of all, the equation (17) of special relativity is a transposition of Newton's second law, while discrete mechanics translates a conservation of accelerations. Thus the mass can vary but it does not tend *a priori* towards infinity, while the velocity tends towards the celerity of the medium (material medium or vacuum). Besides, the material velocity or that of the particle is a notion which is disjointed from that of celerity, and the ratio u/c of the Lorentz transformation is not self-evident, except the consideration as a principle that velocity is always limited by celerity. In relativity, the velocity of the photon cannot exceed the celerity of light in a vacuum in principle; this is not the case in discrete mechanics where the velocity of the photon is not limited, and if it tends towards celerity, this corresponds to a decrease in the sum of the accelerations which are applied to it. In the problem presented, this is indeed the case because the trajectory does not have any curvature.

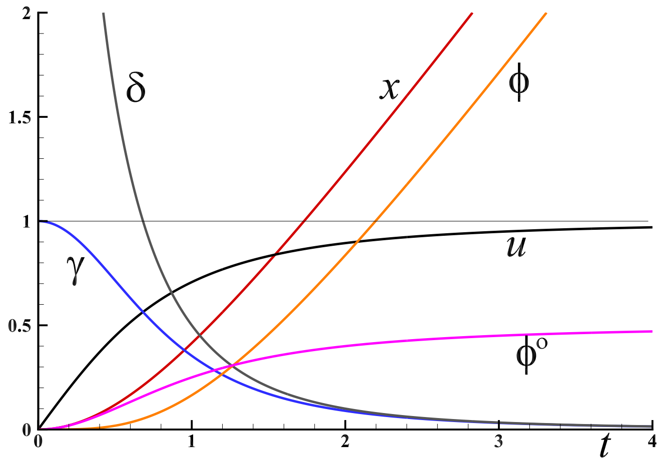

Figure 3. *Acceleration of an isolated particle with or without mass over time: $u(t)$ its velocity, $x(t)$ its position, $\boldsymbol{\gamma}(t)$, its acceleration and $\delta(t) = \nabla \cdot \mathbf{V}$ the divergence of the velocity. The scalar potential is equal to $\phi(t) = x - u^2/2$ and kinetic energy to $\phi^o(t) = u^2/2$.*

The equation of motion of discrete mechanics does not present itself as a relation whose solution we seek directly; it corresponds at each instant to an accumulation of the instantaneous compression energy $dt\, u^2\, \nabla \cdot \mathbf{V}$ within the scalar potential ϕ^o. Thus, for short times, the solution is equal to $u(t) \approx g\, t$, then the term $\nabla \phi$ increases and causes the reduction in the proper acceleration $\boldsymbol{\gamma}$. Gradually the velocity stabilizes towards the celerity of the medium.

The case of the zero mass photon is of particular interest. In discrete mechanics, if it has no mass it has kinetic energy per unit of mass, like massive particles, and its velocity increases gradually; this is not a special case. In relativity, the photon is assumed to always have a velocity equal to the celerity of light because it is already emitted by the source at this velocity. For charged particles, the similarity parameters (10) make it possible to estimate the time constant $\tau \approx c/g$ (independent of mass) so that its velocity is close to the celerity, hence the very high energies used in particle accelerators.

The potential ϕ^o is the energy per unit of mass corresponding to the compression effects. In the case where other gradient effects are imposed by accelerations, the energy is also accumulated in the scalar potential ϕ of the acceleration. The elementary energy corresponding to the compression present in the equation of motion (2), $dt\, c^2\, \nabla \cdot \mathbf{V}$ implicates it in the variable velocity in a similar way to the rotation-shear term. The energy ϕ^o is then upgraded explicitly from the divergence of the velocity.

Analysis of the behavior of the discrete motion equation reveals the physical mechanism of the limitation of velocity to the celerity of the medium. The particle's own acceleration is equal to the sum of the accelerations: that which is imposed, $\mathbf{g}$, of positive sign, and that of the opposite sign which translates the compression of the downstream wave until the divergence velocity is zero. The contraction of lengths and the dilation of time are absolutely necessary concepts in special relativity; they are induced by the initial choice of remaining within the framework of Newton's second law and of associating the mass in motion with the relationship between velocity and celerity. When the velocity increases, the distances contract and the time intervals expand. In discrete mechanics, the length and the time are absolute quantities which do not depend on the velocity. For example, time flows linearly in a discrete way by intervals dt. When the velocity increases, it is the divergence which is reduced and which makes the wave less compressible, until it becomes incompressible when the velocity tends towards celerity. The abstractions of the theory of relativity are replaced here by simple concepts of mechanics, in particular the mechanics of compressible fluids.

3.3 Gravitational deflection of light by the sun

The problem relates to an emblematic case of general relativity, the deflection of light by a massive body. It was responsible for the considerable success of this theory at the beginning of the last century and of its main author, Albert Einstein. The solution of this problem obtained within the framework of Newtonian mechanics and its version in theory of relativity can be easily found in many textbooks [6], [7].

The gravitational contribution to the acceleration of a particle or a material medium $\boldsymbol{h}_g$ is defined by $\phi_g = \mathcal{G}\, M/r$ where $\mathcal{G}$ is the universal constant of gravitation, M the body mass and r the distance from the point considered to the center of gravity of the body. In classical mechanics, only the contribution in $\nabla \phi_g$ is used, which induces an error in the deviation of the trajectory of the light when it crosses the neighborhood of the sun.

Consider first the so-called case of "Newtonian mechanics" where the equation of motion is written in the form:

$$\frac{d\mathbf{V}}{dt} = -\frac{\mathcal{G}\, M_{\odot}}{r^2}\, \mathbf{e}_r \tag{18}$$

where $\mathcal{G} = 6.673848010^{-11} m^3 kg^{-1} s^{-2}$ is the universal constant of gravitation, $M_{\odot} = 1.9910^{30}\, kg$ the mass of the sun and $R = 6.95\, 10^8\, m$ its mean radius. The value of the deviation is equal to:

$$\Phi = \frac{2\, \mathcal{G}\, M_{\odot}}{V_o^2\, R} \tag{19}$$

We note that the celerity of the Light, $c_0 = 2.99792458\, 10^8\, ms^{-1}$ is not present in this equation, which is natural in this simplified context. The celerity c_0 is of course absent from this result, but assuming that the initial velocity V_o is equal to the celerity of the light, we find $\Phi = 4.250\, 10^{-6}\, rd$ is $\Phi = 0.875''$ of arc. The value measured by Eddington in 1919 [2] was $\Phi = 1.75''$ arc or exactly double. The success of the theory of relativity was due mainly to the verification of Einstein's prediction established a few years before. Many other phenomena have been explained and verified since, for example the existence of black holes in the case of massive stars of small diameter; for the sun, the Schwarzschild radius R_s needed to capture light would be $R_s = 2\, \mathcal{G}\, M_{\odot}/c_0^2$ that is $R_s = 2.95\, km$, a value much lower than its radius.

This major result of general relativity is the opportunity to test discrete mechanics and its equation of motion (2) outside its usual field of application. The dual action hypothesis postulates that any scalar potential of physics ϕ is written as the sum of a gradient of the latter and a dual curl of a vector potential equal to $\boldsymbol{\psi} = \phi\, \mathbf{n}$.

The explanation proposed here on the problem posed is of a different nature: the deflection of light by the sun is due to a competition between the inertial and gravitational effects. A particle with a mass or massless has an inertia characterized by the inertial potential $\phi_i = \|\mathbf{V}\|^2/2$; if it is in a gravitational field it is subjected to an acceleration fixed by the potential ϕ_g. The light is deflected in a single direction contained in a plane passing through the center of the sun but with an intensity that is related to the mean curvature of the gravitational potential.

Consider two vectors $\mathbf{t}$ and $\mathbf{m}$ corresponding to the principal directions on an equipotential surface ϕ_g such that $\phi_g = Cte$. These two vectors are orthogonal by definition $\mathbf{t} \cdot \mathbf{m} = 0$ and we show in differential geometry that the mean curvature κ does not depend on the choice of $\mathbf{t}$ and $\mathbf{m}$ on the ϕ_g surface. The source term to integrate into the discrete motion equation (2) is written $\boldsymbol{h}_s = -\nabla \phi_g + \nabla \times \boldsymbol{\psi}_g$

At first order, if the velocity of the particle is always equal to the celerity in vacuum c_0, the deviation is written as the ratio of the gravitational and inertial mean curvatures:

$$\Phi \approx \frac{\kappa_g}{\kappa_i} = \frac{\| - \nabla \phi_g + \nabla \times \boldsymbol{\psi}_g \|}{\|\nabla \phi_i\|} = \frac{2\, \phi_g}{\phi_i} = \frac{4\, \mathcal{G}\, M_\odot}{c_o^2\, R} \tag{20}$$

This is the result predicted by A. Einstein, measured by Eddington in 1919 and confirmed by more recent experiments. The Eulerian vision of the movement of a particle in the vicinity of a massive body leads to explicitly taking into account the inertia independently of the mass of the latter. The result is consistent with that of a Lagrangian vision given elsewhere [1].

The concept of curvature of space-time introduced in general relativity makes it possible to explain the deflection of the light by the sun, replaced here by that of the mean curvatures of the gravitational and inertial potentials. If the result is the same, the formulation of the continuum equations of general relativity is much more complex and requires the use of geodesic calculations and curvature tensor. The problem dealt with in this section corresponds to an unsteady mechanical equilibrium due to the curved trajectory of the photon over a short period of time when it passes near the sun.

3.4 Interferences produced by two coherent light point sources

Application treated here is the case of interference produced by a coherent light source. This phenomenon taught in elementary physics courses are an opportunity to test the law (2) on observations made for centuries. The modeling of these observations does not require complex theories, but the objective here is to use this law without modification by including all the terms, including the inertial effects not present in undulatory optics. The case treated here are classical phenomenon described by the Maxwell equations and whose solutions can be obtained by analytical methods based on the resolution of a Helmholtz equation; the goal here is to use the system directly (2). In order to verify the robustness of this law, it will not be scaled. The times and lengths are characteristic of visible light and the calculations will be done in direct simulation.

Rays of visible light, such as acoustic waves produced by the same source and traveling on different paths, induce interfering fringes materialized by alternately light and dark bands for which the distances which separate them are easily measurable. This phenomenon is one of those that helped understand and model the wave nature of light.

Let us consider two monochromatic synchronous sources of frequency f placed in the planar surface (x, y) in $x_i = \pm d$ and $y = 0$. The areas of equal phase difference, the places of the points P for which the walking difference $\delta = |x_1 P - x_2 P|$ and the phase difference $\varphi = 2\, \pi\, \delta / \lambda$ are constant, are hyperbolas with x_i for foci. The surfaces for which $\delta = k\, \lambda$ ($k \in N$) are ventral surfaces and the surfaces for which $\delta = (2\, k + 1)\, (2\, \lambda)$ are nodal surfaces.

The purpose of this section is to show the results of an unsteady direct simulation of the ignition, from $t = t^o$, of two synchronous sources of visible light of wavelength λ and of frequency f such that $f = c_o / \lambda$. The simulation was performed without upscaling from the system of complete equations (2) including the inertial terms. The light is of wavelength $\lambda = 0.5\, 10^{-6}\, m$, the velocity of the medium (or of the vacuum) is taken equal to $c_0 = 3 10^8 m s^{-1}$ and the frequency equal to $f = 6\, 10^{14}\, s^{-1}$; the two distant sources of $2 \cdot d = 2 \cdot 6\, 10^{-7}\, m$ are of radius $r = 4\, 10^{-8}\, m$ so as to approximate them by a point ($r << d$). The chosen time step is $dt = 10^{-18}\, s$ and the Cartesian mesh is composed of 10^6 cells.

From the initial time at $t = t_o$, the sources emit perfectly cylindrical progressive waves which begin to interfere for a time equal to $t_1 = d/c_0$ ie. $t_1 = 2\,10^{-15}\,s$. The interferences then amplify in all the domain forming a series of stationary fringes which are the places of the nodes where the amplitude remains null. These surfaces appear in a surface as dark lines to an observer who does not distinguish the very fast temporal variations of the amplitudes of the ventral surfaces. In three dimensions of space, these surfaces become hyperboloid. The increase of the distance between the two sources $2 \cdot d$, keeping the same characteristics as those fixed, would make it possible to multiply the fringes of interference between them.

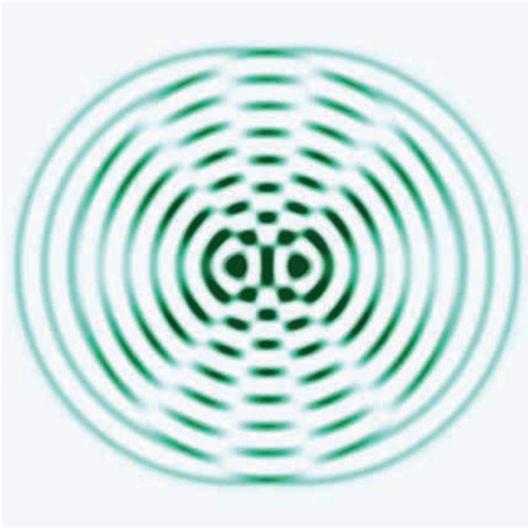

Figure 4. *Stationary interference fringes produced by two coherent light point sources: instantaneous wave field obtained by a direct simulation for a time $t = 1.4\,10^{-14}\,s$.*

The figure (4) shows the scalar potential field ϕ^o. The colored lines are progressive waves of wavelength λ; over time, they form fringes of stationary hyperbolic interference. The result is conventionally obtained by other techniques, especially from complex potentials. Here, the unsteady direct simulation is carried out from null fields. The system (2) is integrated without upscaling.

4 Conclusions

The disparate laws of physics developed by different authors for very specific fields have sometimes been unified, such as the laws of Ampére, Orsted, and Faraday which were unified within the equations of electromagnetism by Maxwell [4]. One reason for this is the multiplicity of physical quantities used and the units with which they are expressed. The choice by I. Newton to connect the force to the mass is not in contradiction with the intuitions of Galileo on the principle of equivalence, but at that time the mass could already be removed from his second law. This choice was not questioned by A. Einstein, who introduced the velocity dependence of the moving mass. Even today, what prevails is the conservation of momentum. The conservation of the mass is of course a legitimate principle which must be respected, but none of the cases treated above shows a result which depends on the proper mass of the particle or the material medium. Yet the theory of relativity is largely based on this quantity.

The discrete motion equation (2) was developed from physical models based on experiments. It allows us to find the most significant results from the Navier-Stokes and Navier-Lamé equations in fluid and solid mechanics. The emblematic cases of special and general relativity retained here, namely the acceleration of a stream of particles, of an isolated particle with or without mass and the deflection of light by a gravitational effect, show that the discrete equation yields the solutions sought without considering mass. We can of course affirm that the equation of discrete motion is relativistic, but the question of whether it is invariant by a Lorentz transformation does not arise directly; moreover, this transformation is not used and is replaced by conservation of kinetic energy. Indeed, the equation of motion (2) is a law of conservation of acceleration but also of the energy, which is its scalar potential. If the acceleration and the energy are conserved then the momentum and the mass are also conserved.

The difference in the formulations of relativity and of discrete mechanics raises a certain number of questions which will be analyzed in more detail in the future. One of them concerns the limitation of the velocity of light to its celerity in a vacuum c_0 imposed by the Lorentz factor. In fluid mechanics, the velocity can exceed the celerity of sound provided that the nozzle has a neck followed by a divergent, or that the profile of the airplane is properly studied. It is possible to relate this phenomenon to the local spatial curvature of the flow introduced by the increase in section of the divergent part of the nozzle or that of the profile of the wings of the aircraft. The transposition to light would suppose the existence of a gravitational field with very strong curvature. Like the "speed of sound", the "speed of light" is not a forbidden barrier.

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