

For-hire services in a duopoly market

The influence of availability on traffic equilibrium and market equilibrium

Jaafar BERRADA and Fabien LEURENT
VEDECOM/ LVMT



BACKGROUND

For-hire services are emerging

- Reduction of transaction times
 - Reduction of costs to both suppliers and customers
- ⇒ More than one service in the same territory, from several cities
- ⇒ A stream of research to optimize the matching between cabs and riders.

DESIGN ISSUES

- How does the competition between two for-hire operators influence the demand, and also each supplier?
- What are the conditions for profit maximization with respect to tariff and fleet size, under the influence of the competing service?

OBJECTIVE

To propose a **microeconomic analysis** of the **duopoly competition** between **two** for-hire services available in a given territory.

METHODOLOGY

- Based on Leurent (2017) Orbicity model of one cab service in a ring-shaped city => Emphasis on the Availability function
- Here, we adapt the availability function in order to incorporate the competition between services.
- A theoretical framework to depict the influences of supplier behaviors in the duopoly setting.

I. Demand-supply model for a for-hire service

1. Modeling assumptions
2. Supply functions
3. Demand function
4. Traffic and economic equilibrium

II. Duopoly market

1. Update of modeling assumptions
2. Market share formulation
3. Traffic equilibrium
4. Economic equilibrium

SUPPLY- DEMAND MODEL

Monopoly setting

- **Private taxis as vehicles**
- **Vehicle capacity = 1 seat**
- **Operational conditions**
- **Fleet size N**
- **Service speed v and travel time t_R**
- **Tariff fare τ**
- **Access time t_A**
- **Production process**
- **Each customer is assigned to the nearest available vehicle**
- **No detour**
- **Empty vehicles go on running**
- **Mobility demand**
- **Trip generation uniformly distributed along the ring and over time**
- **Demand volume Q during a time period H .**
- **Average trip length is related to ring area A .**
- **Individual values of time (access time and ride).**

The **access time** of a taxi service depends on the number of **available vehicles**, the number of **requests**, and the **occupation time** of taxis (ride time).

$$t_A = F_A(N, t_{IVH}, Q, v)$$

The production costs on a daily basis amounts to:

$$C(N, Q) = \chi(N) + c_u(t_A + t_R)Q$$

Wherein:

- C is the total production cost
- χ includes the costs of vehicle depreciation, driver wages and the cost of the transaction platform.
- c_u is the running cost per cab and per unit of time.

The **demand function** depends on the **quality of service** (access time and travel time) and on the **fare** τ .

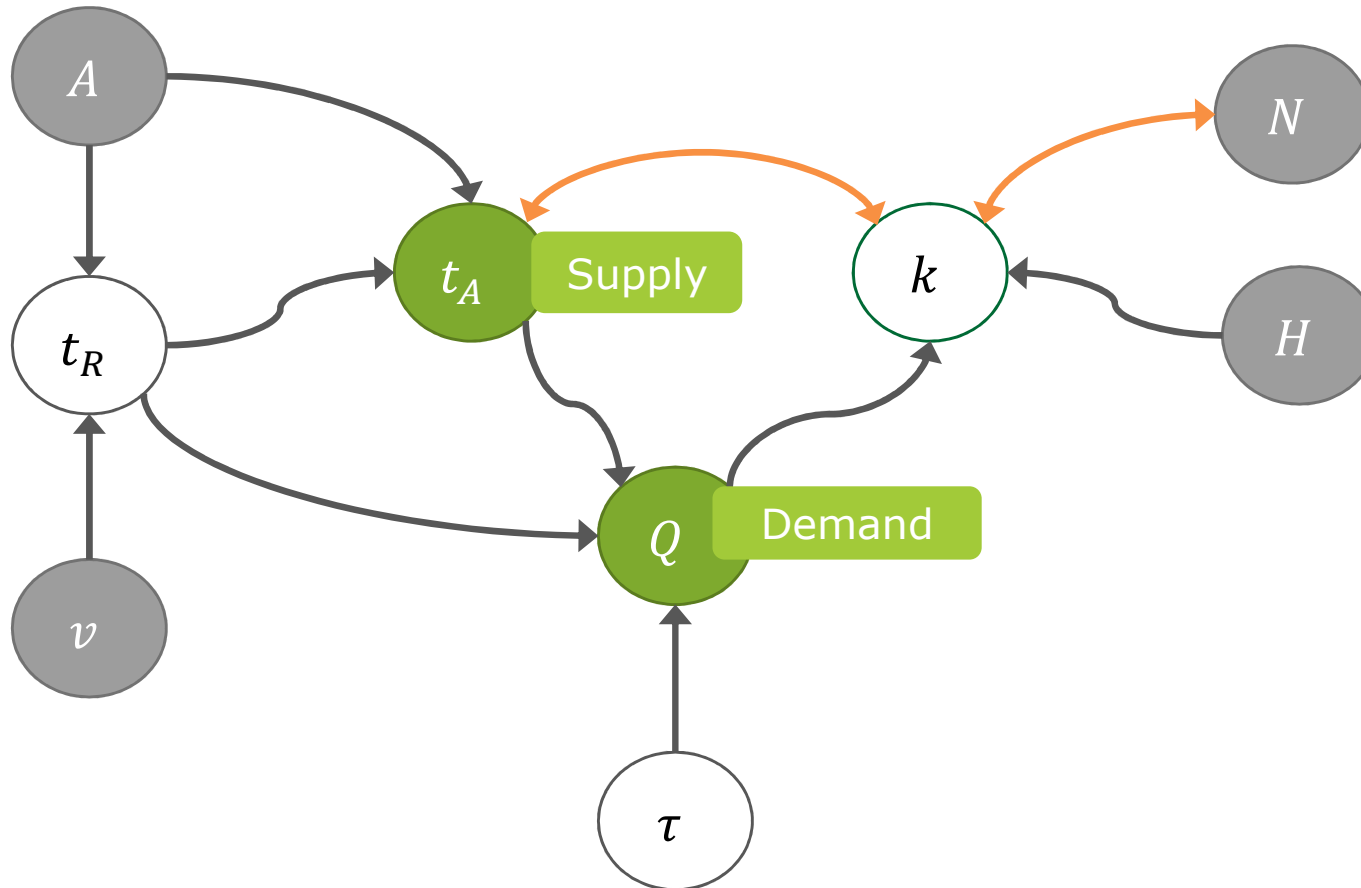
$$Q = D(\tau, t_A, t_R)$$

The **generalized cost** is considered as

$$g = \tau + \alpha(t_R + t_A)$$

We assume that $D(g) = Q_0 \left(\frac{g}{g_0} \right)^\epsilon$

where ϵ is the **elasticity** of demand with respect to the generalized cost



The availability function is established as function with respect to vacant cabs under the form:

$$t_A = \frac{t_0}{k} = \frac{t_0}{N - Q(t_R + t_A)/H}$$

wherein k is the number of vacant taxis and t_0 a coefficient depending on the area form.

(Douglas, 1972; De Vany, 1975; Cairns and Liston-Heyes, 1996; Yang et al., 2002, 2005, Leurent, 2017)

This imposes a condition on the fleet size:

$$N > Q/H \left(t_R + 2 \sqrt{\frac{HQ}{t_0}} \right)$$

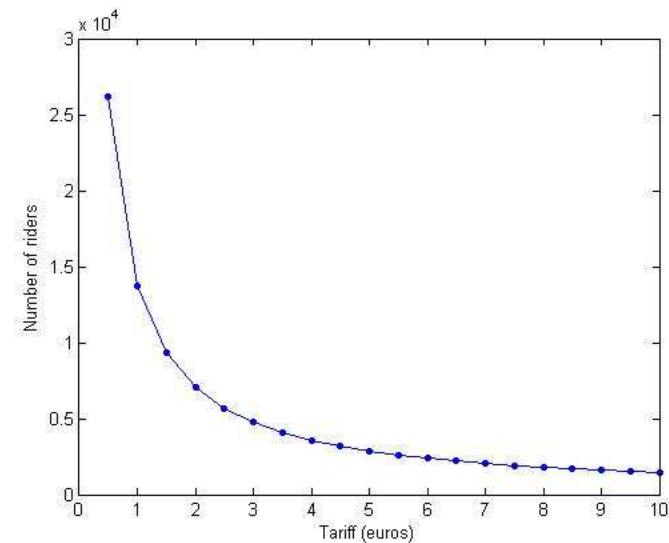
We considered from previous definitions of demand function and supply function that:

$$t_A = \frac{t_0}{k} = \frac{t_0}{N - Q(t_R + t_A)/H} \qquad Q = Q_0 \left(\frac{g}{g_0} \right)^\epsilon = Q_0 \left(\frac{\tau + \alpha(t_R + t_A)}{g_0} \right)^\epsilon$$

These conditions constitute a system of non-linear equations that can be solved using a fixed point algorithm

Effects of attributes on the demand combined function (e.g. tariff fare)

$$\frac{\partial \tilde{D}}{\partial x} = \frac{\frac{\partial D}{\partial x} + \frac{\partial D}{\partial t_A} \frac{\partial T}{\partial x}}{1 - \frac{\partial D}{\partial t_A} \frac{\partial T}{\partial Q}}$$



Equilibrium demand volume w.r.t. tariff

Supplier Behavior

The profit is expressed as $P(\tau, N, Q, t_R, t_A) = \tau Q - C(N, t_R, t_A, Q)$.

The maximization problem is then

$$\begin{aligned} & \max_{\tau, N} P(\tau, N, Q, t_R, t_A) \\ \text{s. t. } & t_A = T(N, Q, t_R) \\ & Q = D(\tau, t_A, t_R) \end{aligned}$$

Profit maximization problem

w.r.t. tariff

$$\tau = \left(\frac{\partial C}{\partial Q} + \frac{\partial C}{\partial t_A} \frac{\partial T}{\partial Q} \right) + Q / \frac{\partial \tilde{D}}{\partial \tau}$$

w.r.t. fleet size

$$\begin{aligned} \tau - \left(\frac{\partial C}{\partial Q} + \frac{\partial C}{\partial t_A} \frac{\partial T}{\partial Q} \right) &= \frac{\left(\frac{\partial C}{\partial N} + \frac{\partial C}{\partial t_A} \frac{\partial T}{\partial N} \right) \left(1 - \frac{\partial D}{\partial t_A} \frac{\partial T}{\partial Q} \right)}{\frac{\partial D}{\partial t_A} \frac{\partial T}{\partial N}} \end{aligned}$$

DUOPOLY

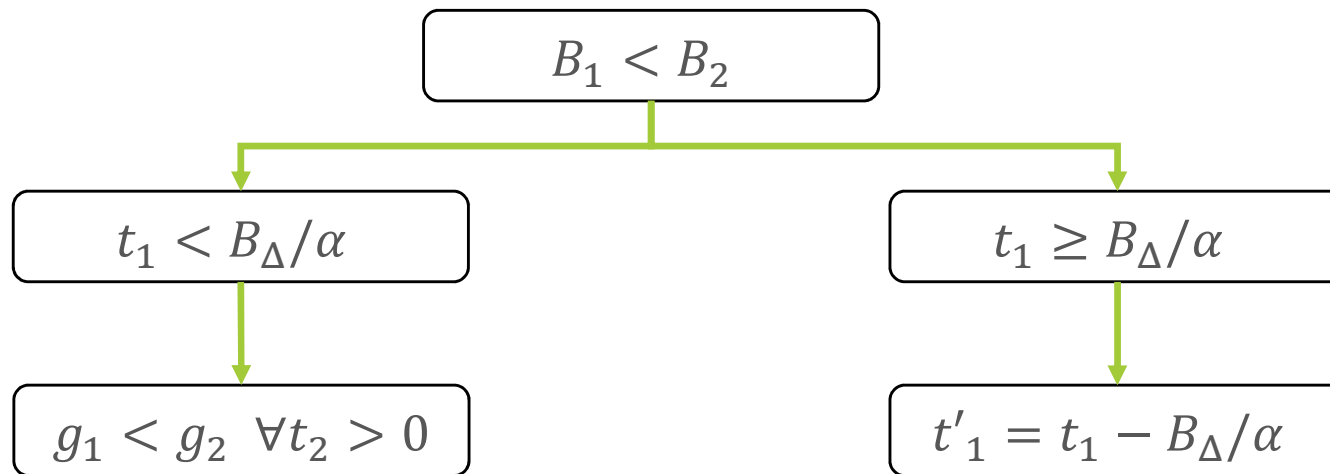
Duopoly setting

- **Two for-hire companies**
- **Services indexed by $i \in \{1, 2\}$**
- **Same operating periods H**
- **Operational conditions**
- **Fleet sizes N_1 and N_2**
- **Tariff fares τ_1 and τ_2**
- **Temporality of the service quality**
- **At instant h , each service has a particular availability time.**
- **Availability times t_1 and t_2 are assumed as independent Random Variables (RV)**
- **They have parameters λ_i and means θ_i**
- **Mobility demand**
- **Each user is a rational decision-maker and selects the service i with minimal generalized cost ($g_i \leq g_j$)**

The market share of service i aggregates the outcomes of all individual situations :

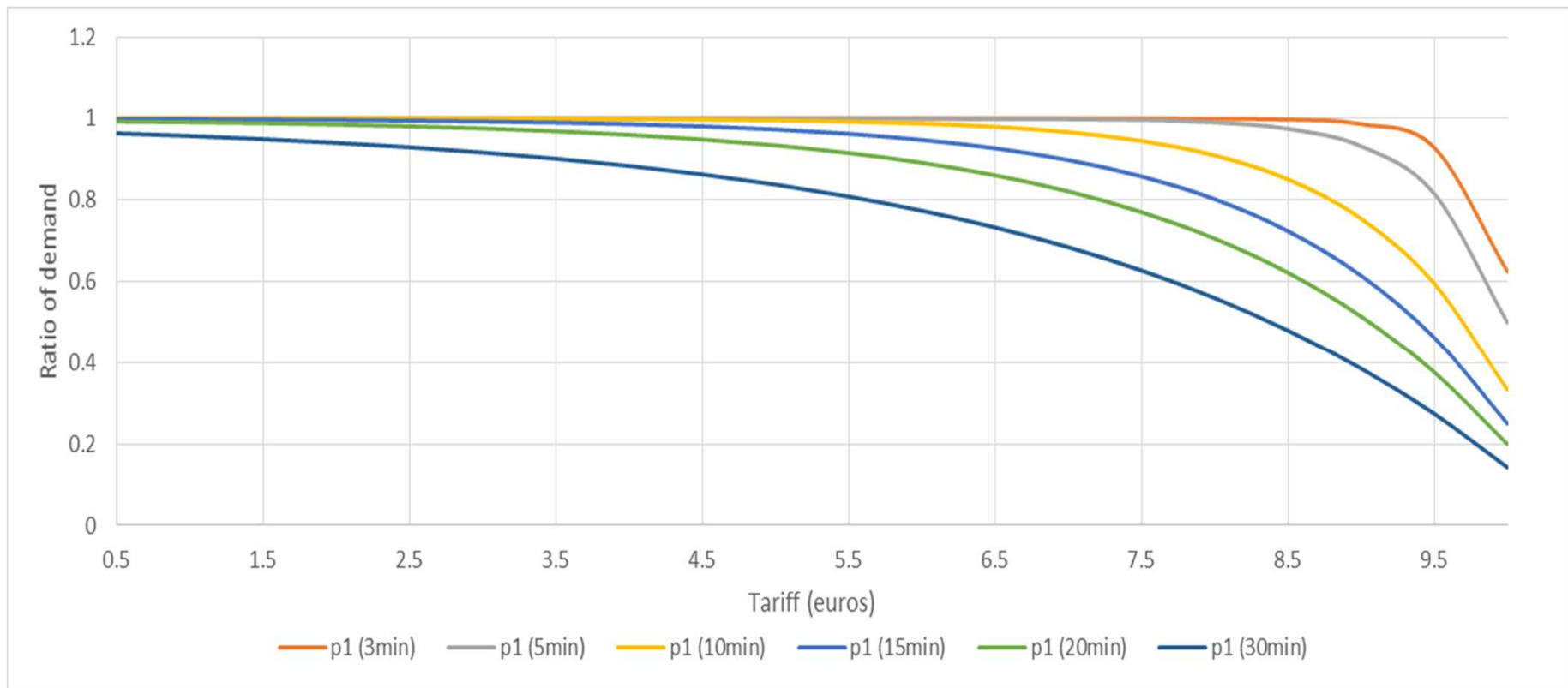
$$p_i = \Pr\{\text{service } i \text{ is selected}\} = \Pr\{g_i \leq g_j\}$$

Consider $B_i = \tau_i + \alpha t_R$ and $B_\Delta = B_2 - B_1$



$$\Pr(t_1 < B_\Delta/\alpha) = 1 - \exp(- B_\Delta/\alpha\theta_1)$$

$$\Pr(t_1 \geq B_\Delta/\alpha) = \frac{\theta_2}{\theta_1 + \theta_2} \exp(- B_\Delta/\alpha\theta_1)$$



Market share of service 1 w.r.t. its tariff for several availability times of it

Availability times are in turn affected by the choice of users.

If service 1 selected:

Service 2

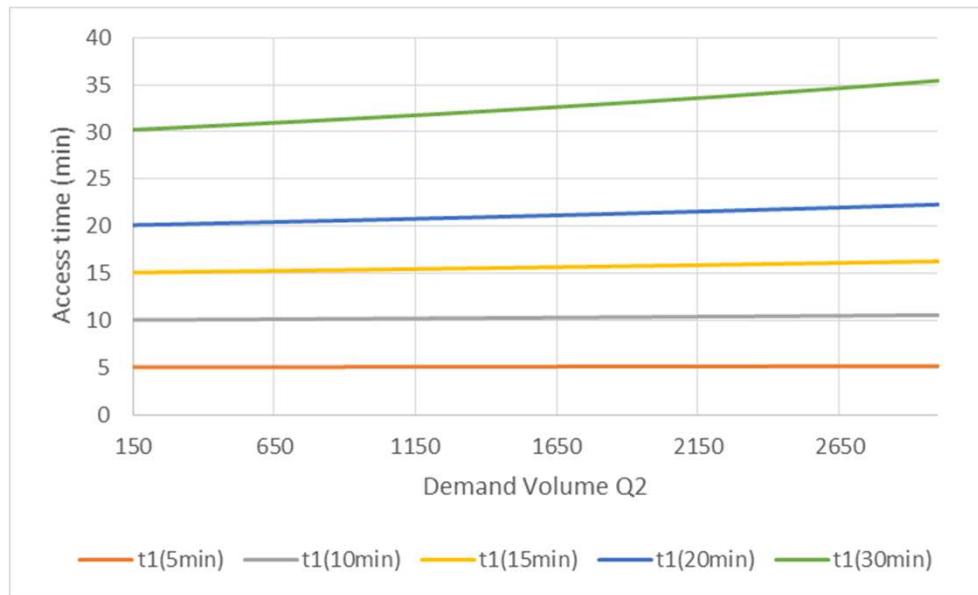
$$\theta'_2 = \frac{\theta_1 \theta_2}{\theta_1 + \theta_2} \rightarrow \theta_2 = \frac{t_0}{N_2 - Q_2(t_{R_2} + \frac{\theta_1 \theta_2}{\theta_1 + \theta_2})/H}$$

Service 1

$$\theta'_1 = \left(1 - \exp\left(-\frac{B_\Delta}{\alpha \theta_1}\right)\right) \left(\theta_1 - \frac{B_\Delta/\alpha}{1 - \exp\left(-\frac{B_\Delta}{\alpha \theta_1}\right)}\right) + \frac{\theta_1 \theta_2}{\theta_1 + \theta_2} \exp\left(-\frac{B_\Delta}{\alpha \theta_1}\right)$$

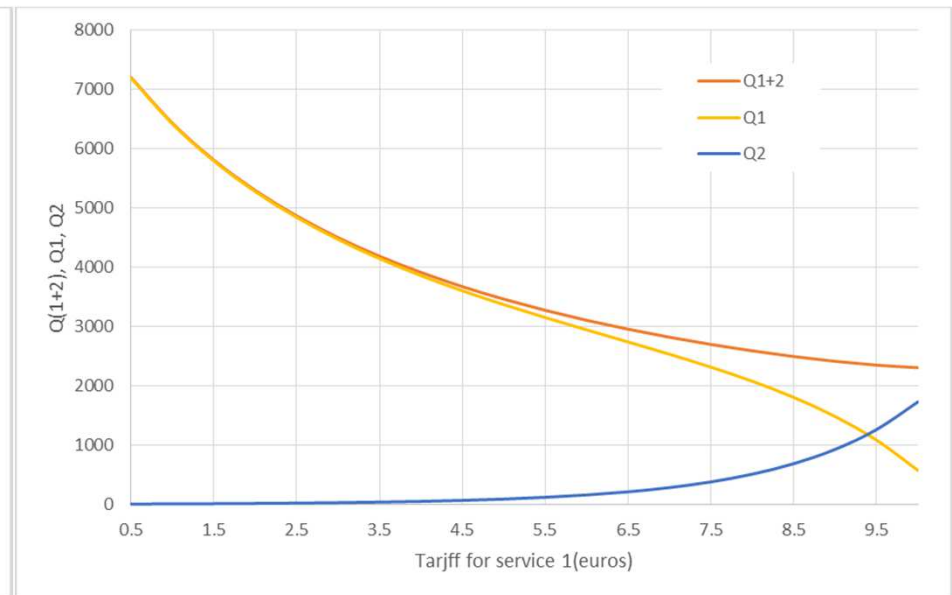
$$\theta_i = T_i(N_i, t_{R_i}, Q_i, \tau_i, t_{R_j}, \tau_j, \theta_j)$$

Fixed demand



Availability time of service 2 w.r.t. service demand volume for several values of θ_1

Variable demand



Traffic equilibrium with variable demand w.r.t. demand volumes

Cost function

$$C(N_i, Q_i, \theta'_i) = \chi(N_i) + c_u(\theta'_i + t_{R_i})Q_i$$

Supplier Behavior

The new profit maximization problem is written as

$$\begin{aligned} & \max_{\tau, N} P_i(\tau_i, \tau_j, N_i, Q_i, \theta_i, \theta_j) \\ \text{s. t. } & \theta_i = T_i(N_i, Q_i, t_{R_i}, \tau_i, \theta_j) \\ & Q_i = D_i(\tau_i, \theta_i, t_{R_i}, \tau_j, \theta_j, t_{R_j}) \end{aligned}$$

Profit maximization problem

w.r.t. tariff

$$\tau_i = \left(\frac{\partial C_i}{\partial Q_i} + \frac{\partial C_i}{\partial \theta_i} \frac{\partial T_i}{\partial Q_i} \right) + Q_i / \frac{\partial \widetilde{D}_i}{\partial \tau_i}$$

w.r.t. fleet size

$$\begin{aligned} \tau_i - \left(\frac{\partial C_i}{\partial Q_i} + \frac{\partial C_i}{\partial \theta_i} \frac{\partial T_i}{\partial Q_i} \right) = & \\ \left(\frac{\partial C_i}{\partial N_i} + \frac{\partial C_i}{\partial \theta_i} \frac{\partial T_i}{\partial N_i} \right) \left(1 - \frac{\partial D_i}{\partial \theta_i} \frac{\partial T_i}{\partial Q_i} \right) / & \frac{\partial D_i}{\partial \theta_i} \frac{\partial T_i}{\partial N_i} \end{aligned}$$

- The availability is a major factor of service quality and then of demand choice from among competing services.
- The joint availability of alternatives services yields benefits to users.
- The influences of the competition on the production processes are traced out in the issues of traffic and economic equilibrium.