

Event-triggered control of Korteweg-de Vries equation under averaged measurements

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Abstract

This work addresses distributed event-triggered control law of 1-D nonlinear Korteweg-de Vries (KdV) equation posed on a bounded domain. Such a system, in a continuous framework, is exponentially stabilizable by a linear state feedback as a source term. Here we consider the situation where the feedback is sampled in time and piecewise averaged in space, and an event-triggering mechanism is designed to maintain stability of this infinite dimensional system. Both well-posedness of the closed-loop system and avoiding the Zeno behaviour issues are addressed. Sufficient LMI-based conditions are constructed to guarantee the regional exponential stability. A numerical example illustrates the efficiency of the method.

Key words: Korteweg-de Vries equation, event-trigger, LMIs.

1 Introduction

In fluid mechanics, the Korteweg-de Vries (KdV) equation is a mathematical model of waves on shallow water surfaces in a rectangular channel, equation in which the effects of dispersion, dissipation and non-linearity are taken into account. When adding a diffusion term, the KdV equation becomes Korteweg-de Vries Burgers (KdVB) equation. The study of KdV/ KdVB systems has been an active research topic because of its potential applications, see e.g. [2–4,6,14,16]. In the field of automatic control, a backstepping approach has been applied in [4,6,16] for the feedback stabilization of KdV equation, and Lyapunov-based arguments have been employed to ensure the stability of the original system under the proposed control law. On the other hand, the survey paper [3] gives a detailed overview of bound-

ary controllability and internal stabilization approaches and results for the KdV equation. One can read in [2] two different approaches (from a Lyapunov functional or from an observability inequality) employed to exponentially stabilize the nonlinear KdV equation via delayed boundary damping terms.

In [14], distributed control of KdVB system has been suggested under point or averaged localised measurements in space but the proof rely strongly on the presence of a diffusion term that is missing in the KdV equation. Such distributed control was introduced for heat equation under point [9] and under averaged [10] measurements. In the latter papers, sampled-data control via time-delay approach and Lyapunov-Krasovskii functionals were studied, and the results of [9,10] were extended to event-triggered control in [23]. However, since the Lyapunov-Krasovskii functionals for sampled-data control depend on the state-derivative (see Chapter 7 of [11]), this method cannot be applied to sampled-data control of KdVB equation. So [14] considered the constant input delay case.

To the best of our knowledge, no event-triggered control of KdV equation has been studied yet. The goal of event-triggering mechanism to a sampled control law is to update the control input only at meaningful instants.

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ness and regularity of the solution. We proceed by induction.

(i) *Initialization.* On the first time interval, (3.1) reads

$$\begin{cases} \partial_t z + z \partial_x z + \partial_x z + \alpha \partial_{xxx} z - \lambda z = -K \sum_{j=1}^N \bar{z}_j^0 \mathbb{1}_{\Omega_j}(x), \\ \forall x \in (0, L), t \in [0, t_1], \\ z(0, t) = z(L, t) = 0, \partial_x z(L, t) = 0, \quad \forall t \geq 0, \\ z(x, 0) = z^0(x), \quad \forall x \in (0, L), \end{cases}$$

where $\bar{z}_j^0 = \bar{z}_j(0) = \frac{1}{|\Omega_j|} \int_{\Omega_j} z^0(x) dx$, and $K > \lambda$. This is a nonlinear KdV equation with initial data $z^0 \in L^2(0, L)$

and source term $f = -K \sum_{j=1}^N \bar{z}_j^0 \mathbb{1}_{\Omega_j} \in L^1(0, t_1; L^2(0, L))$.

Lemma 1 allows to conclude that there exists a unique solution $z \in C([0, t_1]; L^2(0, L)) \cap L^2(0, t_1; H_0^1(0, L))$ to the latter system.

(ii) *Heredity.* Let us only highlight that the previously obtained solution satisfies $z(t_1) \in L^2(0, L)$ so that system (3.1) considered on the next time interval $[t_1, t_2[$ has an initial condition $z(t_1) \in L^2(0, L)$ and a source term $-K \sum_{j=1}^N \bar{z}_j^1 \mathbb{1}_{\Omega_j} \in L^1(t_1, t_2; L^2(0, L))$ where $\bar{z}_j^1 = \bar{z}_j(t_1)$.

Therefore, the same argument using Lemma 1 holds again and the heredity is proved similarly at any step $k \in \mathbb{N}$.

(iii) *Conclusion.* By induction, for any $k \in \mathbb{N}$, $z \in C([t_k, t_{k+1}]; L^2(0, L)) \cap L^2(t_k, t_{k+1}; H^1(0, L))$. Therefore, from the extension by continuity at the instants t_k , one can conclude that (3.1) has a unique solution $z \in C([0, T]; L^2(0, L)) \cap L^2(0, T; H^1(0, L))$.

We aim at showing that the event-triggering mechanism (3.4) rules out the Zeno behaviour, where an infinite number of updates may occur in a finite amount of time. It is actually sufficient to show that for a given $T > 0$, there exists $\tau^* > 0$ such that all the sampling instants $t_k \leq T$ complying to (3.4) satisfy $t_{k+1} - t_k \geq \tau^*$.

Let us denote by e_k the deviation from the continuous time position: for any $x \in [0, L]$ and $t \in (0, T)$, there exists $k \in \mathbb{N}$ such that $t \in [t_k, t_{k+1})$, and we set

$$e_k(x, t) \triangleq z(x, t) - z(x, t_k). \quad (4.1)$$

Since the solution of closed-loop system (3.1) satisfies $z \in C([0, T]; L^2(0, L))$ and $[0, T]$ is a compact set, this error function e_k is uniformly continuous in time with values in $L^2(0, L)$. This means that for any $\epsilon > 0$ there exists $\tau^* > 0$ such that for all $t, s \in [0, T]$, if $|t - s| < \tau^*$ then we have $\|e_k(\cdot, t) - e_k(\cdot, s)\|_{L^2(0, L)} < \epsilon$.

Thus, the following reasoning by contraposition holds: $\forall \epsilon > 0, \exists \tau^* > 0, \forall t, s \in [0, T]$,

$$\|e_k(\cdot, t) - e_k(\cdot, s)\|_{L^2(0, L)} \geq \epsilon \implies |t - s| \geq \tau^*. \quad (4.2)$$

Since $e_k(t_k) = 0$, we have

$$\|e_k(\cdot, t_{k+1}) - e_k(\cdot, t_k)\|_{L^2(0, L)} = \|e_k(\cdot, t_{k+1})\|_{L^2(0, L)}.$$

Next the substitution $t \rightarrow t_{k+1}$ and $s \rightarrow t_k$ into (4.2), together with the definition of t_{k+1} in (3.4), leads to

$$\begin{aligned} \|e_k(\cdot, t_{k+1})\|_{L^2(0, L)} &\geq \gamma E(t_{k+1}) + \gamma_0 E(0) e^{-2\theta t_{k+1}} \\ &\geq \gamma_0 \|z^0\|_{L^2(0, L)}^2 e^{-2\theta T} \end{aligned}$$

implying that $|t_{k+1} - t_k| \geq \tau^*$. Indeed, given $z^0 \neq 0$, we choose $\epsilon = \gamma_0 \|z^0\|_{L^2(0, L)}^2 e^{-2\theta T}$ so that there exists $\tau^* > 0$, depending on $z^0, \theta, \gamma_0, \gamma$ and T for which for any k such that $t_k, t_{k+1} \in [0, T]$, one has $t_{k+1} - t_k \geq \tau^*$, so that the Zeno behaviour is avoided. \square

4.2 Regional stability analysis

Now we focus on the regional stability analysis of the closed-loop system and prove Theorem 1. Let us mention two things.

On the one hand, the event-triggering mechanism (3.4) yields that the event-triggering error function is bounded on each time sub-interval as follows: $\forall t \in [t_k, t_{k+1})$

$$\|e_k(\cdot, t)\|_{L^2(0, L)}^2 \leq \gamma E(t) + \gamma_0 E(0) e^{-2\theta t}. \quad (4.3)$$

On the other hand, $f(x, t)$ defined by (2.3) can be rewritten as

$$\begin{aligned} f(x, t) &= -K \sum_{j=1}^N \mathbb{1}_{\Omega_j}(x) [z(x, t) - f_j(x, t) - \rho_j(t)], \\ \forall x \in [0, L], \forall t \in [t_k, t_{k+1}), \forall k \in \mathbb{N} \end{aligned} \quad (4.4)$$

where

$$\begin{aligned} f_j(x, t) &= z(x, t) - \bar{z}_j(t) = z(x, t) - \frac{1}{|\Omega_j|} \int_{\Omega_j} z(x, t) dx, \\ \rho_j(t) &= \bar{z}_j(t) - \bar{z}_j(t_k) = \frac{1}{|\Omega_j|} \int_{\Omega_j} e_k(x, t) dx. \end{aligned}$$

Proof of Theorem 1

Writing $V_\mu(t) = \mu \int_0^L x |z(x, t)|^2 dx$ for $\mu > 0$, we define the following functional (using simplified notations [2]):

$$V(t) \triangleq E(t) + V_\mu(t) = \int_0^L |z(x, t)|^2 dx + \mu \int_0^L x |z(x, t)|^2 dx. \quad (4.5)$$

First, this Lyapunov functional candidate $V(t)$ is equivalent to the energy of the system $E(t)$ in the sense that

$$E(t) \leq V(t) \leq (1 + \mu L)E(t). \quad (4.6)$$

Then, let us estimate its time derivative. For $t \in [t_k, t_{k+1})$, substituting (4.4) into (3.1) and differentiating $V(t)$ along (3.1), one gets

$$\begin{aligned} \dot{V}(t) &= \dot{E}(t) + \dot{V}_\mu(t) \\ &= 2 \int_0^L z(x, t) \partial_t z(x, t) dx + 2\mu \int_0^L x z(x, t) \partial_t z(x, t) dx \\ &= 2 \int_0^L (1 + \mu x) z(x, t) [-\alpha \partial_{xxx} z(x, t) - z(x, t) \partial_x z(x, t) \\ &\quad - \partial_x z(x, t) + \lambda z(x, t) - K z(x, t)] dx \\ &\quad + 2K \sum_{j=1}^N \int_{\Omega_j} (1 + \mu x) z(x, t) [f_j(x, t) + \rho_j(t)] dx. \end{aligned}$$

Hence,

$$\begin{aligned} \dot{V}(t) &= -\alpha |\partial_x z(0, t)|^2 - 3\mu\alpha \int_0^L |\partial_x z(x, t)|^2 dx \\ &\quad + \mu \int_0^L |z(x, t)|^2 dx + \frac{2}{3}\mu \int_0^L z^3(x, t) dx \\ &\quad - 2(K - \lambda) \int_0^L (1 + \mu x) |z(x, t)|^2 dx \\ &\quad + 2K \sum_{j=1}^N \int_{\Omega_j} (1 + \mu x) z(x, t) [f_j(x, t) + \rho_j(t)] dx. \end{aligned}$$

Using (4.3) we can deduce that

$$\begin{aligned} \dot{V}(t) &\leq \dot{V}(t) + [\gamma E(t) + \gamma_0 E(0) e^{-2\theta t} - \|e_k(\cdot, t)\|_{L^2(0, L)}^2] \\ &\leq -3\mu\alpha \int_0^L |\partial_x z(x, t)|^2 dx - (2K - \mu - \gamma - 2\lambda) \int_0^L |z(x, t)|^2 dx \\ &\quad + \frac{2}{3}\mu \int_0^L z^3(x, t) dx - (2K - 2\lambda)\mu \int_0^L x |z(x, t)|^2 dx \\ &\quad + 2K \sum_{j=1}^N \int_{\Omega_j} (1 + \mu x) z(x, t) [f_j(x, t) + \rho_j(t)] dx \\ &\quad + \gamma_0 E(0) e^{-2\theta t} - \|e_k(\cdot, t)\|_{L^2(0, L)}^2. \end{aligned} \quad (4.7)$$

Several estimates can now be obtained to deal with each of these terms and bring this into a quadratic form. First, Cauchy-Schwarz inequality and Sobolev's inequality (see Lemma A.3) leads to

$$\begin{aligned} \int_0^L z^3(x, t) dx &\leq \|z(\cdot, t)\|_{L^\infty(0, L)}^2 \int_0^L |z(x, t)| dx \\ &\leq L\sqrt{L} \|\partial_x z(\cdot, t)\|_{L^2(0, L)} \|z(\cdot, t)\|_{L^2(0, L)} \end{aligned} \quad (4.8)$$

Then from Lemma A.4, Wirtinger's inequality yields

$$\|z(\cdot, t)\|_{L^2(0, L)}^2 \leq \frac{L^2}{\pi^2} \|\partial_x z(\cdot, t)\|_{L^2(0, L)}^2.$$

Thus, for any $\lambda_1 > 0$,

$$\lambda_1 \left[\|\partial_x z(\cdot, t)\|_{L^2(0, L)}^2 - \frac{\pi^2}{L^2} \|z(\cdot, t)\|_{L^2(0, L)}^2 \right] \geq 0. \quad (4.9)$$

Moreover, since $\int_{\Omega_j} f_j(x, t) dx = 0$, from Lemma A.5, Poincaré's inequality rewrites

$$\|f_j(\cdot, t)\|_{L^2(\Omega_j)}^2 \leq \frac{\Delta^2}{\pi^2} \|\partial_x z(\cdot, t)\|_{L^2(\Omega_j)}^2,$$

bringing for any $\lambda_2 > 0$

$$\lambda_2 \sum_{j=1}^N \left[\|\partial_x z(\cdot, t)\|_{L^2(\Omega_j)}^2 - \frac{\pi^2}{\Delta^2} \|f_j(\cdot, t)\|_{L^2(\Omega_j)}^2 \right] \geq 0. \quad (4.10)$$

Applying the Cauchy-Schwarz inequality, we obtain

$$\begin{aligned} \sum_{j=1}^N \int_{\Omega_j} \rho_j^2(t) &= \sum_{j=1}^N \rho_j^2(t) |\Omega_j| \leq \sum_{j=1}^N \frac{1}{|\Omega_j|} \left(\int_{\Omega_j} e_k(x, t) dx \right)^2 \\ &\leq \sum_{j=1}^N \int_{\Omega_j} e_k^2(x, t) dx = \int_0^L e_k^2(x, t) dx. \end{aligned} \quad (4.11)$$

Set

$$\eta(x, t) = \text{col}\{z(x, t), f_j(x, t), \rho_j(t)\}.$$

Substituting (4.8) and (4.11) into (4.7), and adding (4.9) and (4.10) to $\dot{V}(t)$, we obtain

$$\begin{aligned} \dot{V}(t) + 2\delta V(t) &\leq \sum_{j=1}^N \int_{\Omega_j} \eta(t)^\top \Phi(x) \eta(t) + \gamma_0 E(0) e^{-2\theta t} \\ &\quad - \left(3\mu\alpha - \lambda_1 - \lambda_2 - \frac{2\mu}{3} L\sqrt{L} \|z(\cdot, t)\|_{L^2(0, L)} \right) \|\partial_x z(\cdot, t)\|_{L^2(0, L)}^2 \\ &\quad - 2(K - \lambda - \delta)\mu \int_0^L x |z(x, t)|^2 dx, \end{aligned} \quad (4.12)$$

where

$$\Phi(x) = \begin{bmatrix} \phi_{11} & K(1 + \mu x) & K(1 + \mu x) \\ * & -\lambda_2 \frac{\pi^2}{\Delta^2} & 0 \\ * & * & -1 \end{bmatrix}$$

and $\phi_{11} = -2K + 2\lambda + \mu + \gamma - \lambda_1 \frac{\pi^2}{L^2} + 2\delta$ as in (3.7).

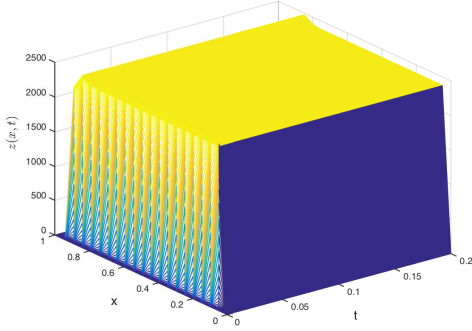


Fig. 1. State of the open-loop system with $\lambda = 0.5$

For the event-triggered control law (2.3) under averaged measurements, we verify LMI conditions of Theorem 1 with $K = L = 1$, $\lambda = 0.5$, $\alpha = 0.3$, $\delta = 0.4$, $\Delta = 0.1$, $R = 0.5$. We find that the closed-loop system under event-triggering mechanism (3.4) with $\theta = 2$, $\gamma = 0.00029$ and $\gamma_0 = 0.02$ is exponentially stable for $\mu = 0.5401$ and for any initial values satisfying $\|z^0\|_{L^2(0,1)} < \frac{1}{\sqrt{6.1997}} \approx 0.4$.

A finite difference method is used to illustrate the effect of the proposed event-triggered control law. The steps of space and time are chosen as 0.05 and 0.0001, respectively. Fig. 1 illustrates the evolution of the state of the open-loop KdV system. It is seen that the open-loop system is unstable. Fig. 2 illustrates the evolution of the state of the closed-loop KdV system under the event-triggering mechanism

$$t_{k+1} = \inf \left\{ t \geq t_k \mid \|e_k(\cdot, t)\|_{L^2(0,L)}^2 \geq 0.00029 \cdot E(t) + 0.02E(0)e^{-4t} \right\}.$$

with the control law (2.3) where

$$\bar{z}_j(t_k) = 10 \int_{\Omega_j} z(\zeta, t_k) d\zeta, \quad t \in [t_k, t_{k+1})$$

subject to $x_j - x_{j-1} = |\Omega_j| = \Delta = 0.1$. It shows that the state of closed-loop KdV system under the event-triggered controller converges exponentially to zero. Fig. 3 shows that the release time and release interval by event-triggering for $t \in [0, 0.2]$. Fig. 4 demonstrates the time evolution of $\ln(E(t))$ for the open-loop system, the closed-loop system under continuous-time controller, and the closed-loop system under the event-triggered controller. The simulations show that the event-triggered controller improves the performance.

6 Conclusion

The present work discusses event-triggered control of the nonlinear KdV equation. An event-triggering mech-

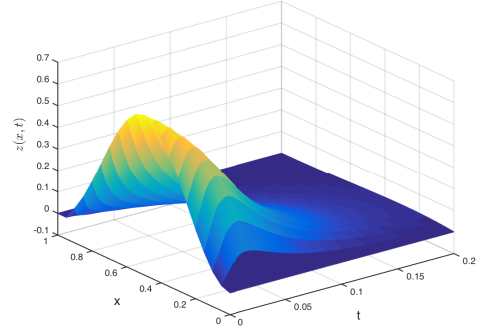


Fig. 2. State of the closed-loop system with the event-triggered control law under averaged measurements

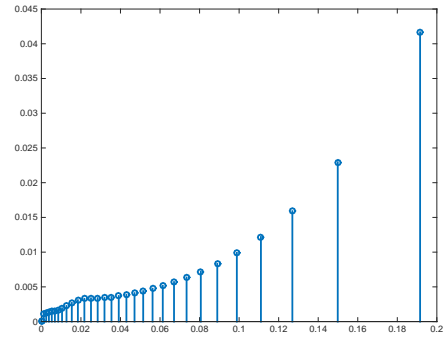


Fig. 3. Release instants and release interval by event-triggering

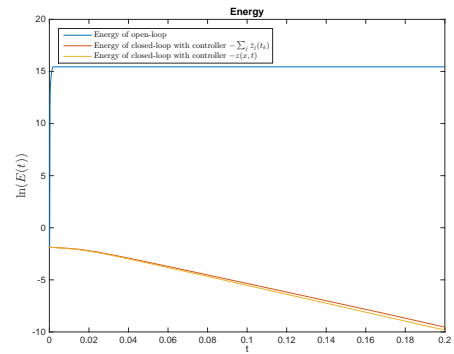


Fig. 4. $\ln(E(t))$ of the open-loop system, closed-loop system under continuous-time/event-triggered controller

anism has been proposed to reduce the number of control update. By constructing an appropriate Lyapunov functional, sufficient LMI-based conditions have been investigated while ensuring that the closed-loop system is regionally exponentially stable. The avoidance of Zeno behaviour is guaranteed. The presented method gives efficient tools for various event-triggered controller and observer design problems for nonlinear PDEs.

Appendix A

Lemma A.3 (Sobolev embedding and inequality). *The embedding $H^1(0, L) \subset C([0, L])$ is compact and for any $g \in H_0^1(0, L)$, it holds $\|g\|_{L^\infty(0, L)} \leq L\|g'\|_{L^2(0, L)}$.*

Lemma A.4 (Wirtinger inequality [12]). *Assume that $g \in H^1(0, L)$ with $g(a) = 0$ or $g(b) = 0$. Then*

$$\|g\|_{L^2(0, L)}^2 \leq \frac{4L^2}{\pi^2} \|g'\|_{L^2(0, L)}^2.$$

Moreover, if $g \in H_0^1(0, L)$, then

$$\|g\|_{L^2(0, L)}^2 \leq \frac{L^2}{\pi^2} \|g'\|_{L^2(0, L)}^2.$$

Lemma A.5 (Poincaré inequality [17]). *Assume that $g \in H^1(0, L)$ with $\int_0^L g(x)dx = 0$. Then*

$$\|g\|_{L^2(0, L)}^2 \leq \frac{L^2}{\pi^2} \|g'\|_{L^2(0, L)}^2.$$

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