Low-density in-medium effects on light clusters from heavy-ion data

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A new low-density nuclear matter equation of state from an experimental data analysis including in-medium effects

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The modification of the ground state properties of light atomic nuclei in the nuclear and stellar medium is addressed, using chemical equilibrium constants evaluated from a new analysis of the intermediate energy heavy-ion (Xe+Sn) collision data measured by the INDRA collaboration. Three different reactions are considered, mainly differing by the isotopic content of the emission source. The thermodynamic conditions of the data samples are extracted from the measured multiplicities allowing for an in-medium correction, with the single hypothesis that the different nuclear species in a given sample correspond to a unique common value for the density of the expanding source. We show that this new correction, which was not considered in previous analyses of chemical constants from heavy ion collisions, is necessary, since the observables of the analysed systems show strong deviations from the expected results for an ideal gas of clusters. This experimental data set is further compared to a relativistic mean-field model, and seen to be reasonably compatible with a universal correction of the attractive σ-meson coupling.

Nuclear clusters are expected to be ubiquitous in warm and low density nuclear matter [1–5], and to affect the neutrino mean free path, with important possible consequences on different astrophysical phenomena such as the core-collapse supernova explosion dynamics, the subsequent cooling of proto-neutron stars, and the initial conditions for r-process nucleosynthesis in those explosive events, just to mention a few. A reliable estimation of the cluster abundancies in the different thermodynamic conditions is therefore needed in the hydrodynamical calculations of supernova and merger dynamics, and in the cooling simulations of proto-neutron stars [6].

The problem of evaluating those abundances arises from the fact that their ground state properties are expected to be modified in a dense medium. Mass shifts arising from in-medium correlations were calculated in the framework of quantum-statistical approaches, but only in a limited density domain, and for a limited number of nuclear species [7]. For this reason, phenomenological models were developed, where the interactions with the medium are governed by coupling constants that must be fixed through comparison with experiment [8, 9].

Very recently [10], the INDRA collaboration presented new sets of data, completing the unique constraint which was previously available from NIMROD data [11]. However, as already observed in Ref. [10], the weak point of both experimental works is that the thermodynamic parameters, in particular the baryon density ρ_B and the temperature T, are not directly measured, but they are deduced from the experimental multiplicities using analytical expressions that explicitly assume that the physical system under study can be modeled as an ideal gas of clusters. This is in contradiction with the very purpose of the analysis, which is to extract the in-medium modifications with respect to the ideal gas limit.

In this Letter, we propose to solve this methodological inconsistency by modifying the ideal gas expressions relating the thermodynamical parameters to cluster yields. This correction is estimated using Bayesian techniques, under the unique condition that, for an experimental sample corresponding to a thermodynamic equilibrium, the volume associated to the thermal motion of each cluster species should be the same.

The new chemical constants evaluated from the INDRA data [10] using this improved data analysis technique are then compared to the relativistic mean field model of Ref. [9], in order to extract the in-medium modifications. We show that a single parameter, expressing a universal reduction of the scalar attractive field to the nucleons bound in clustered states, can be tuned so as to obtain a reasonably good description of the chemical constants. The suppression effect is smaller than the one obtained from the comparison to the experimental data of Ref. [11], where ideal gas expressions were used to extract the thermodynamical parameters, but still corresponds to important in-medium modifications of the binding energies, with important potential effects on astrophysical observables.

Under well-defined thermodynamic conditions, as given by the temperature T, total baryon density ρ_B and proton fraction y_p, equilibrium chemical constants $K_c(A,Z)$ of a cluster of mass (charge) number A (Z), are defined in terms of particle densities ρ_{AZ}, or of mass fractions ω_{AZ} as :

$$K_c(A,Z) = \frac{\rho_{AZ}}{\rho_{11}^{Z}\rho_{10}^{A-Z}} = \frac{\omega_{AZ}}{\omega_{11}^{Z}\omega_{10}^{A-Z}}\rho_B^{-(A-1)} . \quad (1)$$

An experimental measurement of such constants requires the detection of particles and clusters from a statistical ensemble of sources, and an estimation of the associated thermodynamic parameters (T, ρ_B, y_p).

Under the assumption that chemical equilibrium holds at the different time steps of the emission from the expanding source produced in central ^{136,124}Xe+^{124,112}Sn collisions detected with the INDRA apparatus [10], the Coulomb corrected particle velocity v_{surf} in the source frame can be used to select statistical ensembles of particles corresponding to different emission times, and therefore different thermodynamic conditions [11]. A detailed comparison between the four different reactions was performed in Ref. [12], verifying the statistical character of the emission. A strong argument confirming the crucial hypothesis of chemical equilibrium as a function of time was given in Ref. [10], observing that the extracted thermodynamic parameters as a function of v_{surf} are independent of the entrance channel of the reaction.

The detected multiplicities $Y_{AZ}(v_{surf})$ allow a direct experimental determination of the mass fractions as well as of the total source mass $A_T(t)$ as a function of the emission time, but the measurement of the baryonic density $\rho_B(t) = A_T/V_T$ additionally requires an estimation of the source volume, at the different times of the expansion. This latter is given by the free volume V_f with the addition of the proper volume V_{AZ} of the clusters which belong to the source at a given time, $V_T = V_f + \sum_{AZ} V_{AZ}\omega_{AZ}A_T/A$.

The free volume can be extracted from the differential cluster spectra $\tilde{Y}_{AZ}(\vec{p}) = Y_{AZ}(v_{surf})/(4\pi p^2 \Delta p)$, which can be related to differential cluster densities as $\rho_{AZ}(\vec{p}) = A\tilde{Y}_{AZ}(\vec{p})/V_f$, after a correction from the Coulomb boost [10, 11]. Supposing an ideal gas of classical clusters with binding energies B_{AZ} in thermodynamic equilibrium at temperature T in the grand-canonical ensemble, the differential mass densities read:

$$\rho_{AZ}^{id}(\vec{p}) = \frac{A}{h^3} g_{AZ} \exp\left[\frac{1}{T}\left(B_{AZ} - \frac{p^2}{2M_{AZ}} + Z\mu_p + N\mu_n\right)\right], \quad (2)$$

with $M_{AZ} = Am - B_{AZ}$, $g_{AZ} = 2S_{AZ}+1$ the mass and spin degeneracy of cluster (A,Z), and m the nucleon mass. In-medium effects are expected to suppress the cluster densities [7], with respect to the ideal gas limit given by eq.(2), $\rho_{AZ} = C_{AZ}\rho_{AZ}^{id}$, where the in-medium correction $C_{AZ} < 1$ can depend on the thermodynamic conditions, the cluster species and their momentum [7]. If we normalize the cluster spectrum by the proton and neutron spectra at the same velocity, the unknown chemical potentials $\mu_{n,p}$ cancel, and the free volume V_f can be independently estimated from the different cluster species as:

$$V_f = R_{np}^{\frac{A-Z}{A-1}} C_{AZ} \exp\left[\frac{B_{AZ}}{T(A-1)}\right]\left(\frac{g_{AZ}}{2^A}\frac{\tilde{Y}_{11}^{A}(\vec{p})}{\tilde{Y}_{AZ}(A\vec{p})}\right)^{\frac{1}{A-1}}, \quad (3)$$

where the free neutron-proton ratio R_{np} is estimated from the multiplicities of the $A = 3$ isobars , $R_{np} = (Y_{31}/Y_{32})\exp[(B_{32}-B_{31})/T]$.

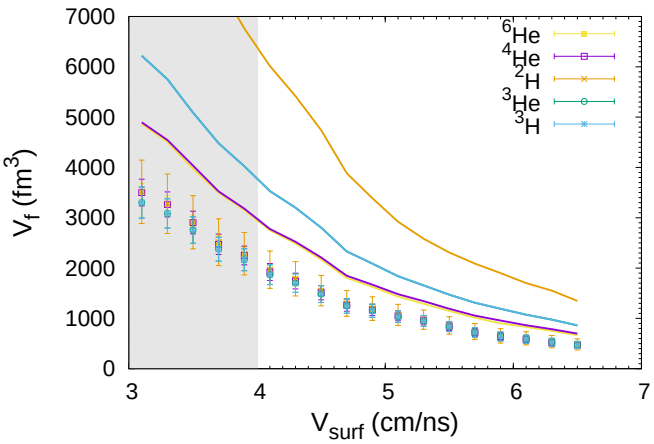

FIG. 1. (Color online) System ^{136}Xe+^{124}Sn: free volume estimated from the different clusters as a function of v_{surf} from Eq. (3). Lines: ideal gas limit $C_{AZ} = 1$. Symbols: bayesian determination of the in-medium correction (see text). The grey band shows the velocity domain where data might be polluted by spectator decay according to Ref. [10].

The presence of in-medium corrections is clearly confirmed by the experimental data, as shown by Fig. 1, which displays the value of the free volume obtained from Eq. (3) as a function of the sorting variable v_{surf} for the ^{136}Xe+^{124}Sn system, using different particle species. A clear hierarchy is observed as a function of the cluster mass if $C_{AZ} = 1$ is assumed, corresponding to the ideal gas limit. It is clear from Eq. (3) that to have consistent estimations of the volume, the deuteron requires a more important correction with respect to the heavier He isotopes. The deviations from the different volume estimations, when we suppose $C_{AZ} = 1$, increase with decreasing v_{surf}, showing that the in-medium effects additionally depend on the thermodynamic conditions.

These considerations lead to the following parametrization for the in-medium correction:

$$C_{AZ} = \exp\left[-\frac{a_1 A^{a_2} + a_3|I|^{a_4}}{T_{HHe}(A-1)}\right], \quad (4)$$

where the temperature is estimated through the isobaric double isotope ratio Albergo formula [13], and it is indicated as T_{HHe}. The unknown parameters $\vec{a} = \{a_i(\rho_B, y_p, T), i = 1-4\}$ are taken as random variables,

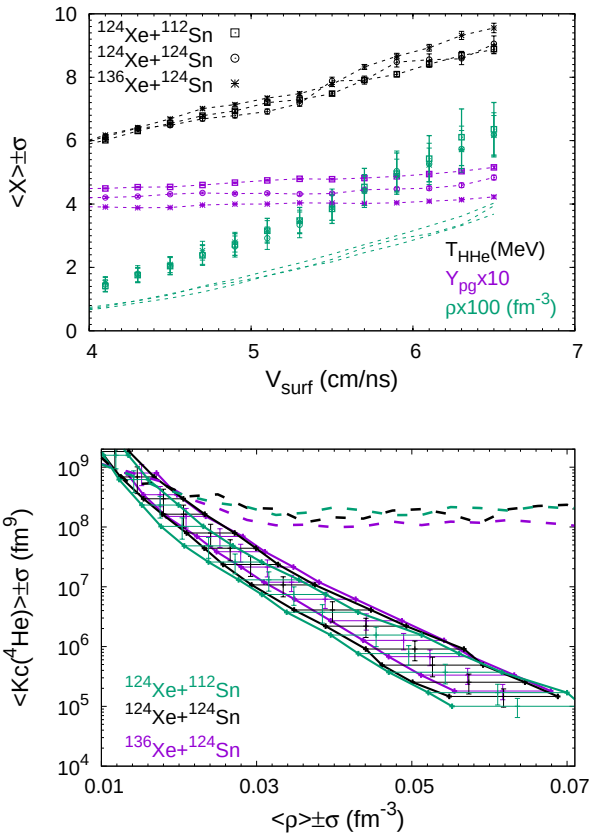

FIG. 2. (Color online) Top: double isotope ratio temperature T_{HHe} (black), global proton fraction (multiplied by a factor of 10) (purple), density (multiplied by a factor of 100) (green) as a function of the experimental quantity v_{surf} for the three experimental systems. Bottom: The chemical equilibrium constant of the α cluster as a function of the density for the three experimental systems. The dashed lines represent the ideal gas limit.

with a probability distribution fixed by imposing that the volumes obtained from the experimental spectra $\tilde{Y}_{AZ}$ of the different (A, Z) nuclear species in a given v_{surf} bin, correspond to compatible values. We take in each v_{surf} bin flat priors, $P_{\rm prior}(\vec{a}) = \theta(\vec{a}_{\rm min} - \vec{a}_{\rm max})$, within an interval largely covering the physically possible reduction range of the binding energy, $0 \leq a_1 \leq 15$ MeV, $0 \leq a_3 \leq a_1$ MeV, $-1 \leq a_2 \leq 1$, $0 \leq a_4 \leq 4$.

The posterior distribution is obtained by imposing the volume observation with a likelihood probability as follows:

$$P_{post}(\vec{a}) = \mathcal{N} \exp\left(-\frac{\sum_{AZ}(V_f^{(AZ)}(\vec{a}) - \bar{V}_f(\vec{a}))^2}{2\bar{V}_f(\vec{a})^2} \right) . \quad (5)$$

Here, $\mathcal{N}$ is a normalization, $V_f^{(AZ)}(\vec{a})$ is the free volume obtained from the (A, Z) cluster using Eq. (3) with the specific choice $\vec{a}$ for the parameter set of the correction, and $\bar{V}_f(\vec{a})$ is the volume corresponding to a given parameter set $\vec{a}$, averaged over the cluster species.

The posterior expectation values of the volume as estimated from the multiplicities of each cluster from Eq. (3), with the associated standard deviations, are shown as symbols in Fig. 1. It is clear that when we include the correction, the volumes decrease and the estimations obtained from the different cluster species are compatible within error bars. Different functional expressions were tried for the argument of the exponential in Eq. (4), and we have checked that they do not impact the results presented in this Letter.

The evolution of the thermodynamic conditions as a function of the sorting variable v_{surf} are displayed in the top panel of Fig. 2. We can see that the existence of an in-medium correction goes in the direction of increasing the density with respect to the ideal gas limit $C_{AZ} = 1$, and the effect is the same for the three systems. We can see that the different collisions explore very similar trajectories in the (T_{HHe}, ρ_B) plane, the only difference being in the global proton fraction as expected.

The chemical equilibrium constants of the α-particles measured in the three different experimental data sets are displayed in the bottom panel of Fig. 2. The constants are seen to increasingly deviate from the ideal gas calculation, as the density increases, reflecting the increased in-medium suppression. The results of the different systems almost perfectly overlap, confirming the expectation that chemical constants are isospin-independent. The same is true for the chemical constants of all the other clusters (not shown).

Still, it is important to stress that the Albergo thermometer used [13] is only valid under the assumption that the in-medium corrections to the expression of the ideal gas of clusters, Eq. (2), cancel in double isobar ratios, which is, in principle, not the case, if the correction does not scale linearly with the particle numbers. Our Bayesian analysis does not allow us to determine the deviation of the observable T_{HHe} from the true thermodynamic temperature, and this can only be done in the framework of a specific model, as we do below.

In Ref. [9], a novel approach for the inclusion of in-medium effects in the equation of state for warm stellar matter with light clusters was introduced. This model includes a phenomenological modification in the scalar cluster-meson coupling, and includes an extra term in the effective mass of the clusters, derived in the Thomas-Fermi approximation, which acts as an exclusion-volume effect. The scalar coupling acting on nucleons bound in a cluster of mass A is defined as $g_s(A) = x_s A g_s$, with g_s the scalar coupling of homogeneous matter, and x_s a free parameter. A constraint on this parameter was obtained in the low-density regime from the Virial EoS, but a precise determination of x_s needs an adjustment on experimental measurements at densities close to the Mott density corresponding to the dissolution of clusters in the medium. The chemical equilibrium constants obtained with this model were compared with the NIMROD data [11] analyzed assuming an ideal gas expression for the determination of the nuclear density [9, 14], and a

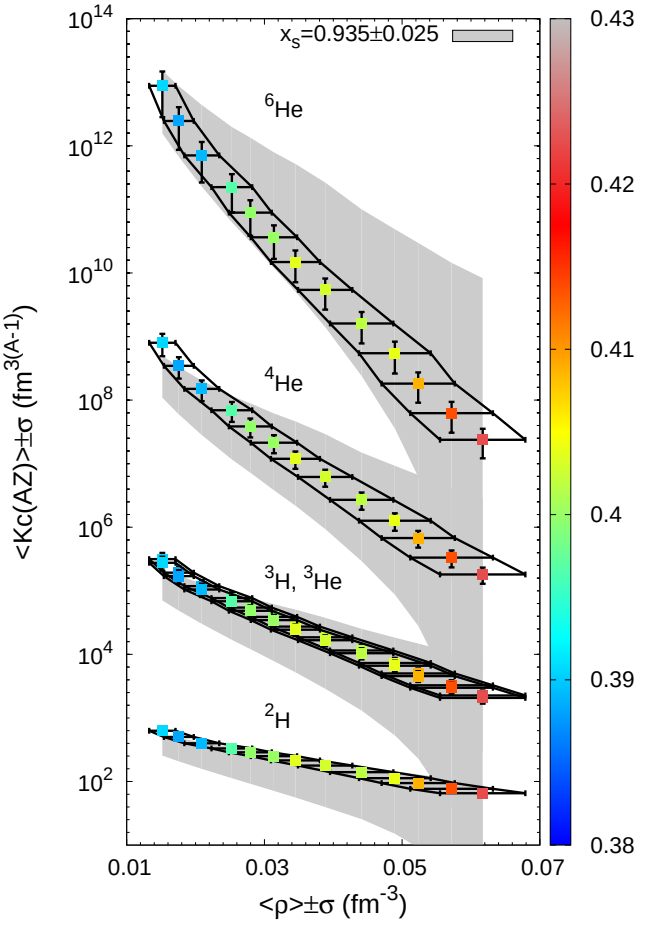

FIG. 3. (Color online) System ^{136}Xe+^{124}Sn: The chemical equilibrium constants as a function of the density. The grey bands are the chemical equilibrium constants from a calculation [14] where we consider homogeneous matter with five light clusters, calculated at the average value of (T, $\rho_{\rm exp}$, $y_{pg_{\rm exp}}$), and considering different cluster-meson scalar coupling constants $g_{s_i} = x_{s_i} A_i g_s$, with $x_{s_i} = 0.935 \pm 0.025$. The color code represents the global proton fraction.

satisfactory agreement was obtained for all clusters but the deuteron using the scalar cluster-meson coupling parameter $x_s = 0.85 \pm 0.05$.

Comparing the model of Refs. [9, 14] with this new analysis allows to determine the value of the in-medium parameter x_s in a more consistent way, and, at the same time, can provide an estimate of the effect of the correction we have introduced, with respect to the analysis method of Refs. [10, 11]. The parameter x_s measures how much the medium affects the binding of the cluster. If $x_s = 1$, the nucleons in the cluster behave as the unbound nucleons. The smaller the x_s, the stronger the in-medium effect, and the smaller the dissolution density of the cluster.

In order to make this comparison, we fix the temperature in each (ρ_B, y_p) point by imposing that the isotopic thermometer T_{HHe} evaluated in the model, correctly reproduces the measured T_{HHe} value. A small correction, which does not exceed 10% at the highest temperature, is obtained. The resulting chemical constants are compared to the experimental ones in Fig. 3. As we can observe, the chemical constants of ^{3}He and ^{3}H are almost superposed, and very similar results are obtained for all the different experimental entrance channels (not shown).

In Refs. [9, 14], we used $x_s = 0.85$ in order to reproduce the data of Qin *et al.* [11]. In the present work, we must take $x_s > 0.85$ in order to fit the INDRA data, corresponding to smaller corrections and a larger dissolution density. An optimal value can be extracted as $x_s = 0.935 \pm 0.025$. This value seems to reproduce very reasonably the whole set of experimental data, and we have checked that it is still within the Virial EoS limits.

In conclusion, a new analysis was performed based on INDRA data presented in Ref. [10]. We have shown that the presence of in-medium effects suppressing the cluster yields is necessary to explain the experimental data, giving rise to larger baryonic densities compared to the ideal gas limit. The reduction factors were directly extracted from the data, under the unique condition that the different nuclear species in a given sample must correspond to a unique common value for the density of the expanding source. We have verified that the three different data sets lead to fully compatible results for the corrections. In the framework of a relativistic mean-field theoretical model [9, 14], these corrections can be interpreted as a stronger scalar meson coupling of the nucleons bound in clusters, which shifts the dissolution to higher densities.

In a future work, it would be extremely interesting to perform a new analysis of the experimental data of Ref. [11], with the same method as the one presented in this Letter, in order to check the consistency of the different data sets, and to settle the model dependence of the results.

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