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► **To cite this version:**

Guillaume Bouchez, Chi-Hak Uy, Brice Macias, Delphine Wolfersberger, Marc Sciamanna. Wideband chaos from a laser diode with phase-conjugate feedback. *Optics Letters*, 2019, 44 (4), pp.975. 10.1364/OL.44.000975 . hal-02447460

**HAL Id: hal-02447460**

**<https://hal.science/hal-02447460>**

Submitted on 21 Jan 2020

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# Wideband chaos from a laser diode with phase-conjugate feedback

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January 21, 2020

**W**e analyze experimentally and theoretically the chaotic dynamics generated by a laser diode subjected to phase-conjugate feedback. Phase-conjugation is obtained from four-wave mixing in a  $BaTiO_3$  photorefractive crystal. We demonstrate that the chaos bandwidth first increases linearly with the feedback ratio but then saturates to relatively high values. A chaos bandwidth up to about 18 GHz is achieved, which is more than five times larger than the free-running laser diode relaxation oscillation frequency. Numerical simulations confirm our experimental observations and unveil that the finite depth penetration into the crystal is responsible for the observed saturation.

Semiconductor lasers with feedback or injection are known to exhibit various non-linear dynamics, such as self-pulsation, quasi-periodicity and chaos [1,2]. The resulting laser chaos finds applications, in e.g. chaos-based cryptography [3], high-frequency random number generation [4] or chaotic lidar [5]. A crucial characteristic in those applications is the chaos bandwidth, which is generally defined as the span of frequencies of the chaotic signal [6,7]. The larger is the chaos bandwidth, the better is typically the system performance: random numbers can be generated at higher rate with wide-band chaos source [8], wide-band chaos encrypted messages are harder to break [9], lidars with wide-band chaos source have better resolution [5]. In general the chaos obtained by a single laser diode with optical feedback has a bandwidth limited by the laser relaxation oscillation frequency [9]. To increase the chaos bandwidth beyond that limit, several sophisticated setups have been proposed. Experimentally, wideband chaotic semiconductor lasers have been achieved using optical feedback from two external cavities (16 GHz) [10], optical injection in a slave laser with feedback (16 GHz and 17 GHz) [11, 12], dual-wavelength injection into a

laser with optical feedback (32 GHz) [13] or three lasers with optical feedback, injecting each others (35 GHz) [14]. Recently, replacing a conventional mirror by a phase-conjugate mirror, i.e. introducing the so-called phase-conjugate feedback (PCF) was found to significantly improve the chaos bandwidth of a laser diode [15]. Ref. [15] reports an enhancement of 30 % of the chaos bandwidth thanks to PCF and reports of a chaos bandwidth of about 10.9 GHz for a phase-conjugate mirror reflectivity of about 14 %, using SPS photorefractive crystal in a self-pumped ring cavity. Nonetheless, the detector bandwidth was limited to about 12 GHz. From Ref. [15] even larger values of chaos bandwidth can be expected for higher values of the phase-conjugate mirror reflectivity, hence justifying further analysis.

In this letter, we therefore extend our previous experiment on PCF-induced optical chaos. Using a PCF in a CAT configuration with a  $BaTiO_3$  crystal and not a self-pumped ring cavity with a SPS crystal, we achieve a wide-band chaos when increasing the reflectivity of the mirror. Up to 18 GHz chaos bandwidth is found for specific values of the feedback strength: this is more than five times larger

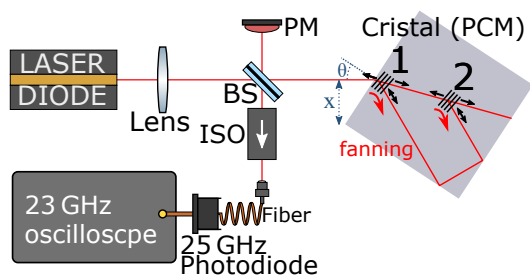


Figure 1: Experimental PCF setup using a CAT configuration.

than the free-running laser diode relaxation oscillation frequency. When the phase-conjugate mirror reflectivity is further increased, the chaos bandwidth saturates. A theoretical analysis confirms that the saturation of the chaos bandwidth originates from the finite penetration depth of the crystal, which induces a filtered phase-conjugate feedback.

The experimental setup is presented in Fig. 1. We use an edge-emitting semi-conductor laser diode. Its wavelength is  $\lambda = 852$  nm and its injection current threshold is  $J_0 = 14.9$  mA. The operating current ranges from 20 mA to 80 mA. The free-running laser relaxation oscillation frequency is measured by analysis of the relative intensity noise and is equal to 3.9 GHz at 50 mA. To generate a phase-conjugate feedback (PCF), we use a phase-conjugate mirror (PCM) in the so-called CAT configuration [16]. The output laser beam is focused by a lens into a *Rh*-doped  $BaTiO_3$  photorefractive crystal. The dimensions of this crystal are 5 mm  $\times$  5 mm  $\times$  5 mm. The beam enters the crystal near the frontal edge. Due to the fanning, part of the light deviates from the straight propagation direction and this deviated beam is reflected by total internal reflection on the crystal sides. Four-wave mixing in zones 1 and 2 generates a backward wave which is the conjugate of the beam entering the crystal. A beam splitter (BS) (transmission of 79 %, reflection of 18 %) is placed to reflect a part of the incident beam into a fiber coupler. A Faraday isolator (ISO) is placed behind the coupler to avoid parasitic feedback into the laser. The multimode fiber is then connected to a 25 GHz photodiode (PD), Newport 1414-50 High Speed Multimode Fiber-Optic Detector, whose electric signal is analyzed by an ultrafast oscilloscope (Tektronix DPO72304SX). The sampling rate of the oscilloscope is 50 GS/s while bandwidth of acquisition is 23 GHz. 18 % of the backward beam is reflected and measured by a powermeter (PM), thus enabling to compute the feedback ratio, defined as the ratio of the free-running power of the laser over the power coming back into the laser.

The cavity is 53.5 cm long, so the round-trip time  $\tau$  is 3.57 ns and the frequency associated with the external cavity is  $1/\tau$  i.e. 280 MHz. Two parameters are introduced to describe the geometry of the beam entering the PCM:

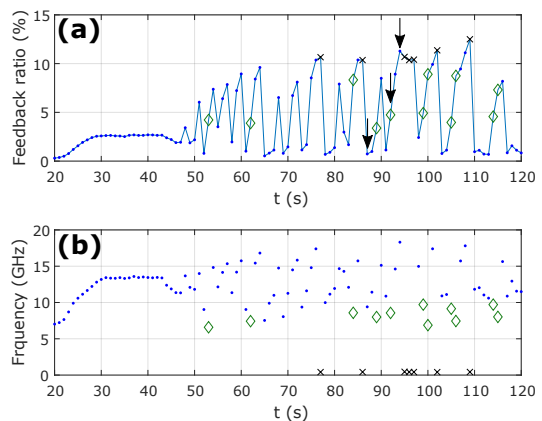


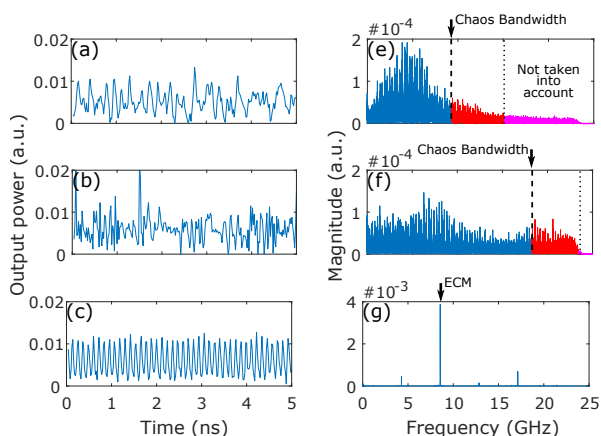
Figure 2: Evolution of (a) the feedback ratio and (b) the chaos bandwidth versus time. Blue dots are for chaotic dynamic states, green diamonds are for self-pulsing states and black crosses are for stationary states. For self-pulsing states (diamonds), the frequency is not the chaos bandwidth but the frequency of the main ECM.

$\theta$ , the angle between the c-axis of the non-linear crystal and the input beam and  $x$  the distance between the edge and the point of incidence in the crystal.

A laser diode with PCF is known to exhibit successive bifurcations of dynamic states, depending on the feedback ratio  $\gamma$ : steady-state for zero or very weak optical feedback, self-pulsing dynamics at frequency near the relaxation oscillation frequency for low levels of feedback [17–20], chaos for higher values of feedback ( $> 0.03$  %) [18–20]. A further increase of the feedback strength results in a chaos crisis leading to self-pulsing [19, 20] and finally re-stabilization for very high values of the feedback strength [17, 18, 20]. Self-pulsing states are oscillating at multiples of the frequency of the external cavity and hence have been named ECMs or external-cavity modes [20]. The power spectrum of this external cavity laser output shows components at multiples of the external-cavity frequency. We chose to consider as self-pulsing dynamics all time series whose spectrum main energy is observed at discrete frequencies. Stationary time series are characterized by a signal AC spectrum equal to noise levels and chaotic states are characterized by broadband power spectrum encompassing many frequencies.

The reflectivity of a CAT-PCM is known to be varying against time on a slow time-scale as a result of the dynamics of the photorefractive induced gratings [21]. In Fig. 2 (a) we therefore follow the evolution of the PCM reflectivity versus time. For each reflectivity value, the laser exhibits therefore a specific dynamic state. The dynamics of the PCM is slow enough to allow us performing time series analysis. Each second, we record the achieved reflectivity value and simultaneously record and analyze a 2  $\mu$ s-long power time-trace. The injection current is fixed at 50 mA.

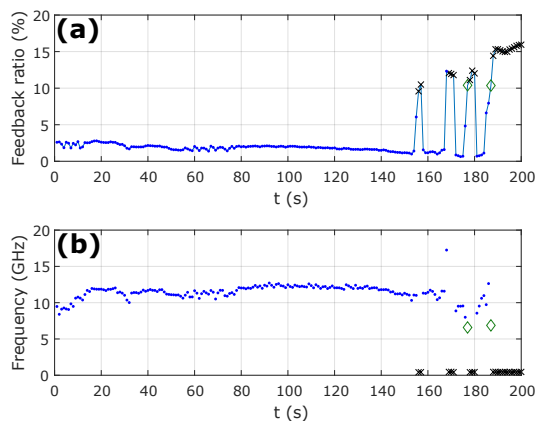
Depending on the observed dynamic, either the self-



**Figure 3:** Time-traces of laser output power (left) and corresponding RF spectra (right) for different phase-conjugate feedback strengths. (a) has been recorded at  $t = 87$  s ( $\gamma = 0.46$  %), (b) at  $t = 94$  s ( $\gamma = 11.01$  %) and (c) at  $t = 92$  s ( $\gamma = 4.47$  %). The dashed line indicates the chaos bandwidth and the thinner dotted line indicates the frequencies on which chaos bandwidth was computed.

pulsing frequency (for regular self-pulsation) or the chaos bandwidth (for chaos) is reported in Fig. 2 (b). Chaos bandwidth is defined as the upper frequency of the RF spectrum that contains 80 % of the signal linear energy [6]. To have snapshots of the RF spectrum of the signal, we use the Fourier transform of time series measured by the oscilloscope. Time series are 100 kS long which is enough to study the high-frequency bandwidth. For that analysis we neglect frequency components whose power is close to the noise floor. More specifically, the frequencies from which the average energy of ten consecutive samples is 50 % above the noise floor power level are not considered to compute the total energy.

For illustration, time series and RF spectrum obtained at different times (black arrows in Fig. 2) are plotted in Fig. 3. The RF spectrum of these time traces show frequency components at multiples of the cavity round-trip frequency (280 MHz) since the dynamics originate from bifurcations on the ECM solutions. The chaos bandwidth is shown with an arrow. As mentioned before, the violet part of the spectrum is considered to be below the noise floor and is not taken into account for the chaos bandwidth analysis. At low feedback (i.e.  $\gamma = 0.46$  %, see Fig. 3 (a)) the laser exhibits a chaotic behavior. The achieved chaos bandwidth is equal to 9.4 GHz (Fig. 3 (e)) and is already larger than the relaxation oscillation frequency (3.9 GHz). Fig. 3 (f) presents the RF spectrum at a higher feedback ratio ( $\gamma = 4.6$  %). The laser signal is still chaotic, but has a significantly larger chaos bandwidth, equal to 18,3 GHz. It is close to the measurement limit: as the oscilloscope has an analog bandwidth of 23 GHz, the limit chaos bandwidth will be around 18.4 GHz (80 % of 23 GHz). At a moderately large feedback ratio, the

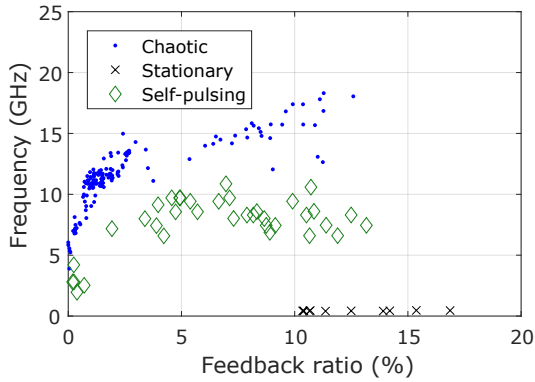


**Figure 4:** Evolution of (a) the feedback ratio and (b) the chaos-bandwidth versus time when attempting to stabilize the reflectivity of the PCM. For regular self-pulsing states (diamonds), the frequency is not the chaos bandwidth but the frequency of the ECM.

system can also present self-pulsing ECMs. Figs. 3 (c) and (g) shows the system locked to an ECM at a frequency of 8.56 GHz (around 31 times the frequency of the cavity) when  $\gamma = 4.47$  %.

As the feedback ratio is not constant and is varying versus time, the system will not remain in the desired dynamical state for a long time. Some experiments to stabilize the reflectivity of undoped  $BaTiO_3$  PCMs at visible wavelengths have been reported, but in experiments where an optical isolator prevents the laser to get destabilized by the PCF [22, 23]. As suggested in Ref. [23], we scanned the behavior of crystal reflectivity when changing distance  $x$  between the edge and the impact point. Whatever fixed angle  $\theta$  is chosen, we do not find any long-term (over several tens of minutes) stable position for the PCM reflectivity, therefore contrasting with previous experiments at visible wavelengths and with stationary laser dynamics. Fig. 4 shows that a nearly stationary reflectivity can be achieved with similar external parameters. During more than two minutes, from  $t = 17$  s to  $t = 147$  s, feedback ratio was stabilized around 4 %, and the laser diode remains chaotic with a bandwidth fluctuating between 10 GHz and 12.7 GHz. However, at  $t = 145$  s, the reflectivity begins to oscillate, and then the variations of the PCM reflectivity destabilize the laser dynamics to either chaotic states with different bandwidths or even regular self-pulsating states or steady-states. Our systematic analysis of the PCM reflectivity variations over time and the resulting consequences on the laser diode dynamics therefore tells us that wideband chaotic dynamics with bandwidths up to five times the relaxation oscillation frequency can be stabilized over at least several minutes.

In Fig. 5 we summarize the evolution of either the chaos bandwidth (for chaotic states) or the main self-pulsating frequency (for ECMs) versus the feedback ratio, by concate-



**Figure 5:** Experimental evolution of the chaos bandwidth versus the feedback ratio. For regular self-pulsing states (diamonds), the reported frequency is not the chaos bandwidth but the frequency of the ECM.

nating the data obtained during a 200 s long measurement. As the reflectivity of the PCM is varying continuously, there is some uncertainty in the measurement of the feedback ratio.

Still, that figure shows undoubtedly that chaos bandwidth shows a first rapid linear increase with the increase of the feedback ratio until reaching a value of about 15 GHz at a feedback ratio of about 3 %. In a range of less than 3 % of feedback ratio, the laser pulsating dynamics therefore increases its bandwidth by a factor 4 relatively to its free-running relaxation oscillation frequency. Then for moderately large feedback ratio, the chaos bandwidth shows a much slower increase with the feedback ratio and a maximum value of about 18 GHz - i.e. close to the measurement limit - is measured at a feedback ratio of about 10 %. In that range of feedback ratios, the dynamics shows several stable ECMs at very high harmonic frequencies of the external cavity frequency, as also suggested by previous theoretical analyses [20]. For still larger feedback ratios, the laser diode re-stabilizes to a steady-state. It is also worth mentioning that, as again suggested by previous theoretical works, the dynamics of a laser diode with PCF shows significant multi-stability between chaotic attractors of different properties and even chaos and more regular self-pulsating solutions. This is also reflected in Fig. 5.

At first sight, the conclusions drawn from Fig. 5, i.e. a first rapid linear increase then almost saturation of chaos bandwidth with feedback ratio, contradict earlier theoretical predictions from our group: in Ref. [24] numerical simulations of a PCF laser model found a monotonic linear increase of the chaos bandwidth with the feedback ratio, as is also observed in a conventional optical feedback configuration but with a much faster increase. However, this simulation was performed on a PCF laser model with instantaneous phase-conjugation and therefore did not consider the filtering effect of the phase-conjugate mirror

related to its finite penetration depth. In Refs. [20, 25, 26] the PCF laser model has been extended to account for the finite penetration time  $\tau_r$  of light inside the PCM.

In this paper, we simulate the filtered PCF laser model but more specifically analyzing the chaotic dynamics and their corresponding bandwidths. The model writes as [26]:

$$\dot{Y} = (1 + i\alpha)YZ + \kappa U \quad (1)$$

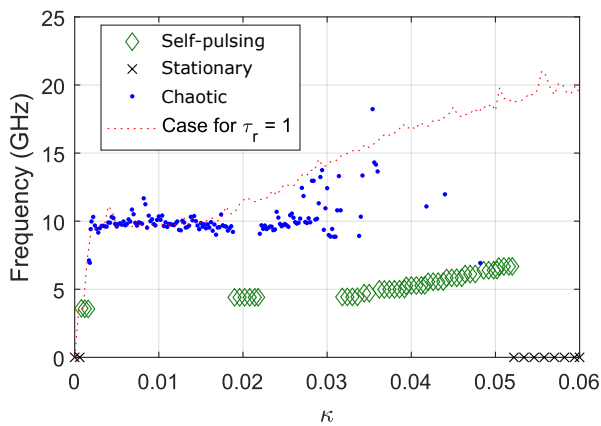
$$T\dot{Z} = P - Z - (1 + 2Z)|Y|^2 \quad (2)$$

$$\tau_r\dot{U} = Y^*(t - \tau) - U \quad (3)$$

where  $Y(t)$  is the complex normalized electric field of the laser,  $U(t)$  is the complex normalized feedback field and  $Z(t)$  is the real normalized carrier density.  $t$  is the time,  $\tau$  is the time delay,  $\tau_r$  is the penetration time and  $T$  is the electron lifetime. All these times are normalized by the photon lifetime  $\tau_p = 1.4$  ps.  $\kappa$  is the dimensionless feedback strength,  $\alpha$  is the linewidth enhancement factor and  $P$  is the pump above threshold. We choose  $\alpha = 2$ ,  $P = 0.6016$ ,  $T = 1200$  and  $\tau_r = 50$  which are taken from our previous study of the same laser system [27]. Considering our experimental cavity round-trip length (1.07 m), we take  $\tau = 2548$ .

We plot in Fig. 6 the chaos bandwidth versus the feedback rate. The model generates different dynamics: stable, self-pulsing and chaotic states. We analyzed the periodicity of the laser dynamics : if the laser output power is stationary, we consider the frequency to be equal to 0 (black crosses), if the output power is time-periodic, the displayed frequency is the main period (green diamonds) and if the signal is aperiodical, the frequency is its chaos bandwidth (blue dots). We also plot in red dotted line the evolution of the chaos bandwidth for a laser with a PCF with  $\tau_r = 1$  (instantaneous feedback). For  $\kappa < 0.015$ , the chaos bandwidth is similar for filtered and unfiltered PCFs. From  $\kappa = 0.009$  to  $\kappa = 0.024$ , the chaos bandwidth stays around 10 GHz. At higher feedback strengths, many high-frequency stable ECMs are found and for specific feedback rates the achieved chaos bandwidth is extremely high reaching about 18 GHz. The difference between the unfiltered PCF and the filtered PCF cases gets clear at large feedback rates: while the unfiltered PCF case predicts a monotonic linear increase of the chaos bandwidth with the feedback ratio, the filtered PCF case predicts saturation of the chaos bandwidth at large feedback rates. Furthermore the re-stabilization to the steady-state is only found theoretically in the filtered PCF case and not in the instantaneous PCF case, as already reported in Ref. [20]. The theory therefore matches qualitatively very well with the experimental findings and confirms the importance of accounting for the filtering properties of the PCM.

In summary, we report on the generation of wideband chaos from a laser diode with a single optical feedback. The achieved chaos bandwidth extends up to about five times the laser relaxation oscillation frequency. This



**Figure 6:** Theoretical variation of chaos-bandwidth (for chaotic state) and oscillation frequency (for self-pulsing states) of a laser with filtered PCF, versus the feedback rate. The red dotted line is the chaos bandwidth of an instantaneous PCF.

achievement is obtained thanks to the peculiar dynamical properties of phase conjugate optical feedback and does not require any additional feedback nor injection from a second laser. Interestingly and in contrast to earlier predictions, the chaos bandwidth increases in a non monotonic way with the increase of the feedback ratio. We demonstrate that this property results from the filtering property of the phase-conjugate mirror owing to its finite penetration depth.

**Acknowledgements** The authors thank the founders of the Chair in Photonics: Ministère de l'Enseignement Supérieur et de la Recherche, Région Grand-Est, Département Moselle, European Regional Development Fund (ERDF), Metz Métropole, Airbus GDI Simulation, CentraleSupélec and Fondation CentraleSupélec.

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