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Probabilistic Indoor Position Determination via Channel Impulse Response

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Abstract—Location Fingerprinting (LF) is a promising localization technique that enables many commercial and industrial Location-based Services (LBS). In this paper, a Channel Impulse Response (CIR) based indoor localization system is proposed. To fully exploit the most location-specific multipath information, we first conduct a power-based time tap filtering for the received CIR measurements. Furthermore, we experimentally observe that the filtered CIR data exhibits a multivariate circularly-symmetric Gaussian feature, which hints that fingerprinting positioning can be implemented by using a more accurate probabilistic method with a less computational complexity. In the online position determination phase, the Kullback-Leibler Distance (KLD) is adopted to quantify the similarities between the received measurements of target and the fingerprint database. Afterwards, we employ a probability kernel based regression approach to accurately infer the estimated target's location. Through extensive experiments performed on CRAWDAD database, the efficiency of our proposed scheme is validated.

Index Terms—Indoor localization, channel impulse response, multivariate complex Gaussian, Kullback-Leibler distance.

I. INTRODUCTION

Indoor Location-based Services (ILBS) have drawn much attraction in recent years due to its huge potential values for industrial and commercial applications [1], such as tracking of products through manufacturing lines, shop advertising for target customers, security surveillance in banking system, first-responder navigation at medical center, etc.. Despite that Global Navigation Satellite System (GNSS) offers a high-accuracy position information, it can not penetrate well and locate the device since the signal is blocked by the buildings. Consequently, alternative signals have been largely investigated for localization purpose. Examples include Wi-Fi, Bluetooth, Radio Frequency Identification (RFID), Ultra-wideband (UWB), sound, visible light, magnetic field and so forth [2] [3]. Indoor position determination can be then operated in these communication systems with the firmware upgrades or software implementations.

Traditional indoor localization is commonly conducted by the trilateration-based or the Fingerprinting (FP)-based methods. The trilateration scheme requires Line-of-Sight (LoS) measurement, which can not be well acquired indoors due to complex obstacles and room

partitions. This, to a great extent, limits the accuracy of indoor positioning. Thanks to the ubiquitous presence of Wireless Local Area Network (WLAN) and off-the-shelf Wi-Fi devices, Wi-Fi fingerprinting can be functionally carried out by measuring the Received Signal Strength (RSS) or other kinds of signal properties [4] [5].

In IEEE 802.11a/g/n standard, Wi-Fi networks use Orthogonal Frequency Division Multiplexing (OFDM) technique to modulate data on different orthogonal sub-channels and transmit them over multiple transmit-receive antenna pairs simultaneously. The channel response in time domain, which is estimated at the receiver through Inverse Fourier Transform (IFT), denotes Channel Impulse Response (CIR). Unlike the RSS as the indicator of link quality, the PHY layer CIR measurement can serve as a signature of the location characterized by the small-scale multipath effect, which significantly deteriorates the quality of its RSS counterpart. Furthermore, CIR indicates channel qualities in the level of multiple time taps and thus provides richer location information than RSS-based methods [6].

A major flaw lying in the CIR-based Wi-Fi fingerprinting approach is that the generation of radio map is labor-intensive and time-consuming. In this paper, a time efficient CIR-based fingerprint positioning system is proposed. To reduce the computational requirements of the algorithm, we introduce a power-based tap filtering scheme to preserve the location-specific CIR time taps, which contains the most relevant information related to multipath effects. Moreover, we successfully model the filtered CIR taps as multivariate circularly-symmetric Gaussian (also known as complex Gaussian) random variables [7], which inspires us to leverage the probabilistic Kullback-Leibler Distance (KLD) based kernel regression method to accurately infer the target's location. To the best of our knowledge, this is the first work to statistically characterize multidimensional CIR data and leverage KLD-based kernel regression approach in Wi-Fi fingerprint localization system. The experimental results also demonstrate the superior performance of our proposed system over other channel response based localization schemes.

The rest of this paper is organized as follows. In

Section II, we present the related work. The proposed CIR statistical feature extraction and other system designs are discussed in Section III, and the experimental results are provided in Section IV. Conclusions are drawn in Section V.

II. RELATED WORK

Generally, Wi-Fi fingerprint localization approaches mainly exploit two types of wireless signal property: the received signal strength and the channel response. We present related work based on these two categories.

A. RSS based Localization

RADAR [8] performed comprehensive site surveys for the first time and utilized RSS recorded by wireless infrastructure to generate position fingerprints. Subsequently, the authors of RADAR used the deterministic k-Nearest Neighborhood (kNN) technique to estimate the user's location with an accuracy of $3m$. In contrast, Youssef et al. employed the probabilistic approach in Horus [9] by using joint clustering algorithm and provided an accuracy improvement of $2.1m$, which outperformed RADAR even with less computational complexity. However, the instability of RSS still remains challenging.

B. Channel Response based Localization

Sen et al. proposed the PinLoc [10], in which the localization process was conducted on a set of $1m \times 1m$ spots. The main observation in PinLoc is that the probability density function of channel frequency response on a single sub-carrier at the same location demonstrates clustered distributions on the complex plane. Thus, the channel response on each sub-carrier was modeled as a Gaussian mixture distribution. The experimental result of PinLoc shows a mean accuracy of 89% for 100 spots. In FIFS [11], the authors leveraged the spatial and frequency diversity of the channel response for Wi-Fi fingerprinting. In addition, FIFS used the power summation over all independent sub-carriers as location fingerprint and applied Maximum A Posteriori (MAP) algorithm to achieve an improved accuracy over RSS based Horus system.

III. SYSTEM DESIGN

In this section, we lay out the detailed architecture of the proposed indoor fingerprint positioning system.

A. Overview

The proposed localization system exploits the suitable features of CIR from commodity Wi-Fi device to achieve target position determination. To fully characterize the individual multipaths, the wireless propagation channel is modeled as a temporal linear filter, known as Channel Impulse Response. Mathematically, it is denoted as

$$h(t) = \sum_{i=1}^T \alpha_i e^{-j\varphi_i} \delta(t - \tau_i), \quad (1)$$

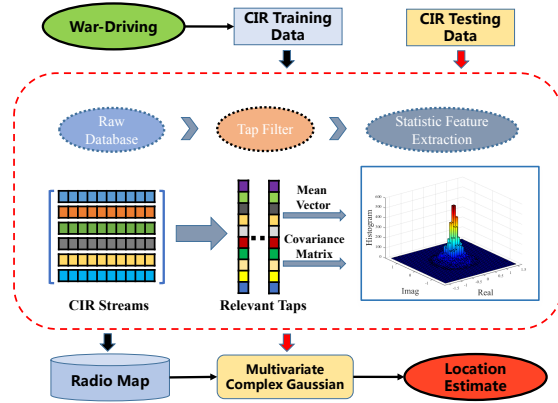


Fig. 1. System Architecture

where α_i , φ_i and τ_i are the amplitude, phase and time delay spread of the i^{th} path, respectively. T is the total number of multipaths and $\delta(\cdot)$ is the Dirac delta function. It is worth mentioning that, based on the current IEEE 802.11n standards, wireless receivers periodically receive the raw OFDM beacon messages from access points. We approximate the CIR by applying inverse Fourier transform to the channel frequency estimation at the receiving end. Hence, the different subcarriers of channel frequency response are then transformed into the same number of CIR time taps, depicting the multipath information in the time domain.

The overall architecture of the proposed system is illustrated in Figure 1. In general, it consists of two major functionality blocks: the CIR fingerprint map construction block and the target's location estimation block. In CIR fingerprint map construction block, once getting the received CIR time series, we first employ a power relevant filtering to extract the front CIR time taps which contains the most informative CIR measurements in the multipath-rich environment. Subsequently, we learn and model the statistical features for the selected CIR taps and build a representative radio map in the map construction block. Afterwards, in location estimation block, when a mobile target arrives into the area of interest, it matches against the learned attributes by using a probabilistic KLD based kernel regression scheme and finally figures out its physical position. In what follows, we will interpret each block of the proposed system at length in a divide-and-conquer manner.

B. Offline Training Phase

As the cornerstone of fingerprinting based localization system, the empirical fingerprinting approach requires a previous set of calibration for CIR measurements in an offline phase (also denoted as site survey). The CIRs are observed at various Reference Positions (RPs) in the area of interest and stored together with ground-truth locations in a database known as "radio map". Without

loss of generality, we consider a typical WLAN indoor scenario where CIR streams are measured in sequence at each of M reference locations from K available access points. In addition, the two-dimension physical space coordinate of one sample position ℓ_m is denoted as $\ell_m = (\ell_{m,x}, \ell_{m,y}), m = 1, 2, \dots, M$.

In the offline phase, let $\mathbf{h}_{m,k}(i)$ be an $F \times 1$ dimensional vector containing complex CIR values of F time taps for the i^{th} CIR sampling packet which is observed at the specific RP position ℓ_m from the k^{th} AP. Then, let $\mathbf{H}_{m,k}$ be an $N \times F$ dimensional matrix containing the CIR values over F time taps between this link for N consecutive CIR packets. This CIR matrix can be expressed by the following equation.

$$\mathbf{H}_{m,k} = [\mathbf{h}_{m,k}(1), \dots, \mathbf{h}_{m,k}(i), \dots, \mathbf{h}_{m,k}(N)]^T \quad (2)$$

As noticed, the columns of matrix $\mathbf{H}_{m,k}$ denote the CIR time series for each sampled time tap. Furthermore, the whole measurement database can be constructed with each entry $\mathbf{H}_{m,k}$ representing one CIR sampling matrix for the m^{th} RP position from the k^{th} AP.

The motivation of leveraging the spatial diversity stems from the fact that CIR benefits from the multipath effects, in which case the received signals at different positions are the combination of different reflections. Therefore, these complex CIR values will reflect the combination result ignoring the large scale fading. Here it's worth noting that we fix the number N for every CIR time samples in order to express conveniently for the following processes.

1) *Time Tap Filtering*: Recall that channel impulse response completely characterizes the multipath channel and preserves the location dependency, which makes it a good choice for location fingerprint. Figure 2 exhibits the overall CIR amplitude change during a given consecutive time sequence. By using the commodity Wi-Fi with center frequency of $2.4GHz$, the time range of the CIR vector is not efficiently large. The bandwidth of the system is therefore $20MHz$ in this case, yielding a time resolution of $50ns$. Since typical indoor maximum excess delay τ_{max} is smaller than $500ns$, given a time resolution of $50ns$, approximately only the first 10 out of the 30 accessible CIR time taps are relevant to multipath propagation. In other words, the remaining 20 taps are irrelevant for localization purpose. Moreover, when the Signal-to-Noise-Ratio (SNR) is not high enough, the receiver Additive White Gaussian Noise (AWGN) at these time taps will only make the accuracy worse.

Hence, based on the system bandwidth, a reasonable number of relevant time samples should be chosen for the sake of computation efficiency and accuracy. In this paper, we design a power-based tap filtering method to preserve the most informative CIR features for fingerprinting. For each CIR matrix $\mathbf{H}_{m,k}$, we first calculate the average channel power for each time tap, denoted

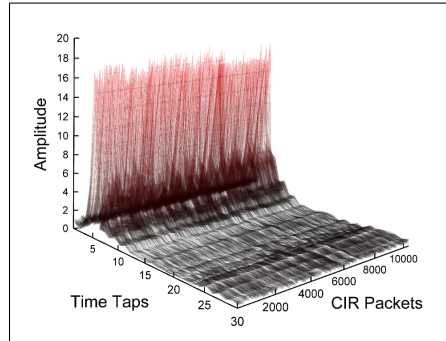


Fig. 2. CIR time samples at 20MHz

by $\mathbf{W} = (w_1, w_2, \dots, w_F)$, where F is the number of time taps. Then, we define a cumulative contribution rate of the first i taps as $C_i = \sum_{j=1}^i w_j / \sum_{j=1}^F w_j$. If the cumulative contribution rate of the first R taps, i.e. C_R , is greater than the predefined threshold C , we regard the first R taps as the extracted fingerprints. Hereby, the CIR matrix can be renewed as $\mathbf{H}_{m,k}^\dagger = \mathbf{H}_{m,k}^{\{1:R\}}$, where the superscript denotes the index of the first R columns of matrix $\mathbf{H}_{m,k}$.

In the following, we will proceed to study the statistical features of this simplified CIR data.

2) *Statistical Feature Modeling*: Previous researches show a superior performance of the probabilistic approaches such as Maximum Likelihood (ML) over deterministic ones in indoor environment [9] [11]. This motivated us to take into account the statistical model of CIR measurements and adopt probability-based location estimation scheme in our work.

The simplest probabilistic model for the CIR time taps is based on the assumption that there are a large number of statistically independent reflected and scattered paths with random amplitudes corresponding to a single tap. By the Central Limit Theorem, it can reasonably be modeled as a circularly-symmetric Gaussian (complex Gaussian) random variables [12]. It is noteworthy that, if the channel has a fixed LOS component then the real and imaginary parts of CIR taps are not zero-mean. In this case the received signal equals the superposition of a complex Gaussian component and this LOS component. However, through our extensive indoor office testing, we observed that CIR time taps exhibit circularly-symmetric Gaussian property in both LOS and NLOS scenarios. The underlying explanation resorts to the complexity of certain indoor environments, which mitigate the LOS component effect under strong NLOS conditions.

To further verify the theoretical assumption of CIR tap statistic, an analytical experiment is conducted to observe the statistical features of CIR samples. For the sake of generality, we collect around 10000 CIR packets from a specific AP over nearly half an hour of a general working time. Given the CIR real and imaginary parts

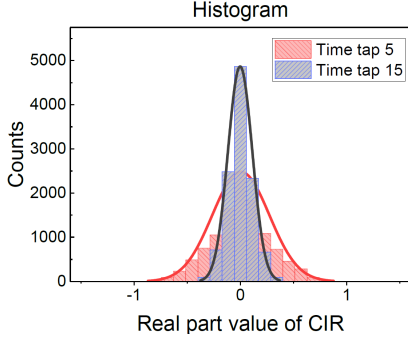


Fig. 3. CIR real part histogram of 5th and 15th tap

are independent identically distributed (i.i.d.), for the purpose of clear presentation, we choose two time taps (5 and 15) and only depict the histogram of CIR real part values in Figure 3. As can be observed, both of them appear to share zero-mean Gaussian distributions, of which the 5th tap has a larger variance than the 15th time tap.

Given that this is only a visual indication, we then utilize QQ-plot to carefully test the Gaussian distribution hypothesis for aforementioned CIR taps. Figure 4 (a) and (b) illustrate that both taps statistically fit well to normal distribution and hence verify that these complex CIR samples can be properly modeled as circularly-symmetric Gaussian random variables.

Finally, it comes to extract the statistic features from $\mathbf{H}_{m,k}^\dagger$ and then construct a radio map, of which the R -dimensional mean vector of each tap $\boldsymbol{\mu}_{m,k}$ and covariance matrix $\boldsymbol{\Sigma}_{m,k}$ of filtered CIRs are stored as fingerprint database for the next online localization stage.

C. Online Position Estimation

For the online location determination, the target is required to be accurately mapped to the pre-designed radio map. To quantitatively measure the similarity between stored fingerprints and observed CIR measurements, we employ the Kullback-Leibler Distance which is also known as relative entropy capable of measuring the difference between two probability distributions in information theory. The general definition of the KLD for two continuous probability densities p and q is shown as follows

$$\text{KLD}(p(x)||q(x)) = \int_x p(x) \log \frac{p(x)}{q(x)} dx \quad (3)$$

Recall that in the offline phase, we model the multiple filtered CIR time taps $\mathbf{H}_{m,k}^\dagger$ as complex Gaussian random variables with mean vector $\boldsymbol{\mu}_{m,k}$ and covariance matrix $\boldsymbol{\Sigma}_{m,k}$. The Probability Density Function (PDF)

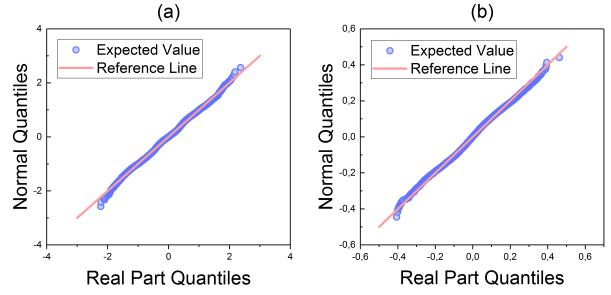


Fig. 4. CIR Gaussian Hypothesis Tests. (a) The 5th time tap. (b) The 15th time tap.

p for the multivariate complex Gaussian distribution is shown as follows:

$$p(x) = \frac{1}{(2\pi)^{R/2} \det(\boldsymbol{\Sigma}_{m,k})^{1/2}} \exp\left(-\frac{1}{2}(x - \boldsymbol{\mu}_{m,k})^T \boldsymbol{\Sigma}_{m,k}^{-1} (x - \boldsymbol{\mu}_{m,k})\right) \quad (4)$$

where $\det(\cdot)$ signifies the determinant of a square matrix. In the online phase, let us suppose we observe a CIR time series $\mathbf{Q}_k = [\mathbf{q}_k(1), \mathbf{q}_k(2), \dots, \mathbf{q}_k(s), \dots, \mathbf{q}_k(S)]^T$ from the k^{th} AP. Here S is denoted as the number of testing samples. Likewise, after time taps filtering, we then have the reduced version of $\mathbf{Q}_k^\dagger \in \mathbb{C}^{S \times R}$ with mean vector $\hat{\boldsymbol{\mu}}_k$ and covariance matrix $\hat{\boldsymbol{\Sigma}}_k$.

In our location determination scheme, we only focus on the statistic features extracted from CIR measurements. As discussed in Section III-B, each time tap is zero-mean complex Gaussian distributed so that the negligible mean vector $\boldsymbol{\mu}_{m,k}$ can be omitted. In order to further simplify the computational process, according to [13], the multivariate KL distance between two Gaussian distributions has the following closed-form solution:

$$\mathcal{D}_{m,k} = \frac{1}{2} \left(\log \frac{\det(\hat{\boldsymbol{\Sigma}}_k)}{\det(\boldsymbol{\Sigma}_{m,k})} + \text{tr}(\hat{\boldsymbol{\Sigma}}_k^{-1} \boldsymbol{\Sigma}_{m,k}) - R \right) \quad (5)$$

where $\mathcal{D}_{m,k}$ is defined as the KL distance between the fingerprint $\mathbf{H}_{m,k}^\dagger$ and testing measurement \mathbf{Q}_k^\dagger from the k^{th} AP, and $\text{tr}(\cdot)$ denotes the trace of a matrix. Moreover, by using the chain rule for the relative entropy [14], it can be proved that the KL distance of a joint distribution of independent variables is equal to the sum of the KL distance for each variable's distribution. Under K independent AP assumption, we therefore have the KL distance for all available APs at the m^{th} RP location as follows

$$\mathcal{D}_m = \sum_{k=1}^K \mathcal{D}_{m,k} \quad (6)$$

In order to properly obtain the location estimation of the target, we choose to perform the weighted kernel regression by using the KLD based kernel function \mathcal{K} and

the whole set of known reference points. The estimated location can be derived by the following equation

$$\hat{\ell} = \frac{\sum_{m=1}^M \ell_m \mathcal{K}_m}{\sum_{m=1}^M \mathcal{K}_m} \quad (7)$$

Here \mathcal{K}_m is defined as the probability kernel of the m^{th} RP position by exponentiating its corresponding KL distance [15], which is presented as follows:

$$\mathcal{K}_m = \exp(-\rho \mathcal{D}_m) \quad (8)$$

where ρ is a training parameter which is chosen for optimally minimizing the fingerprint matching error. It is notable that the kernel \mathcal{K}_m is equal to one if the given two distributions exhibit identically ($\mathcal{D}_m = 0$) and decays to zero as the dissimilarity of the two distributions increases. In other words, this probability kernel provides a flexible way to naturally handle the CIR data and hence takes full advantage of our probabilistic complex Gaussian model, which can then lead to an improved localization performance.

The performance of the proposed fingerprinting approach will be evaluated in the following section.

IV. EVALUATION

In this section, we present the experimental evaluation of our proposed localization system. First, we start by introducing the experimental setup and detailed implementation methodology. Then, the results of localization performance will be exhibited in Section IV-B.

A. Experimental Presentation

The entire experiments are conducted on the open wireless network data resource CRAWDAD [16]. It provides access to over 9300 measured temporal CIR data in a 44-node office scenario as shown in Figure 5. Multiple measurements (typically 5) are provided for each of the $44 \times 43 = 1892$ pairwise links between the nodes. The measurement transmitter is a 40 MHz chip rate DS-SS signal generator with the center frequency of 2.443 GHz. The receiver is a software radio (Sigtek model ST-515) designed to receive the signal. What's more, the transmitter and receiver are closely synchronized, within 1-2 nanoseconds of each other.

For the experimental methodology, we randomly choose 3 nodes appointed as APs and the other 40 nodes' locations as reference points. Therefore, the CIR measurements received from these 3 APs are stored as original radio map for the training phase. The measurements received at the remaining one node (target) naturally serve as testing samples. Subsequently, we change the target's location from a chosen set of 30 different node points and evaluate the positioning performance. In addition, for the purpose of comparison of different AP deployment, we extend to conduct a data training and then select the optimal ρ in multivariate KLD based kernel function for all different AP configurations. The evaluation results are displayed in the following section.

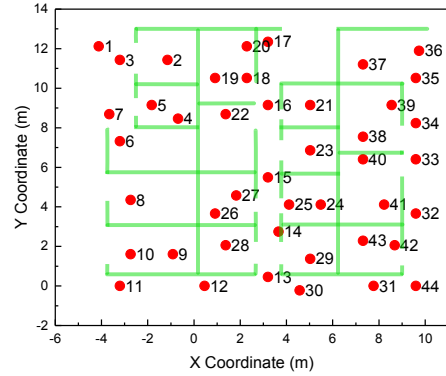


Fig. 5. Office scenario map of node locations

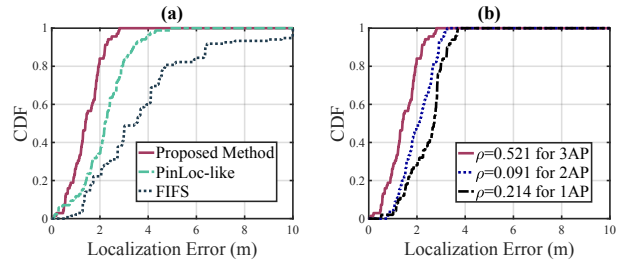


Fig. 6. Empirical CDF of localization errors. (a) Three positioning scheme performances under 3 AP configuration. (b) The proposed localization system under different AP settings.

B. Localization Performances

In this part, we compare our results with FIFS and PinLoc. Considering that the original PinLoc system conducted war-driving procedure in a set of predefined $1m \times 1m$ grids, known as spots, in order to provide a fair comparison, we modify PinLoc to use the same training set that we use in the proposed localization system.

Figure 6 (a) depicts the cumulative error distribution curves for three schemes under 3 AP configuration. As observed, our proposed positioning system provides a 90th percentile error of 2.21 m, which outperforms the PinLoc-like approach and FIFS with respective 55% and 29% accuracy within the same error. As can be viewed from Figure 6 (b), the CDF curves of localization errors for our proposed localization scheme are presented when the number of AP varies from 1 to 3. It indicates that the localization accuracy can be considerably enhanced by using more access points. This makes sense since more integrated CIR information leads to a higher accuracy. The parameter ρ for different AP settings are optimally chosen by data training as 0.214 for 1 AP, 0.091 for 2APs and 0.521 for 3 APs, respectively. When deploying 3 APs, our approach can maintain no more than 2.21m distance error with the probability of 0.9. However, for 2 and 1 AP settings, the accuracy percentage falls to 58% and 36%, respectively.

Moreover, as shown in Table I, we provide more

TABLE I
LOCALIZATION ACCURACY OF DIFFERENT METHODS USING 3 APs

Methods	Max. err.	Min. err.	Mean err.	Acc. at 90%
Proposed	2.81m	0.08m	1.44m	2.21m
PinLoc-like	4.92m	0.10m	2.21m	3.72m
FIFS	8.91m	0.56m	3.89m	6.38m

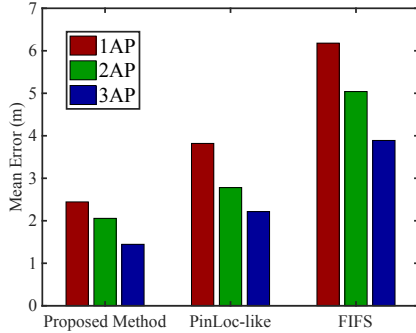


Fig. 7. The average distance error bar of three schemes under different AP settings.

detailed accuracy comparison for the three methods using 3 APs. The proposed scheme yields a mean error of 1.44m by combining 3 APs' information, which is better than PinLoc-like method and FIFS by 2.21m and 3.89m, which indicate an accuracy improvement of 34.8% and 62.9%, respectively. As for the maximal and minimal error, our system also achieves superior accuracy of 2.81m and 0.08m over its counterparts PinLoc-like approach and FIFS, respectively.

Figure 7 portrays the mean error bar graph for 3 schemes with different AP numbers. It can be seen that for each scheme, combining more APs' information contributes to less localization errors. Furthermore, the performance of our proposed approach sufficiently surpasses the PinLoc-like and FIFS system when considering the same number of access points. To be specific, with AP number increasing from 1 to 3, our method can achieve the average distance error of around 2.44m, 2.05m and 1.44m, respectively.

V. CONCLUSION

This paper has presented an indoor probabilistic fingerprint localization system which validates the multivariate circular-symmetric Gaussian hypothesis of the most informative time taps within the CIR measurements. The reduced dimensional fingerprint in radio map significantly reduces the computational complexity of the proposed scheme without loss of essential location information. By using KLD-based kernel regression method, the evaluation results show that our proposed system can improve localization average accuracy of about 34.8% and 62.9% compared with PinLoc-like scheme and FIFS, respectively.

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