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Special issue in honour of Denis Bosq's 80th birthday

Robust statistical signal processing in semi-Markov nonparametric regression models*

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Abstract: We develop model selection methods for robust processing of non-parametric periodic signals observed in continuous time with noises containing impulse components defined by a non Gaussian semi - Markov processes. In particular, we apply the developed model selection methods for the detection problem of the number of signals in multi-path information transmission observed with complex dependent impulse semi-Markov noises. As an example of semi-Markov noises, we consider the signals models with the noises defined through fractional Poisson processes. For this problem we show non asymptotic sharp oracle inequalities for robust risks, i.e., we show that the constructed procedures are optimal in the sense of sharp oracle inequalities.

1. Introduction

1.1. Motivations

One of the most important problems in the statistical signal processing theory is the detection of the number of signals observed in continuous time with random noises in multi-path connection channels (see, for example, [5, 24, 26, 29] and the references therein). Usually, in the framework of the statistical radio-physics, the telecommunication and navigation systems are modelled by the following formal equation

$$y_t = \sum_{j=1}^q \theta_j \phi_j(t) + \nu_t, \quad 0 \leq t \leq n,$$

where $(\nu_t)_{t \geq 0}$ is the Gaussian white noise, $(\theta_j)_{1 \leq j \leq q}$ are energetic parameters and $(\phi_j)_{j \geq 1}$ are known orthonormal signal functions, i.e., $\int_0^1 \phi_i(t) \phi_j(t) dt = \mathbf{1}_{\{i=j\}}$. To

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set this problem in the statistical analysis framework, one has to use the corresponding stochastic differential equations

$$(1.1) \quad dy_t = \left(\sum_{j=1}^q \theta_j \phi_j(t) \right) dt + dw_t,$$

where $(w_t)_{t \geq 0}$ is the standard Brownian motion. Note that this is the well-known "signal+white noise" model which is well studied in many papers (see, for example, [12, 19, 20, 27] and the references therein). In this paper we study the estimation problem of the number of the signals q , when the parameters $(\theta_j)_{1 \leq j \leq q}$ are unknown. In the white noise case, the logarithm of the likelihood function of the model (1.1) can be represented as

$$\ln L_n = \sum_{j=1}^q \theta_j \int_0^n \phi_j(t) dy_t - \frac{1}{2} \sum_{j=1}^q \theta_j^2.$$

Therefore, the maximum likelihood estimators of $(\theta_j)_{1 \leq j \leq q}$ and q , can be obtained as

$$\max_{1 \leq q \leq q^*} \max_{\theta_j} \ln L_n = \frac{1}{2} \sum_{j=1}^{q^*} \left(\int_0^n \phi_j(t) dy_t \right)^2,$$

i.e., the maximum likelihood estimator $\hat{q} = q^*$; note that we obtain $\hat{q} = \infty$ for $q^* = \infty$. For this reason, we study this problem in nonparametric setting proposed by Bosq in [6] for the general theory of stochastic processes, i.e., we represent the signal in (1.1) in the following form

$$(1.2) \quad dy_t = S(t)dt + d\xi_t, \quad 0 \leq t \leq n,$$

where $S(\cdot)$ is an unknown 1-periodic nonparametric function from $\mathbf{L}_2[0, 1]$.

Our goal is to introduce in the observation model noise components with jumps given by semi-Markov processes as in [2]. In particular, such models enable us to consider the dependence in the observations during a long time, which significantly expands the possible applications of the statistical methods. These models allow, for example, estimating the signals observed under long impulse noise impact having a memory or under "nuisance signals" action.

1.2. Model

In this paper, we study the model (1.2) in the framework of the general semimartingale regression model introduced in [14], in which the noise process $(\xi_t)_{t \geq 0}$ is a non Gaussian semimartingale with jumps of special form defined as

$$(1.3) \quad \xi_t = \varrho_1 w_t + \varrho_2 L_t + \varrho_3 z_t,$$

where ϱ_1 , ϱ_2 and ϱ_3 are unknown coefficients, $(w_t)_{t \geq 0}$ is a standard Brownian motion, $(L_t)_{t \geq 0}$ is a jump Lévy process and the pure jump process $(z_t)_{t \geq 1}$ is a semi-Markov impulse noise introduced in [2]. This means that, instead of the Gaussian white noise, in the radio-electronic stochastic systems we assume that the useful signal S is distorted by random impulse noises. Note that the impulse noises for the signal estimation problems have been used in many papers (see, for example, [5, 13, 18, 25] and the references therein). It should be noted also that the use of additional non Gaussian noise components in the models poses also the question of how the estimation quality depends on the noise distribution; in other words, this naturally raises the question about the stability and robustness of the proposed procedures. For these reasons, the estimation quality will be studied with respect to the robust risks that were introduced by Galtchouk and Pergamenschikov in [9] for nonparametric models. This allows constructing the estimation procedures having stable fixed-accuracy properties with respect to changes of the noise distributions in a fairly wide range. Such procedures provide the high-noise immunity of the algorithms synthesized on their basis. Therefore, we also assume that the distribution Q of the noise process $(\xi_t)_{t \geq 0}$ is unknown. It is only known that this distribution belongs to the distribution family Q_n^* that will be defined in the next section. We set the robust risk as

$$(1.4) \quad \mathcal{R}_n^*(\hat{S}_n, S) = \sup_{Q \in Q_n^*} \mathbf{E}_{Q,S} \|\hat{S}_n - S\|^2 \quad \text{and} \quad \|S\|^2 = \int_0^1 S^2(t) dt,$$

where \hat{S}_n is an estimator of S (i.e., any measurable function of $(y_t)_{0 \leq t \leq n}$).

1.3. Methods

To estimate the unknown signal S we use the sharp model selection methods developed for such problems in [2]. The interest in these statistical procedures can be explained by the fact that they provide adaptive solutions for a nonparametric estimation through non asymptotic sharp oracle inequalities which give an upper bound for quadratic risks including the minimal risk over a chosen family of projection estimators with a coefficient close to one. This means that this statistical procedure is the best in the chosen family of estimators, i.e., it is optimal in the sharp oracle inequality sense. Such inequalities were obtained, for example, in [10] for non Gaussian regression models in discrete time and in [14] for general regression semimartingale models in continuous time. It should be noted that the model selection approach was proposed for the first time in [1, 23] for parametric models. Then, using the oracle inequality tool, these methods had been developed for nonparametric estimation in [4, 15] for Gaussian regression models and in [8] for non Gaussian models. Moreover, in [15] such methods were developed for the signal estimation problem for the Gaussian color noise model in continuous time. Unfortunately, the oracle inequalities obtained in these papers are not sharp and, therefore, they cannot provide

the efficiency property in the asymptotic setting (see, for example, [11, 16, 17]). To obtain the sharp oracle inequality for the models (1.2) – (1.3), one needs to use a special analytical tool based on nonlinear renewal theory developed in [2]. In this paper we use this method for the detection problem of the number of signals in (1.2) – (1.3).

1.4. Plan of paper

The rest of the paper is organized as follows. In Section 2, we give the main conditions which will be assumed for the model (1.2). In Section 3, we construct a model selection procedure and we give the sharp oracle inequalities for the detection problem of the number of signals.

2. Main conditions

In the model (1.3) we assume that the jump Lévy process L_t is defined as

$$(2.1) \quad L_t = \int_0^t \int_{\mathbb{R}_*} x(\mu(ds, dx) - \tilde{\mu}(ds, dx)),$$

where $\mu(ds, dx)$ is the jump measure with the deterministic compensator $\tilde{\mu}(ds, dx) = ds\Pi(dx)$, where $\Pi(\cdot)$ is the Lévy measure on $\mathbb{R}_* = \mathbb{R} \setminus \{0\}$ for which

$$(2.2) \quad \Pi(x^2) = 1 \quad \text{and} \quad \Pi(x^8) < \infty,$$

where we use the usual notation $\Pi(|x|^m) = \int_{\mathbb{R}_*} |z|^m \Pi(dz)$ for any $m > 0$.

Note that $\Pi(\mathbb{R}_*)$ may be equal to $+\infty$. Moreover, as in [2], we assume that the pure jump process $(z_t)_{t \geq 0}$ in (1.3) is a particular case of a semi-Markov process, with the following form

$$(2.3) \quad z_t = \sum_{i=1}^{N_t} Y_i,$$

where $(Y_i)_{i \geq 1}$ is an i.i.d. sequence of random variables with

$$\mathbf{E}_Q Y_i = 0, \quad \mathbf{E}_Q Y_i^2 = 1 \quad \text{and} \quad \mathbf{E}_Q Y_i^4 < \infty.$$

Here N_t is a general counting process defined as

$$(2.4) \quad N_t = \sum_{k=1}^{\infty} \mathbb{1}_{\{T_k \leq t\}} \quad \text{and} \quad T_k = \sum_{l=1}^k \tau_l,$$

where $(\tau_l)_{l \geq 1}$ is an i.i.d. sequence of positive integrated random variables with distribution η and mean $\tilde{\tau} = \mathbf{E}_Q \tau_1 > 0$. We assume that the processes $(N_t)_{t \geq 0}$ and

$(Y_i)_{i \geq 1}$ are independent between them and are also independent of $(L_t)_{t \geq 0}$. Note that the process $(z_t)_{t \geq 0}$ is a special case of a semi-Markov process (see, e.g., [3] and [22]).

Note that for any function f from $\mathbf{L}_2[0, n]$, $f : [0, n] \rightarrow \mathbb{R}$, for the noise process $(\xi_t)_{t \geq 0}$ defined in (1.3), with $(z_t)_{t \geq 0}$ given in (2.3), the integral

$$(2.5) \quad I_n(f) = \int_0^n f(s) d\xi_s$$

is well defined, with $\mathbf{E}_Q I_n(f) = 0$ and $\mathbf{E}_Q I_n^2(f) \leq \kappa_Q \int_0^n f^2(t) dt$ for some constant $\kappa_Q > 0$.

In the sequel we assume that the measure η has a density g which satisfies the following conditions from [2].

H₁) Assume that, for any $x \in \mathbb{R}$, there exist the finite limits

$$g(x-) = \lim_{z \rightarrow x-} g(z) \quad \text{and} \quad g(x+) = \lim_{z \rightarrow x+} g(z)$$

and, for any $K > 0$, there exists $\delta = \delta(K) > 0$ for which

$$\sup_{|x| \leq K} \int_0^\delta \frac{|g(x+t) + g(x-t) - g(x+) - g(x-)|}{t} dt < \infty.$$

H₂) For any $\gamma > 0$,

$$\sup_{z \geq 0} z^\gamma |2g(z) - g(z-) - g(z+)| < \infty.$$

H₃) There exists $\beta > 0$ such that $\int_{\mathbb{R}} e^{\beta x} g(x) dx < \infty$.

For the next condition we need to introduce the Fourier transform of any function f from $\mathbf{L}_1(\mathbb{R})$, $f : \mathbb{R} \rightarrow \mathbb{R}$, defined as

$$(2.6) \quad \widehat{f}(\theta) = \frac{1}{2\pi} \int_{\mathbb{R}} e^{i\theta x} f(x) dx.$$

H₄) There exists $t^* > 0$ such that the function $\widehat{g}(\theta - it)$ belongs to $\mathbf{L}_1(\mathbb{R})$ for any $0 \leq t \leq t^*$.

One can check directly that Conditions **H₁**)–**H₄**) hold true for any two times continuously differentiable density g defined on \mathbb{R}_+ , for which $g(0) = 0$ and there exists $\beta > 0$ such that

$$\int_0^{+\infty} e^{\beta x} (g(x) + |g'(x)| + |g''(x)|) dx < \infty,$$

$$\lim_{x \rightarrow \infty} e^{\beta x} g(x) = 0 \quad \text{and} \quad \lim_{x \rightarrow \infty} e^{\beta x} g'(x) = 0.$$

Indeed, Conditions $\mathbf{H}_1)$ – $\mathbf{H}_3)$ are obvious and Condition $\mathbf{H}_4)$ can be obtained by integrating two times by parts. For example, we can take a Gamma density with the shape parameter greater than one. Moreover, one can check directly that these conditions hold true also for the density of $X_0^{H/3} \wedge X_1$, where X_0 is a Mittag-Leffler waiting time with parameter $0 < H < 1$ (see, for example, [21, 28]) and X_1 is a Gamma random variable with the shape parameter greater than one. So, in this case, the process N_t is a truncated fractional Poisson process.

Remark 2.1. *It should be noted that the fractional Poisson processes is very popular in the different applied areas such that radiophysics, insurance (see, for example, [7, 21, 28] and the references therein). This is explained by the fact that these processes keep a long dependence structure on large time intervals. Therefore, one can use this property to describe the dependent impulse noise influence in the connection channels. It should be noted also that the direct use of fractional Poisson processes themselves in signal models is not reasonable, since the time between jumps in fractional Poisson processes is very large, i.e., the noise impulses will be very rare. It will be more convenient to use truncated fractional Poisson processes for signal processing problems. They continue to keep the strong dependence, but at the same time the intervals between jumps are significantly smaller than the ones of fractional processes, i.e., they will be more frequent even compared to impulses modelled by compound Poisson processes.*

Remark 2.2. *It should be noted that if τ_j are exponential random variables, then $(N_t)_{t \geq 0}$ is a Poisson process and, in this case, $(\xi_t)_{t \geq 0}$ is a Lévy process for which this model has been studied in [14] and [16]. But, in the general case when the process (2.3) is not a Lévy process, as for example, truncated fractional Poisson processes, this process has a memory and cannot be treated in the framework of semi-martingales with independent increments. In this case, we need to use the new special analytical tool based on renewal theory arguments developed in [2].*

Let us now define the family of the noise distributions for the model (1.2) which is used in the robust risk (1.4). In our case the distribution family \mathcal{Q}_n consists in all the distributions on the Skorokhod space $\mathcal{D}[0, n]$ of the process (1.3) with the parameters satisfying Conditions (2.7) and (2.8). Note that any distribution Q from \mathcal{Q}_n is defined by the unknown parameters in (1.3) and (2.1). We assume that

$$(2.7) \quad \varsigma_* \leq \sigma_Q \leq \varsigma^*,$$

where $\sigma_Q = \varrho_1^2 + \varrho_2^2 + \varrho_3^2/\tilde{\tau}$, the unknown bounds $0 < \varsigma_* \leq \varsigma^*$ are functions of n , i.e., $\varsigma_* = \varsigma_*(n)$ and $\varsigma^* = \varsigma^*(n)$, such that for any $\epsilon > 0$,

$$(2.8) \quad \lim_{n \rightarrow \infty} n^\epsilon \varsigma_*(n) = +\infty \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{\varsigma^*(n)}{n^\epsilon} = 0.$$

Remark 2.3. *Note that in general (but this is not necessary), the parameters ϱ_1 , ϱ_2 and ϱ_3 can depend on n . Condition (2.8) means that we consider all possible*

cases, i.e. these parameters may go to infinity, or may be constant, or may go to zero as well. See, for example, Conditions (3.32) in [17].

3. Model selection

Let $(\phi_j)_{j \geq 1}$ be an orthonormal uniformly bounded basis in $\mathbf{L}_2[0, 1]$, i.e., for some constant $\phi_* \geq 1$, $\sup_{0 \leq j \leq n} \sup_{0 \leq t \leq 1} |\phi_j(t)| \leq \phi_*$; the constant ϕ_* may depend on n , such that $\lim_{n \rightarrow \infty} \phi_*(n)n^{-\epsilon} = 0$ for any $\epsilon > 0$. We extend the functions $\phi_j(t)$ by periodicity, i.e., we set $\phi_j(t) := \phi_j(\{t\})$, where $\{t\}$ is the fractional part of $t \geq 0$. For example, we can take the trigonometric basis (see, for example, (3.2) in [2]). To estimate the function S we use the model selection procedure from [2] based on the weighted least square estimators defined as

$$(3.1) \quad \widehat{S}_\lambda(t) = \sum_{j=1}^n \lambda(j) \widehat{\theta}_{j,n} \phi_j(t),$$

where the coefficients $\lambda = (\lambda(j))_{1 \leq j \leq n}$ belong to some finite set Λ from $[0, 1]^n$ and $\widehat{\theta}_{j,n}$ are the estimators for the Fourier coefficients $\theta_j = \int_0^1 \phi_j(t) S(t) dt$ defined as

$$(3.2) \quad \widehat{\theta}_{j,n} = \frac{1}{n} \int_0^n \phi_j(t) dy_t,$$

which can be represented, in view of the model (1.2), as

$$(3.3) \quad \widehat{\theta}_{j,n} = \theta_j + \frac{1}{\sqrt{n}} \xi_{j,n} \quad \text{and} \quad \xi_{j,n} = \frac{1}{\sqrt{n}} I_n(\phi_j).$$

Now, to choose a weight sequence λ in the set Λ we use the empirical quadratic risk, defined as

$$\text{Err}_n(\lambda) = \|\widehat{S}_\lambda - S\|^2,$$

which in our case is equal to

$$(3.4) \quad \text{Err}_n(\lambda) = \sum_{j=1}^n \lambda^2(j) \widehat{\theta}_{j,n}^2 - 2 \sum_{j=1}^n \lambda(j) \widehat{\theta}_{j,n} \theta_j + \sum_{j=1}^{\infty} \theta_j^2.$$

Since the Fourier coefficients $(\theta_j)_{j \geq 1}$ are unknown, we replace the terms $\widehat{\theta}_{j,n} \theta_j$ by

$$(3.5) \quad \widetilde{\theta}_{j,n} = \widehat{\theta}_{j,n}^2 - \frac{\widehat{\sigma}_n}{n},$$

where $\widehat{\sigma}_n$ is an estimate for the variance proxy σ_Q defined in (2.7). If this variance proxy is known, we take $\widehat{\sigma}_n := \sigma_Q$; otherwise, we can choose it, for example, as it is

defined in (3.11) in [2]. Finally, in order to choose the weights, we will minimize the following cost function

$$(3.6) \quad J_n(\lambda) = \sum_{j=1}^n \lambda^2(j) \widehat{\theta}_{j,n}^2 - 2 \sum_{j=1}^n \lambda(j) \widetilde{\theta}_{j,n} + \rho \frac{\widehat{\sigma}_n |\lambda|^2}{n}.$$

where $\rho > 0$ is some threshold which will be specified later. We define the model selection procedure as

$$(3.7) \quad \widehat{S}_* = \widehat{S}_{\widehat{\lambda}} \quad \text{and} \quad \widehat{\lambda} = \operatorname{argmin}_{\lambda \in \Lambda} J_n(\lambda).$$

We recall that the set Λ is finite so $\widehat{\lambda}$ exists. In the case when $\widehat{\lambda}$ is not unique, we take one of them.

Theorem 3.1. *Assume that Conditions \mathbf{H}_1) – \mathbf{H}_4) hold and that the unknown function S is continuously differentiable. Then, for the robust risk defined in (1.4) through the distribution family (2.7) – (2.8), the procedure (3.7) for any $n \geq 1$ and $0 < \rho < 1/6$, satisfies the following oracle inequality*

$$(3.8) \quad \mathcal{R}_n^*(\widehat{S}_*, S) \leq \frac{1+3\rho}{1-3\rho} \min_{\lambda \in \Lambda} \mathcal{R}_n^*(\widehat{S}_\lambda, S) + \frac{\mathbf{U}_n^*(S)}{n\rho},$$

where the term $\mathbf{U}_n^*(S) > 0$ is such that for any $r > 0$ and $\epsilon > 0$

$$(3.9) \quad \lim_{n \rightarrow \infty} \sup_{\|\widehat{S}\| \leq r} \frac{\mathbf{U}_n^*(S)}{n^\epsilon} = 0.$$

Remark 3.1. *We recall that, the inequality (3.8) is called sharp oracle inequality because the main coefficient in the right side goes to one as $\rho \rightarrow 0$. If the chosen estimator family $(\widehat{S}_\lambda)_{\lambda \in \Lambda}$ contains an efficient estimator which can be computed directly, then, asymptotically, the model selection procedure will have the same properties as the efficient estimator. To do this, it is sufficient to choose the parameter ρ as a function of n , i.e., $\rho = \rho(n)$ such that*

$$\lim_{n \rightarrow \infty} \rho(n) = 0 \quad \text{and, for any } \epsilon > 0, \quad \lim_{n \rightarrow \infty} n^\epsilon \rho(n) = +\infty.$$

For example, we can take $\rho(n) = (6 + \ln(n))^{-1}$. This is the main idea of using sharp oracle inequalities for the study of efficiency properties in the adaptive setting (see, for example, [11, 16, 17]).

Now, let us consider the model (1.2) with the unknown function S defined as

$$(3.10) \quad S(t) = \sum_{j=1}^q \theta_j \phi_j(t).$$

For this problem we use the LSE family $(\widehat{S}_d)_{1 \leq d \leq m}$ defined as

$$(3.11) \quad \widehat{S}_d(x) = \sum_{j=1}^d \widehat{\theta}_{j,n} \phi_j(x).$$

This estimate can be obtained from (3.1) with the weights $\lambda_d(j) = \chi\{j \leq d\}$.

As a risk for the number of signals we use

$$(3.12) \quad \mathbf{D}_n(d, q) = \mathcal{R}_n^*(\widehat{S}_d, S),$$

where the risk $\mathcal{R}_n^*(\widehat{S}, S)$ is defined in (1.4) and d is an integer number (maybe random) from the set $\{1, \dots, \iota\}$, where the maximal signal number ι is assumed to be a function of n , i.e. $\iota = \iota(n)$, such that $\iota(n) \rightarrow \infty$ and, for any $\epsilon > 0$, the sequence $n^\epsilon \iota(n) \rightarrow 0$ as $n \rightarrow \infty$. In this case the cost function (3.6) has the form

$$(3.13) \quad J_\epsilon(d) = \sum_{j=1}^d \widehat{\theta}_{j,n}^2 - 2 \sum_{j=1}^d \widetilde{\theta}_{j,n} + \rho \frac{\widehat{\sigma}_n |\lambda|^2}{n}.$$

So, for this problem the LSE model selection procedure is defined as

$$(3.14) \quad \widehat{q}_n = \operatorname{argmin}_{1 \leq d \leq \iota} J_n(d).$$

Note that Theorem 3.1 implies the next result.

Theorem 3.2. *Assume that Conditions \mathbf{H}_1 – \mathbf{H}_4 hold. Then, the robust risks of the procedure (3.7), for any $0 < \rho < 1/6$, satisfy the following oracle inequality*

$$(3.15) \quad \mathbf{D}_n(\widehat{q}_n, q) \leq \frac{1 + 3\rho}{1 - 3\rho} \min_{1 \leq d \leq \iota} \mathbf{D}_n(d, q) + \frac{\mathbf{U}_n^*(S)}{n\rho},$$

where the last term satisfies the property (3.9).

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References

- [1] AKAIKE, H. (1974). A new look at the statistical model identification. *IEEE Trans. on Automatic Control* **19** 716–723.
- [2] BARBU, V. S., BELTAIEF, S. and PERGAMENSHCHIKOV, S. M. (2019). Robust adaptive efficient estimation for semi-Markov nonparametric regression models. *Statistical inference for stochastic processes* **22**(2) 187–231.

- [3] BARBU, V. S. and LIMNIOS, N. (2008). *Semi-Markov Chains and Hidden Semi-Markov Models toward Applications - Their use in Reliability and DNA Analysis*. Lecture Notes in Statistics **191** Springer, New York.
- [4] BARRON, A., BIRGÉ, L. and MASSART, P. (1999). Risk bounds for model selection via penalization. *Probab. Theory Relat. Fields* **113** 301–415.
- [5] BELTAIEF, S., CHERNOYAROV O. and PERGAMENSHCHIKOV, S. M. (2019). Model selection for the robust efficient signal processing observed with small Lévy noise. *Annals of the Institute of Statistical Mathematics*. DOI: 10.1007/s10463-019-00726-2, <https://doi.org/10.1007/s10463-019-00726-2>
- [6] BOSQ, D. (1996). *Nonparametric Statistics for Stochastic Processes. Estimation and Prediction*. Lecture Notes in Statistics, Springer.
- [7] BIARD, R. and SAUSSEREAU, B. (2014). Fractional Poisson process: long-range dependence and applications in ruin theory. *Journal of Applied Probability* **51**(3) 727–740.
- [8] FOURDRINIER, D. and PERGAMENSHCHIKOV, S. M. (2007). Improved selection model method for the regression with dependent noise. *Annals of the Institute of Statistical Mathematics* **59**(3) 435–464.
- [9] GALTCHOUK, L. I. and PERGAMENSHCHIKOV, S. M. (2006). Asymptotically efficient estimates for nonparametric regression models. *Statistics and Probability Letters* **76**(8) 852–860.
- [10] GALTCHOUK, L. I. and PERGAMENSHCHIKOV, S. M. (2009). Sharp non asymptotic oracle inequalities for nonparametric heteroscedastic regression models. *Journal of Nonparametric Statistics* **21**(1) 1–16.
- [11] GALTCHOUK, L. and PERGAMENSHCHIKOV, S. M. (2009b). Adaptive asymptotically efficient estimation in heteroscedastic nonparametric regression. *Journal of Korean Statistical Society* **38**(4) 305–322.
- [12] IBRAGIMOV, I. A. and KHASHMINSKII, R. Z. (1981). *Statistical Estimation: Asymptotic Theory* Springer, Berlin-New York.
- [13] KASSAM, S. A. (1988). *Signal detection in non-Gaussian noise* Springer-Verlag, New York.
- [14] KONEV V. V. and PERGAMENSHCHIKOV S. M. (2009a). Nonparametric estimation in a semimartingale regression model. Part 1. Oracle Inequalities. *Tomsk State University Journal of Mathematics and Mechanics* **3** 23–41.
- [15] KONEV V. V. and PERGAMENSHCHIKOV S. M. (2010). General model selection estimation of a periodic regression with a Gaussian noise. *Annals of the Institute of Statistical Mathematics* **62** 1083–1111.
- [16] KONEV V. V. and PERGAMENSHCHIKOV S. M. (2012). Efficient robust nonparametric estimation in a semimartingale regression model. *Ann. Inst. Henri Poincaré Probab. Stat.* **48**(4) 1217–1244.
- [17] KONEV V. V. and PERGAMENSHCHIKOV S. M. (2015). Robust model selection for a semimartingale continuous time regression from discrete data. *Stochastic processes and their applications* **125** 294–326.
- [18] KONEV V., PERGAMENSHCHIKOV, S. M. and PCHELINTSEV, E. (2014). Esti-

- mation of a regression with the impulse type noise from discrete data. *Theory Probab. Appl.* **58**(3) 442–457.
- [19] KUTOYANTS, YU. A. (1984). *Parameter Estimation for Stochastic Processes*. Heldeman-Verlag, Berlin.
- [20] KUTOYANTS, YU. A. (1994). *Identification of dynamical systems with small noise*. Mathematics and its Applications, 300, Kluwer Academic Publishers Group, Dordrecht.
- [21] LASKIN, N. (2003). Fractional Poisson processes. *Communications in Nonlinear Science and Numerical Simulation* **8** 201–213.
- [22] LIMNIOS, N. and OPRISAN, G. (2001). *Semi-Markov Processes and Reliability*. Birkhäuser, Boston.
- [23] MALLOWS, C. (1973). Some comments on C_p . *Technometrics* **15** 661–675.
- [24] PCHELINTSEV, E. A. and PCHELINTSEV, V. and PERGAMENSHCHIKOV, S. M. (2018). Non asymptotic sharp oracle inequality for the improved model selection procedures for the adaptive nonparametric signal estimation problem. *Communications, Scientific Letters of the University of Zilina* **20**(1) 72–76.
- [25] PCHELINTSEV, E. A. and PCHELINTSEV, V. and PERGAMENSHCHIKOV, S. M. (2019). Improved robust model selection methods for the Lévy nonparametric regression in continuous time. *Journal of Nonparametric Statistics* **31**(3) 612–628.
- [26] PCHELINTSEV, E. A., MARCOKOVA, M. and PERGAMENSHCHIKOV, S. M. (2019). Adaptive robust efficient methods for periodic signal processing observed with colours noises *Advances in Electrical and Electronic Engineering*, **17**(3) 270–274.
- [27] PINSKER, M. S. (1981). Optimal filtration of square integrable signals in gaussian white noise. *Problems of Transimission information* **17** 120–133.
- [28] REPIN, O. N. and SAICHEV, A. I. (2000). Fractional Poisson law. *Radiophys. and Quantum Electronics* **43** 738–741.
- [29] TRIFONOV A. P., KHARIN A. V., CHERNOYAROV O. V., KALASHNIKOV K. S. (2016). Estimation of the Number of Orthogonal Signals with Unknown Parameters. *Journal of Communications Technology and Electronics* **61**(9) 1026–1033.

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