Variations in students' reading process when working on mathematics tasks with high demand of reading ability

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The purpose of this study is to deepen the understanding of the relation between features of the text of mathematics tasks and the tasks' demand of reading ability. Variations in students' reading processes when they work with PISA mathematics tasks with high demand of reading ability are identified and analyzed. These variations can be related to linguistic features of words, phrases, or sentences in the tasks, which in turn can be possible sources for the high demand of reading ability.

Keywords: Mathematics tasks, reading comprehension, reading process, PISA.

Introduction

Mathematics tasks are a common way to assess students' mathematical knowledge. They are used in examinations, nationwide tests, and also in international assessments like the Programme for International Students Assessment (PISA). They are meant to assess mathematical ability, that is, acquired proficiency in mathematics, but a written task usually also requires that the student can read natural language in the task text, which can make the task also assess reading ability. Some reading ability can be seen as a necessary part of mathematical communication which is an aspect of mastering mathematics (NCTM, 2000). However, to ensure that the task assesses what it is intended to assess and nothing else, *unnecessary* demand of reading ability not connected to mathematical ability should be avoided. This unnecessary demand of reading ability can be caused by linguistic features of the task text which may be avoidable if they are not connected to mathematics (e.g., use of uncommon words not belonging to mathematical vocabulary or complex sentence structure). Therefore, it is important to identify which linguistic features of mathematics tasks are related to an unnecessary demand of reading ability. This demand may become apparent in variations in students' reading process when working with mathematics tasks (e.g., stumbling at a word or rereading a sentence). Analyzing these variations, and identifying the units of the text where they occur, makes it possible to determine features of the task text potentially causing an unnecessary demand of reading ability.

This study thus detects and analyses variations in students' reading processes when working with mathematics PISA tasks identified as having high unnecessary demand of reading ability. Also, features of the tasks' text and their possible impact on the tasks' demand of reading ability are discussed.

Background

This section presents the theories about reading comprehension and reading of mathematical texts that are used in this study to characterize the variation in the students' reading processes. It will also give a short overview of earlier research concerning sources of (unnecessary) demand of reading ability in mathematics tasks.

Reading comprehension

A simple description of reading comprehension is given by Hoover and Gough (1990) who describe *decoding* and *linguistic comprehension* as necessary for successful reading. Decoding means the ability to recognize words, that is, being able to connect written words to mental lexical information. Linguistic comprehension is the ability to derive meaning out of lexical information. In the concept of *reading comprehension*, these two are combined into the ability to derive meaning out of written information. In addition, Österholm, Bergqvist and Dyrvold (2016) identify three aspects of reading comprehension that can be related to reading difficulties. These are *phonological*, *syntactic*, and *semantic* aspects. The phonological aspect concerns sound and the flow of reading, since phonological memory supports listening comprehension and thus, indirectly, reading comprehension (Perfetti, Landi, & Oakhill, 2005, p. 238). The syntactic aspect concerns the decoding of grammatical structures at the sentence level. Finally, the semantic aspect refers to the depth and breadth of knowledge of word meaning (Nation, 2005, p. 254).

The different parts and aspects of reading comprehension are used in this study to identify possible sources for issues students encounter when reading mathematics texts.

Reading mathematics text

This study concerns reading of mathematics tasks. It is therefore relevant that there exist features that are typical for mathematics texts, like special mathematical words (e.g., differentiate), words that have a different meaning in mathematical context than in everyday language (e.g., product), or the use of numbers and formulas (see, e.g., Schleppegrell, 2007). In this study, an assumption is therefore that reading and understanding a mathematical text requires a certain kind of reading ability that is part of the mathematical ability that a mathematics task intends to assess. But a mathematics task text can also contain features unconnected to mathematical ability, for instance, long or unusual "non-mathematical" words or complex sentences with subordinate clauses. These features might cause an unnecessary demand of reading ability that is unconnected to mathematical ability. In this case, a student needs, besides mathematical ability (including the aforementioned mathematics-specific reading ability), also a reading ability not connected to mathematics to solve the task. Therefore, it is important to separate necessary and unnecessary demands of reading ability of a mathematics task.

Österholm and Bergqvist (2012a) developed and confirmed the reliability and validity of the statistical measure for the unnecessary demand of reading ability (DRA) of a mathematics task used in this study. A principal component analysis of students' results on both mathematics and reading tasks extracts two main and partly overlapping components; a mathematics and a reading component. Mathematics tasks are loading mostly on the mathematics component, but are also, to varying degree, loading on the reading component. The loading on the reading component excludes the effects from the overlap between the components, and can therefore be interpreted as a measure of DRA.

In a mathematics task that is supposed to measure mathematical ability only, DRA should be avoided as far as possible. To be able to avoid DRA, it is necessary to identify not only tasks with high DRA, but also which features of a mathematics task are related to high DRA.

Sources of DRA

Since DRA measures a task's demand of reading ability that is not part of mathematical ability, it is plausible that linguistic features of the task text can be sources of DRA. These could be features connected to readability in general, such as long words (e.g., Lenzner, 2014), unfamiliar words (e.g., Abedi & Lord, 2001), or complex sentence structure (e.g., Dempster & Reddy, 2007). In an earlier study, Österholm and Bergqvist (2012b) found correlations between DRA and both word length and information density for the Swedish mathematics tasks of PISA 2003 and 2006. In a more recent study, Bergqvist, Theens, and Österholm (2018) identified tasks in the Swedish version of the PISA 2012 mathematics assessment having high DRA. In that study, however, the DRA was not correlated to any of the investigated features (word length, sentence length, task length, and information density). Identifying possible sources of DRA in mathematics tasks might help task constructors and teachers to avoid DRA or, at least, to make them aware of these issues.

Purpose of the study

The purpose of this study is to deepen the understanding of the relation between features of the text of mathematics tasks and the tasks' demand of reading ability. Therefore, this study identifies and analyzes variations in students' reading processes when they work with PISA mathematics tasks with high demand of reading ability (DRA) and then relates these variations to the linguistic features of the task texts. Thus, the research question is: Which kinds of variations appear in students' process of reading Swedish mathematics PISA tasks with high DRA?

After having identified the variations, by examining the units of the text where they occurred, I attempt to identify and discuss which features of the text can be possible sources for the DRA

Research Method

To investigate the variations in students' reading process, I collected and analyzed think-aloud-protocols (TAP) of students reading and solving mathematics PISA tasks in Swedish. These TAPs are part of a bigger data collection including students from both Sweden and Germany, several PISA-tasks, and follow-up interviews. In this study, I used the part of the TAPs where Swedish students read the tasks. The different steps of data collection and analysis are described in more detail below.

Task selection

For this study, I chose two tasks from the Swedish version of the 2012 PISA assessment that were identified having high DRA in an earlier study (Bergqvist et al., 2018). Since these PISA-tasks are confidential they cannot be reproduced here. The two tasks differ in several properties: Task 800Q01 (Dataspel/Computer Game) is a single task without subtasks and contains relatively little text. It is a selected-response (multiple choice) task made up of two shorter sentences and a table with numbers. Task 446Q01 (Termometersyrsan/The Thermometer Cricket) is the first subtask of two in a longer task. It contains more text than 800Q01 - three long and two shorter sentences - and a photo showing the insect. It is a closed constructed-response item, that is, the students have to calculate an answer and write it down. They do not need to show their calculations.

Selection of participants and data collection

The students were recruited from three different schools, two from rural areas and one from a bigger city. They voluntarily agreed to participate when their teachers asked the classes. All twelve students spoke Swedish fluently and gave their informed consent to take part in the study. Six students were working with task 446Q01 and seven students with task 800Q01. One of the students worked with both tasks. The eight girls and four boys were between 14 years and 6 months and 16 years and 1 month old, that is, about the age of 15, which is the age for which the PISA tasks are designed.

The students were told to read and think aloud during their attempt to solve the tasks. Everything the students said was recorded with a Livescribe Echo® smartpen, which also recorded everything the students wrote and connected it to the audio data. If the students were silent for a while they were encouraged to continue talking.

Think-aloud-protocols have the advantage that it is possible to follow the students' reading process and to get to know in which order they are reading the task text. Even if reading aloud is a somewhat unnatural way of reading mathematics tasks, it makes visible which words, phrases, or sentences may be problematic for the students, since oral reading fluency is an indicator for reading competence in general (Fuchs, Fuchs, Hosp, & Jenkins, 2001).

Data analysis

To find the variations in students' reading process when solving the tasks, I performed an analysis in two steps. In step 1, I analyzed the TAPs to identify all variations and sorted them in different categories of types of variations. In step 2, I analyzed the categories found in step 1 to find patterns in the variations. The steps are described in more detail below.

Step 1: While listening to the TAPs, I made a note every time a student made any deviation from a "straight-on" reading process. This straight-on process is understood as starting to read at the beginning and reading all text once to the end without any interruptions. The straight-on process is not assumed to be an ideal way of reading, but used as a guiding norm to highlight variations. I also noticed if a student did not make any deviations from the straight-on process. Each time a variation occurred, I labeled it with a category. When this type of variation occurred for the first time, a new category was defined. When a variation of the same category had occurred before, this existing category was used. Examples for the categories are *stumbling*, *misreading*, or *rereading*.

Step 2: To get a clear picture of the nature of the categories of variations, I sorted the categories in two dimensions. The first dimension referred to which kind of unit of the text the category concerned. That is, whether the variations occurred at a single word, a phrase consisting of several words, or a whole clause or sentence. This division facilitated the identification of text features that might trigger variations in the reading process, since, for example, syntactic aspects of reading comprehension concern decoding at the sentence level, whereas phonological aspects concern word level. Some of the categories of variations can occur at several different units, for instance, a student could stumble at a single word or a longer phrase. For other categories, it is more difficult to determine which unit they concern. If a student, for example, makes a filler sound (like "er") in the

beginning of a sentence, this variation could be categorized as being at word level (the first word of the sentence) or at sentence level. In cases like this, I had to interpret within the context which unit the variation concerned, as described in the results paragraph below.

The second dimension referred to which aspect of reading the variation concerns. That is, whether it deals with the accuracy, the flow, or the order of reading. The accuracy of reading concerned what is read, that is, if the student reads exactly what is written in the text, or if he/she leaves out anything, reads anything wrong, or adds words to the text. The flow of reading concerned interruptions in the reading, for example, stumbling or hesitating. It also included repeating a word or phrase directly when reading it, which is a form of stammering. The order of reading concerned which order the student reads the text in, that is, if he/she starts at the beginning or, for example, by reading the question, and if he/she rereads any parts of the text.

Results

The analysis of the think-aloud-protocols (TAPs) revealed several different categories of variations in students' reading process when solving mathematical PISA tasks with high DRA. A very common category was reading words and phrases correctly and with flow. On the other hand, there was only one student who read a whole task (Computer Game, 800Q01) straight-on, that is, read all text just once from beginning to end without any interruptions. In Table 1, the categories of variations that deviated from straight-on reading are sorted by the two dimensions mentioned above. Some of the categories are described in more detail below and exemplified in the discussion.

Variations at word level

Many of the observed variations in the students' reading process were connected to a single word, like the ones categorized as *hesitating* or *stumbling*. Hesitating means that the student paused within a sentence before reading a certain word, interrupting the flow of reading. Stumbling also occurred in relation to phrases but the students mostly stumbled at single words, that is, they started to read the word, eventually read wrong, stopped in the middle of the word, and started over. Other types of variations concerning single words were *reading slowly* and different types of *misreading*. These variations occurred for example in task 800Q01 for the word "kriterium" (English: criterion) and in task 446Q01 for the words "snöträdssyrsan", "termometersyrsan", and "temperatursyrsan" (different names for the thermometer cricket), and "Fahrenheit".

Variations at sentence/clause level

At the sentence/clause level the observed variations concerned *rereading* or making *filler sounds* like "er" or "um" in the beginning of a sentence or clause. Filler sounds almost only occurred in the beginning of a sentence or clause and not within sentences. Therefore, this variation was categorized as sentence/clause level when occurring at the beginning of a sentence or clause.

There was no type of variation where the students changed the order of reading when they were reading the task for the first time. All students started reading the tasks at the beginning and read them until the end, that is, none of them started by reading the question written in the end. For task 446Q01, all students but one reread some part of the text task at some point after having read the

complete text first — either before or after having solved the task, or both. For task 800Q01, four of seven students who worked on the task did not reread any clause or sentence of the task text.

	Word	Phrase	Sentence/clause
Accuracy of reading	• misreading, becomes a different word • misreading, becomes meaningless • misreading, becomes a synonym • misreading a number • skipping a word • adding a word • wrong pronunciation • mumbling	• mumbling	
Flow of reading	• hesitating • filler sounds • stumbling • pausing within a word • reading slowly • repeating	• stumbling • repeating	• filler sounds
Order of reading			• rereading after having read everything • rereading after having solved the task

Table 1: Categories of variations in the students' reading process deviating from straight-on reading

Discussion

Some of the observed variations in students' reading processes may indicate problems for the students when reading the tasks' text. These reading problems can relate to different aspects of reading comprehension (phonological, semantic, or syntactic) depending on features of the word, phrase, or sentence/clause that was problematic. In this section, I discuss some of the variations that may indicate sources of unnecessary demand of reading ability (DRA).

Variations at word level

Several types of variations occurred in relation to the different Swedish words for "the thermometer cricket" ("snöträdssyrsan", "termometersyrsan", "temperatursyrsan") in task 446Q01. Some students were stumbling or hesitating when reading these words. These expressions are long compounds of shorter and more common words such as "snö" (English: snow) or "syrsan" (cricket), which can make it difficult for the students when they read them for the first time. Since the observed categories of variations concern decoding of written text into sound, they relate to the phonological aspect of reading comprehension (Hoover & Gough, 1990; Perfetti et al., 2005). Two other types of variation occurring for these words are related to semantic aspects of reading comprehension, since they refer to the meaning of the words. One type of variation was that some students read "...syran" (acid) instead of "...syrsan" (cricket) the first time they read the word. A possible

explanation is that reading the context and seeing the photo of the cricket eventually helped them realize that the text was about an insect, and they did not make this mistake any more. The other type of variation was that some students read simply "syrsan" (cricket) instead of "thermometersyrsan" (thermometer cricket) when they reread the sentences. A possible explanation is that they had understood that the text was about some kind of cricket and realized that the exact name was less important. Since DRA for a task concerns a demand of reading ability that is not part of mathematical ability, and since knowing and reading these names for the cricket cannot be seen as a part of mathematical competence, the use of these long compound words in this task may be a source of high DRA.

Another word in task 446Q01 where the variations *hesitating*, *stumbling*, and *reading slowly* occurred was "Fahrenheit". For Swedish students, this is not a common word in their everyday language since temperature is measured in degrees Celsius in Sweden. The use of different units can be seen as being a part of mathematical language and, thus, a part of mathematical competence. In this case, the use of "Fahrenheit" is not likely to be connected to high DRA, since DRA concerns the part of reading ability that is *not* included in mathematical ability. In task 800Q01, some students were hesitating and reading slowly the word "kriterium" (English: criterion). This is a shorter word than, for example, "snöträdssyrssan" but not a compound of shorter common words and not common in the students' everyday language either. Since it does not belong to the vocabulary that is necessary or common when communicating mathematics at these students' school level (year 8-9), it might contribute to DRA of the task.

Variations at sentence/clause level

At the sentence/clause level, one variation was that students reread sentences or clauses after having read the whole text first. At task 446Q01, this occurred most frequently at sentence 3, which was a long sentence presenting the way to calculate. It also occurred at sentences 4 and 5, where the question was posed, but never at sentence 1 and 2 that contained general information about the cricket. At task 800Q01, it occurred mostly for sentence 2 that contained the question and more seldom for the first sentence that presented the situation. The same pattern occurred when students were reading sentences or clauses again after having solved the task. I see two aspects as possible explanations for this, either individually or in combination of both. After having read the text once, it is possible that the students had figured out in which sentences the information necessary to solve the task was presented. Based on this conclusion, they could have deemed it unnecessary to reread, for example, the first two sentences in task 446Q01, since these only presented general information about the cricket. This behavior would be an indication that the students mastered the semantic aspect of reading comprehension, which refers to meaning (Nation, 2005).

Another possible explanation for these variations, in particular the common rereading of sentence three in task 446Q01, can be syntactic issues. This sentence is long (37 words) and contains three subordinate clauses. Such sentence complexity could cause DRA, that is, a demand of reading ability, not connected to mathematical competence.

Conclusions

This study has shown that there are different kinds of variations in students' process of reading when they are working with mathematics tasks that have a high demand of reading ability (DRA) not connected to mathematical ability. These variations occur both at single words, longer phrases, and whole sentences and may be of different types. Some of the variations, such as stumbling or hesitating, can indicate reading problems that the students encounter. In this study, I found some examples in the two tasks investigated that may be related to these issues. Further research with more students and tasks can help to identify more text features that may cause DRA.

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