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Classroom communication: Defining and characterizing perpendicular lines in high school algebra

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A study was conducted with a high school algebra teacher and her students with the purpose of gaining insight into the students' difficulties with perpendicular lines. The study reported here focuses on the communication between the teacher and her students, and the nature of their messages. Classroom observations were conducted, and a written task was administered to students. The communication was analyzed using constructs from Hall's theory of culture. The analysis revealed that the teacher's messages were not received by her students in the way intended by the teacher. The students had difficulties defining perpendicular lines, representing graphically and characterizing them algebraically, and connecting between representations.

Keywords: Classroom communication, perpendicular lines, high school algebra.

Introduction

Research on students' difficulties with perpendicular lines and linear equations in the context of high school algebra is scarce (Postelnicu, 2017). Some studies point to students' difficulties with graphical representations and connecting among various representations (Arcavi, 2003; Knuth, 2000), and teachers' failure to identify the nature of their students' difficulties in geometric context (Gal & Vinner, 1997). The purpose of this study is to gain insight into the students' difficulties with perpendicular lines and their equations in the context of high school algebra, as it is taught and learned in a public school in the United States. The focus is on the mathematical knowledge constructed during the classroom interaction (Steinbring, 2005), i.e., on the communication between the teacher and her students, the nature of their messages, and their different interpretations of those messages. Nührenbörger and Steinbring (2009) studied the different interpretations by the teacher and two students of a mathematical statement, and pointed to the dominance of the teacher's ideas during the observed interaction. In the study reported here, the relations between the teacher's language (her messages), the language of the learners (students' messages), and the language of mathematics (textbook's messages) (Planas, Morgan, & Schütte, 2018; Planas & Schütte, 2018) are studied using Hall's (1959) theory of culture.

Theoretical framework: Hall's theory of culture

The teaching and learning of mathematics are cultural activities that take place in various institutions (e.g., the classroom from this study). The theoretical framework used to analyze the classroom culture abides by several principles (Hall, 1959):

i) Culture is equivalent with communication. "Culture is communication and communication is culture" (Hall, 1959, p. 217). This principle of equivalence between classroom culture and classroom communication, allowed us to account for the teaching and learning of mathematics in the observed

classroom by analyzing the communication between the teacher and her students. The idea of equivalence between doing mathematics and communicating mathematically is also found in Sfard (2008).

ii) We communicate through messages. A message is constituted by *sets* communicated in *patterns*. *Sets* are constituted by *isolates*. When one communicates using a language, one uses words (*sets*) that are made of sounds (*isolates*). The words are communicated in some context using a specific syntax (*pattern*) “in order to give them meaning.” (Hall, 1959, p. 124). Old words, communicated using a *pattern*, are used to give meaning to new words. When we learn a new language, the words (*sets*) are perceived first. It takes time to understand the *pattern* of communication in a new language and construct meaningful sentences. As well, it may take a long time to master the sounds (*isolates*) of the new language and pronounce them without the accent from the old language. At the academic level, the mathematical knowledge is communicated in statements like axioms, definitions, or theorems. These statements constitute the *sets*. Examples of such *sets* are the definition of perpendicular lines, or the theorem of algebraic characterization of two perpendicular lines. The *isolates* are the mathematical objects and their properties - like line, slope of a line, lines that intersect to form a right angle. In scholarly mathematics, the *pattern* of communication is the formal proof; it gives meaning to statements/theorems. Old statements, already accepted as true, are used to logically derive new statements, using the accepted rules of inference. Oftentimes, when validating a new statement, the steps of the argument are expressed in natural language and employ previously validated statements (e.g., see Postelnicu (2017) for an example of a proof of the theorem of algebraic characterization of two perpendicular lines described as a “Key Concept” in Table 1, below). The communication *pattern* changes with the cultural context/institution. In school mathematics, the *pattern* of communication may be a justification (e.g., see Wallace-Gomez & Miller (2011) for a visual justification of the “Key Concept” from Table 1, below). In United States high schools, researchers point out that proofs are found mainly in geometry courses, and are almost non-existent in algebra courses (Martinez & Superfine, 2012). In the study reported here, the participants used an Algebra 1¹ textbook (Larson et al., 2007) that contains mathematical statements (e.g., “key concepts”) without proofs. The *pattern* of communication consists of stating the “key concepts” and providing examples of algorithms to execute tasks.

iii) We can only observe a message at one level of communication at a time. When a message is communicated from one cultural context to another, its meaning may change. As observed in this study, the message changes when the level of communication changes (from textbook to teacher, and from teacher to students). Consequently, it is imperative to account for the way the textbook message (with its *pattern* of communication, *sets*, and *isolates*) changes into the teacher’s message, and then into the students’ message.

Methods

¹ In the United States, students take the first algebra course before the geometry course.

The study reported here is part of a larger study with an Algebra 1 high school teacher and her students from a public high school in the United States. The study lasted four weeks and included a teaching experiment with multiple episodes, with the author as a witness (Steffe & Thompson, 2000), classroom observations carried out by the author (Erickson, 1985), and the administration of various written tasks to students. Only 58 students out of 137 agreed to have their answers reported in this study. The focus is on the classroom communication, as observed by the author, the day after the teacher communicated to her students the definition of perpendicular lines and the “Key Concept” about the slopes of perpendicular lines (described in Table 1, below). The author observed all six classes taught by the teacher in one day, took observation notes, and collected copies after the students’ written answers to the following task administered by the teacher, **Task 1**:

- “a) Sketch the graph of the line that is perpendicular to $y = \frac{3}{4}x - 2$ and goes through the point $(2, -6)$. What is the equation of the new line you created?
- b) How did you determine this?
- c) A fellow student does not understand what perpendicular lines are. Explain to them in as much detail as possible what perpendicular lines are and how you find them.”

Before administering Task 1, the teacher assigned a warm-up task on parallel lines, similar to Task 1a², and solved it on the whiteboard. She also included in her the post warm-up communication a review of the knowledge on perpendicular lines, necessary to execute Task 1a. Students’ written answers to Task 1 were collected. Due to space constraints, I analyze only Task 1c. In my analysis I refer to the connections between Tasks 1a, 1b, and 1c.

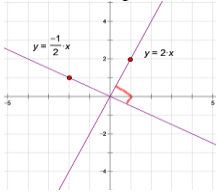
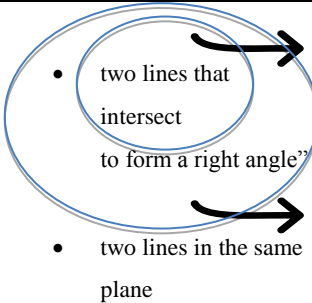
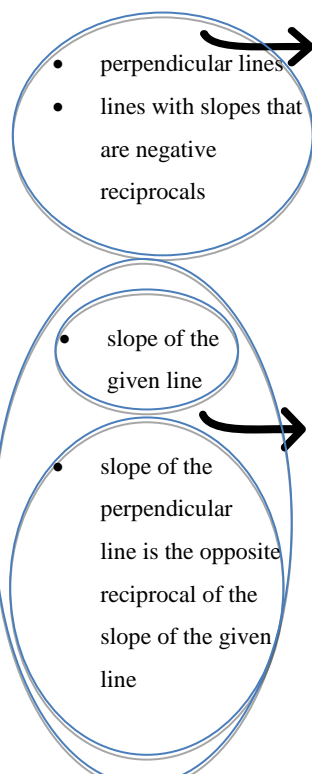
Analysis and results

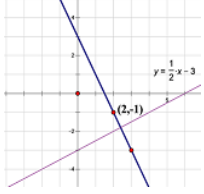
Table 1, below, represents the levels of communication taken from the Algebra 1 textbook (Larson et al., 2007), the author’s observation notes on the teacher’s communication with her students, and the students’ written answers to Task 1c. The analysis considers the messages at each level, as well as the changes in message from one level to another. The messages are decomposed in *sets* (titled in bold, for example **Statements 1-4**, at the student level), and the *isolates* are presented with bullets. The textbook messages are presented in the first column from Table 1, followed by the teacher’s messages, preceded by “**T:**”. The teacher’s messages were subsets of the textbook messages, hence the presentation in the same column from Table 1. The Venn diagrams and the arrows illustrate the change in message from the teacher level to the student level. Table 2, following Table 1, presents examples from the students’ written answers on Task 1c. Each of the **Statements 1-4** is accompanied by **Examples** from the students’ answers. For continuity of presentation, Table 2 restates in its first columns the student messages (*sets* with their *isolates*), exactly like in Table 1. The analysis follows each message (*set* and its *isolates*) horizontally in Tables 1 and 2, respectively. As mentioned before, when one encounters a

² Sketch the graph of the line that is parallel to $y = \frac{1}{2}x - 3$ and goes through $(2, -1)$.

new language, the words (*sets*) are perceived first, therefore the analysis was conducted at the level of *sets*.

It can be seen in Table 1 that in the context of Task 1c, three *sets* were communicated at the textbook and teacher level: the definition of perpendicular lines, the “key concept” stating the theorem of algebraic characterization of two perpendicular lines, and the first step of the algorithm for executing Task 1a, the step referring to identifying the slope of a line perpendicular to a given line. Each of those *sets* are analyzed at the textbook, teacher, and student level. It can be observed in Table 1 that the *sets* at the student level were constituted from the *isolates* at the textbook and teacher level.

Communication at the textbook and teacher level (T)		Communication at the student level	
<i>Sets</i>	<i>Isolates</i>	<i>Sets</i>	<i>Isolates</i>
<p>“Perpendicular Lines: Two lines in the same plane are perpendicular if they intersect to form a right angle.” (Larson et al., 2007, p. 320)</p>  <p>T: “Perpendicular lines intersect and form a right (90°) angle.”</p>	 <ul style="list-style-type: none"> two lines that intersect to form a right angle” two lines in the same plane 	<p>Statement 1 about intersecting lines: Perpendicular lines are lines that intersect. (N=35)*</p> <p>Statement 2 about intersecting lines forming a right angle: Perpendicular lines are two lines that intersect to form a right (90°) angle. (N=6)</p>	<ul style="list-style-type: none"> line two lines two lines that intersect two lines that intersect angle right angle
<p>“Key Concept: If two nonvertical lines in the same plane have slopes that are negative reciprocals, then the lines are perpendicular. If two nonvertical lines in the same plane are perpendicular, then the slopes are negative reciprocals.” (Larson et al., 2007, p. 320)</p> <p>T: “Perpendicular lines have slopes negative (opposite) reciprocals.”</p> <p>Algorithm for Task 1a (Step 1): Identify the slope m of the new line based on the “key concept” (perpendicular lines have slopes negative reciprocals). (Larson et al., 2007, p. 321)</p> <p>T: “You take the slope of the original line, a/b, that is $1/2$, flip it, b/a, that is $2/1$, and change its sign, $-2/1$.” (Note: Given the line and the point from the warm-up activity, the teacher drew the line through $(2, -1)$ with the slope $-2/1$).</p>	 <ul style="list-style-type: none"> perpendicular lines lines with slopes that are negative reciprocals slope of the given line slope of the perpendicular line is the opposite reciprocal of the slope of the given line 	<p>Statement 3 about the slopes of perpendicular lines: Perpendicular lines have slopes that are negative reciprocals. (N=8)</p> <p>Statement 4 about finding the slope of a perpendicular line: To find the slope of a line perpendicular to a given line, you find i) the slope of the given line, and ii) the negative/opposite reciprocal of the slope of the given line. (N=28)</p>	<ul style="list-style-type: none"> two perpendicular lines slope negative reciprocals given line slope of the given line the negative/opposite reciprocal of the slope of the given line perpendicular line

	<ul style="list-style-type: none"> • nonvertical lines 		
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* (N=35), after Statement 1, represents the number of students who communicated that statement.

Table 1: Representation of levels of communication

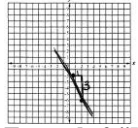
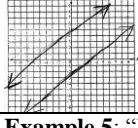
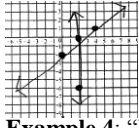
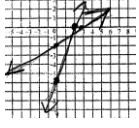
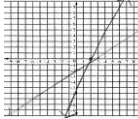
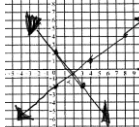
The first *set* in Table 1 (column 1) defines perpendicular lines. The textbook marks one of the right angles formed by two perpendicular lines in a graphical representation similar to the one from Table 1 and points to the “Key Concept” necessary to execute Task 1a (a 90° rotation of a line generates a perpendicular line, and the two lines have slopes that are opposite reciprocals). At the textbook level, the definition contains *isolates* like “two lines in the same plane” and lines that “intersect to form a right angle” (Table 1, column 2). At the teacher’s level, the *isolate* “two lines in the same plane” was omitted, and the right angle was sometimes referred to as a “ 90° angle.” As noticed in Table 1 (columns 2 and 3) at the student level, the teacher’s *isolate* referring to two lines that “intersect to form a right angle” changed into two *sets*, **Statements 1** and **2**, the first referring to intersecting lines and the second referring to intersecting lines that form a right angle. Consequently, more than half of the students (N=35) defined perpendicular lines as lines that intersect. The *isolate* “line” was not problematic for students with respect to their graphical representations (all students graphed lines), but the other *isolates* from **Statement 1**, “two lines” and “two intersecting lines” posed difficulties. The examples accompanying **Statement 1** in Table 2 suggest the following difficulties: i) conceiving of the second line, given one line (**Example 1**, “a perpendicular line is a line that intersects with something”); ii) connecting between representations (parallel lines were represented in **Example 2**, but described as “lines that intersect, they both share the same point,” and parallel lines that “do not ever touch” were described in **Example 3**, where intersecting lines were graphed); iii) conceiving of the intersecting point (the point of intersection was mentioned in three of the students’ answers but was not marked in any of the students’ answers, making questionable the meaning of “intersecting lines” in answers like in **Example 4** where intersecting lines were described as “touching or crossing over each other”).

Continuing with the **Statement 2** in Table 2, only six students mentioned “lines that intersect to form a right (or 90°) angle, three of them did not have matching graphical representations (see **Example 5**, where the lines intersect but do not form right angles), and three of them had matching graphical representations like in **Example 6**. The analysis of the students’ answers including **Statement 2**

indicates that while the *isolate* “two intersecting lines” did not pose difficulties, the *isolate* “right angle” was problematic. This issue raised the question if the difficulty was limited only to the measure of the angle or if it extended to the concept of angle itself.

To execute Task 1a, one needs the theorem of algebraic characterization of two perpendicular lines in a Cartesian system of coordinates (see the “**Key Concept**” in Table 1, column 1), and the algorithm to apply it (see also the **Algorithm for Task 1a (Step 1)**). At the teacher level, the theorem was stated “Perpendicular lines have slopes negative reciprocals,” and sometimes “negative” was replaced by “opposite” to emphasize that the slopes of perpendicular lines have opposite signs. The *isolate* “nonvertical lines” was not mentioned by the teacher during classroom observations. As mentioned in **Methods**, the teacher reminded her students how to determine the slope of the line perpendicular to the given line $y = (1/2)x - 3$, by finding the opposite reciprocal of the slope of the given line: “You take the slope of the original line, a/b , that is $1/2$, flip it, b/a , that is $2/1$, and change its sign, $-2/1$.”

The teacher matched the algebraic and graphical representations (see the last figure in Table 1, column 1). The students used the *isolates* from the teacher level (Table 1, column 2) to constitute their *sets*, **Statements 3** and **4** (see Table 1, column 3). **Statement 3** refers to a characteristic property used by students to define two perpendicular lines, while **Statement 4** refers to the algorithm of finding the slope of a line perpendicular to a given line, $y = mx + b$. None of the students who stated the property from **Statement 3** used it correctly in the algorithm described in **Statement 4**. Although the students mentioned “slope” and “negative reciprocals,” they failed to identify the slope of the given line or its negative reciprocal. Therefore, the students’ answers were reported separately for **Statements 3** and **4**.

Communication at the student level		Examples from the students’ written answers	
Sets	Isolates		
<p>Statement 1 about intersecting lines: Perpendicular lines are lines that intersect. (N=35)</p>	<ul style="list-style-type: none"> line two lines two lines that intersect 	<p>Example 1: “A perpendicular line is a line that intersects with something.”</p>  <p>Example 2: “Perpendicular lines are lines that intersect, they both share the same point.”</p> 	<p>Example 3: “Perpendicular lines are lines that are side by side and do not ever touch each other.”</p>  <p>Example 4: “Perpendicular lines are lines that are straight but are touching or crossing over each other.”</p> 
<p>Statement 2 about intersecting lines forming a right angle: Perpendicular lines are two lines that intersect to form a right (90°) angle. (N=6)</p>	<ul style="list-style-type: none"> two lines that intersect angle right angle 	<p>Example 5: “Perpendicular lines [...], the intersection of the lines form 90° angles.”</p> 	<p>Example 6: “Perpendicular lines- lines that intersect with a 90° angle.”</p> 

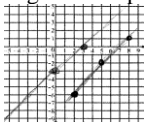
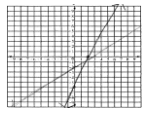
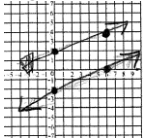
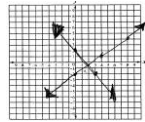
<p>Statement 3 about the slopes of perpendicular lines: Perpendicular lines have slopes that are negative reciprocals. (N=8)</p>	<ul style="list-style-type: none"> perpendicular lines slope negative reciprocals 	<p>Example 7: “A perpendicular line are 2 nonvertical lines that have negative reciprocals.”</p> 	<p>Example 8: “Perpendicular lines are two nonvertical lines that have negative reciprocals. Slopes are a/b and $-b/a$.”</p> 
<p>Statement 4 about finding the slope of a perpendicular line: To find the slope of a line perpendicular to a given line, you find i) the slope of the given line, and ii) the negative/ opposite reciprocal of the slope of the given line. (N=28)</p>	<ul style="list-style-type: none"> given line slope of the given line the negative/ opposite reciprocal of the slope of the given line perpendicular line 	<p>Example 9: “Perpendicular lines touch, you must flip the sign and make it a negative.”</p> 	<p>Example 10: “Perpendicular lines—lines that intersect with a 90° angle. In order to find them you need to plot the original $y = \frac{1}{2}x + 3$, then make everything opposite of that, $y = -2x + 3$, then graph.”</p> 

Table 2: Representation of the communication at the student level

Only one of the eight students who referred to **Statement 3** in their answers had a graphical representation of perpendicular lines. The other answers had graphical representations of parallel lines and non-perpendicular, intersecting lines (see **Examples 7** and **8**, respectively, accompanying **Statement 3** in Table 2). In **Examples 7** and **8** perpendicular lines were described as “lines that have negative reciprocals,” and in **Example 7** “slope” was omitted. The *isolate* “perpendicular lines” remained problematic for students. Likewise, the *isolate* “slope” posed great difficulty, especially in the graphical representation. The *isolate* “negative reciprocals” or “opposite reciprocals” was used by 28 students (see Table 2, **Statement 4** and **Examples 9-10**). However, *isolates* like “perpendicular line” and “given line” and its “slope” posed difficulties for students, as shown in their graphical representations (see **Example 9** with parallel lines). In **Example 10**, the slopes of the two lines are those from the teacher’s example used in the post warm-up communication, the student wrote “make everything opposite,” although the y -intercept of the lines was not changed, and the graphical and algebraic representations did not match. Only 14 students determined correctly the slope of the perpendicular line in Task 1a, but they failed to find the correct y -intercept. Notwithstanding a minor sign error, two students found the equation of the perpendicular line from Task 1a, but their algebraic and graphical representations did not match.

Conclusions

The communication between the teacher and her students, analyzed at the level of *sets*, showed the following features:

- 1) The lessons observed in this study were “ritualistic” and abided by the following “routine” (Lavie, Steiner, & Sfard, 2019): like in the textbook, the teacher stated the “key concepts” and presented a “model” for executing a task, then asked the students to execute a similar task. The *pattern* of communication lacked justifications or proofs.

2) The teacher's messages were not understood by the students in the way the teacher intended. The teacher communicated using *sets* like those from the Algebra 1 textbook (Larson et al., 2007), but only some of the *isolates* of those *sets* were received by students. The students communicated using *sets* that were constituted from the teacher's *isolates*, with a *pattern* lacking justification or proofs.

To overcome the effects of the "ritualization," Lavie et al. (2019) recommend turning rituals into explorations. In our case, another way to think about teaching about perpendicularity would have been to look for *patterns* of *isolates* within each *set*. For example, in the case of the *isolate* "right angle," one such *pattern* might have been constituting all four right angles that are formed by two perpendicular lines, pointing to their vertex and sides (to constitute the *isolate* "angle"), describing them (their measure), giving counterexamples (acute and obtuse angles), and connecting the verbal and graphical representations. An example of an exploration to justify the "**Key Concept**" (see Table 1, column 1) is described by Wallace-Gomez and Miller (2011).

References

- Arcavi, A. (2003). The role of visual representations in the teaching and learning of mathematics. *Educational Studies in Mathematics*, 52(3), 215-241. doi: 10.1023/A:1024312321077
- Erickson, F. (1985). *Qualitative methods in research on teaching*. Occasional Paper No. 81. East Lansing, MI: The Institute for Research on Teaching, Michigan State University.
- Gal, H., & Vinner, S. (1997). Perpendicular lines - What is the problem? Pre-Service teachers' lack of knowledge on how to cope with students' difficulties. In E. Pehkonen (Ed.), *Proceedings of PME21* (Vol. 2, pp. 281-288). Lahti, Finland: PME.
- Hall, E. T. (1959). *The silent language*. Garden City, NY: Doubleday.
- Knuth, E. J. (2000). Student understanding of the Cartesian connection: An exploratory study. *Journal for Research in Mathematics Education*, 31(4), 500-508. doi: 10.2307/749655
- Larson, R., Boswell, L., Kanold, T., & Stiff, L. (2007). *Algebra I*. Evanston, IL: McDougal Litell.
- Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics*, 101(2), 153-176. doi: 10.1007/s10649-018-9817-4.
- Martinez, M. V., & Superfine, A. C. (2012). Integrating algebra and proof in high school: Students' work with multiple variables and a single parameter in a proof context. *Mathematical Thinking and Learning*, 14(2), 120-148. doi: 10.1080/10986065.2012.657956
- Nührenbörger, M., & Steinbring, H. (2009). Forms of mathematical interaction in different social settings: Examples from students', teachers' and teacher-students' communication about mathematics. *Journal of Mathematics Teacher Education*, 12(2), 111-132. doi: 10.1007/s10857-009-9100-9
- Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics and language: Lessons and directions from two decades of research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.),

Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe (pp. 196-210). New York: Routledge.

Planas, N., & Schütte, M. (2018). Research frameworks for the study of language in mathematics education. *ZDM*, *50*(6), 965-974. doi: 10.1007/s11858-018-0997-2

Postelnicu, V. (2017). Didactic transposition in school algebra: The case of writing equations of parallel and perpendicular lines. In T. Dooley & G. Gueudet (Eds.), *Proceedings CERME10* (pp. 480-487). Dublin, Ireland: Dublin City University, Institute of Education and ERME.

Sfard, A. (2008). *Thinking as communicating. Human development, the growth of discourses and mathematizing*. New York: Cambridge University Press.

Steffe, L. P., & Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. In A. E. Kelly, & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 267–307). Mahwah, NJ: Lawrence Erlbaum Associates.

Steinbring, H. (2005). *The construction of new mathematical knowledge in classroom interaction –An epistemological perspective*. New York: Springer.

Wallace-Gomez, P., & Miller, J. (2011). Slopes of parallel and perpendicular lines: A visual justification. *The Mathematics Teacher*, *104*(9), 728-728.