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# Explaining geometrical concepts in sign language and in spoken language – a comparison

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*The explaining of mathematical terms is part of both learning and teaching mathematics. In this study, explanations, in the forms of a video in sign language, a video and an audio recording in spoken language (all on the subject of quadrilaterals), are compared. Therewith, the explanation process is understood as semiotic mediation, as proposed by Hasan (2002, 2005). The question regarding which role the modality plays in transitioning from specific geometric forms to general statements of the geometric forms is also pursued.*

*Keywords: Semiotic mediation, sign language, spoken language, use of representations.*

## Introduction

The explaining of mathematical terms is part of both learning and teaching mathematics. With this, the process of explanation can be understood as semiotic mediation (Hasan, 2002, 2005), which refers to a “mediation by means of the modality of language” (Hasan, 2002, p. 112) This study focuses on the modality of language, or to be more specific, on the differences in explanations in sign language and in spoken language. For the subject matter of the ‘House of Quadrilaterals’ (a hierarchic structure of different quadrilaterals; see figure 1), three medially different explanations were developed along the lines of the same subject analysis: one video and one audio product, both in German spoken language, along with one video in Austrian Sign Language (ÖGS<sup>1</sup>). Thus, this study focuses on the question: What roles do modalities of spoken and sign language play in transitioning from specific geometric forms to general statements of the geometric forms?

## Semiotic mediation of a geometrical concept

Hasan (2002, 2005) considers ‘semiotic mediation’ as a process, in which a mediator mediates something to a mediatee, the addressee. Though, the details of the mediation, regarding modality and where it took place, can differ (cf. Hasan, 2005). For the process of mediation, Hasan (2002, p. 115) distinguishes between acting by doing and acting by saying, which she defines as *material action* and *verbal action*, respectively. In the context of this paper, for example, material action includes writing down or pointing at mathematical inscriptions on blackboard or on pinboard (see Figure 2). Both forms of action can take place independently, though possibly in the same place or even simultaneously. In the last case, Hasan differentiates between *ancillary verbal action*, where “verbal action is assisting in the conduct of the ongoing material activity” (p. 116) and *constitutive verbal action*, where the verbal action is not “running in parallel” (p. 116) with the material action, but is pivotal for the topic of a discussion. Hasan primarily considers daily conversations, wherein a

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<sup>1</sup> ÖGS stands for “Österreichische Gebärdensprache“.

physical action is commented on and, after a *shift* from ancillary to constitutive verbal action, the discussion then distanced from the physical level to then focus on topics, which go beyond the physical level. Hasan considers this shift to be of importance because, after such a change, an additional invisible mediation can take place in which habits of mind can be communicated.

Mathematical inscriptions (on paper, on the chalkboard, etc.) and experimentation with these is a fundamental part of mathematical activity (cf. Dörfler, 2016), which is principally of material nature. With the help of inscriptions, one can not only provide for concrete propositions, but can also generalize these. This is especially visible in geometry when, for example, quadrilaterals are compared and general propositions are made. Dörfler (1991) describes this as an “empirical generalization”, though the “basic process is to find a common quality or property among several or many objects or situations (mostly from sense perceptions) and to notice and record these qualities as being common and general to these objects or situations” (p. 65). For example, the figurative forms of geometric inscriptions can be compared and then generalizations can be made. For instance: ‘Every square is a rectangle, but not every rectangle is a square.’

How can an explanation, which leads to a generalization, differ with different language modalities such as sign language and spoken languages? Sign languages are natural languages, just like spoken languages. Though, these are visual, in contrast to auditory spoken languages. The consequence of this is that different visual levels can be processed simultaneously (cf. Grote & Linz, 2003). Particularly, signs utilize space in a distinct manner: Often times, in narratives, a “singular aspect” is focused on and located in the sign space. This point of focus is referred back to throughout the narration (cf. Grote, 2016, p. 144). One example is the description of a picture: In spoken language, a picture is typically described in a linear fashion, for example, from left to right. Contrasting to this, in sign language, a central object is described and all other objects are then related to it.

A further central characteristic of sign language is the iconicity and indexicality of many signs, especially in terms of mathematical signs (cp. Krause, 2017; Wille, submitted). An icon, understood in the sense of Peirce, represents relations (cp. Hoffmann, 2007, p. 3; Peirce EP II 13). Mathematical signs rather frequently picture either the apparent relational structure of mathematical inscriptions or are similar to operations with the inscriptions. This is not limited to geometric forms only (cp. Wille, submitted), even if such are the focus of this study. To provide for an example, in Austrian Sign Language (ÖGS), the sign RECTANGLE<sup>2</sup> is communicated by tracing the geometrical form of a rectangle with the fingers. The sign ROTATION, on the other hand, resembles the actual operation of rotating a geometric form (cf. Schreiber & Wille, submitted). This principally means that mathematical ÖGS signs often look like concrete inscriptions. Although signs are used symbolically, just as often as words in spoken languages, this “does not imply that the iconic dimension of a linguistic sign becomes completely blended out or deleted” (Grote & Linz, 2003, p. 35). In the following, the question regarding how the transition from specific forms to general propositions can take place in sign language is compared to spoken language.

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<sup>2</sup> In this study, ÖGS signs are written in gloss with capital letters as is common in sign language literature. (cf. Skant et al., 2002).

### Three different explanations regarding quadrilaterals

Three medially different explanations on how a square is related to other quadrilaterals were developed on the basis of the same subject analysis at the universities of Giessen and Klagenfurt:

For the video in spoken language, based on the subject analysis, a script was prepared with different scenes, containing the action of the students, the spoken text and material to use in the video (see the example below). This script was critically discussed with the students and optimized. In the video, reproductions of the geometrical figures were successively pinned to the board. These figures were discussed and relationships between the figures were identified and explained.

For the audio recordings, the script dialogue was developed so that a pair of both male and female speakers reciprocally demonstrated the relationships between the quadrilaterals, which, in this case, are not presented in the form of a picture.

The realization into Austrian Sign Language (ÖGS) took place in cooperation with a deaf colleague from the center for sign language and hearing-impaired communication of University Klagenfurt. For this, a script in German written language served as the basis for a discussion in sign language, concerning how this text could be properly realized. This was then transferred to a teleprompter, in gloss, which was signed with slight variation in the video. Geometrical forms were also attached to a bulletin board. For the spoken language realizations, in the forms of both audio and video, scripts also served as the foundation. Though, these scripts were not changed, due to the fact that there was only a shift from “writteness” to “orality” (see Schreiber, 2013, p. 1598).

These videos and the audio do not have the purpose of replacing lessons, but were rather developed with the intention of comparing various modalities of the mathematical explanation. For this reason, the influence of such explanations on students is not being focused upon.

### Analysis of videos in sign language from the standpoint of Hasan’s theoretical perspective

In contrast to mathematical terminology in spoken language, mathematical signs feature an inherent form of materiality. In ÖGS, forms such as the square, rectangle and rhombus are traced with the fingers. The sign ROTATION not only depicts that something is being rotated, but one can also see the type of object. Furthermore, for the sign PARALLELOGRAM, a right-angled parallelogram is transformed into a non-right-angled parallelogram (see Figure 1). This means that in ÖGS, the difference between material action and verbal action is not as distinguishable as in German spoken language.

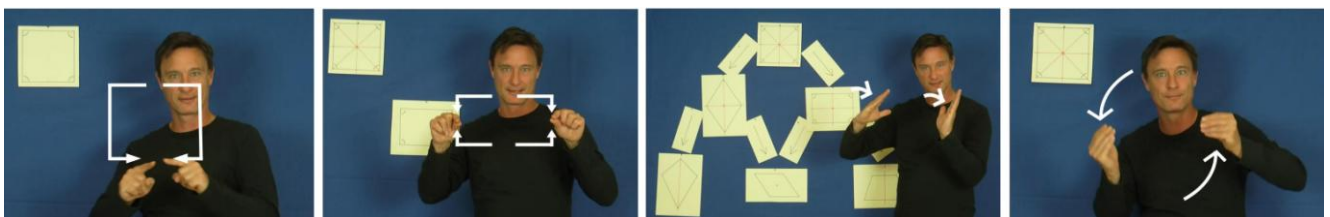


Figure 1: The ÖGS signs SQUARE, RECTANGLE, PARALLELOGRAM and ROTATION

Which shifts from ancillary to constitutive verbal action are thus discernable in the ÖGS video? At first, the ÖGS video will be compared to the corresponding script. In the next section, a comparison to the spoken language video and audio follows.

When comparing the script text for the ÖGS video with the actual transcript for the ÖGS video in gloss, it can be seen that in the ÖGS video, ancillary verbal action increases and, simultaneously, constitutive verbal action decreases. The following section demonstrates this noticeably.

The following text is an excerpt of the script of line 42 to 47:

**Script Text in German**

**English Translation**

|    |   |   |
|----|---|---|
| 42 | Also: Dieses Rechteck ist kein Quadrat.                         | So, this rectangle is not a square.                     |
| 43 | Es besitzt nicht alle Eigenschaften, die das Quadrat hat.       | It does not possess all the properties of the square.   |
| 44 | Aber wie sieht es andersherum aus?                              | But, what about the other way around?                   |
| 45 | Ist ein Quadrat ein Rechteck? – Ja!                             | Is a square a rectangle? – Yes!                         |
| 46 | Jedes Quadrat ist ein Rechteck. Es ist ein besonderes Rechteck. | Each square is a rectangle. It is a specific rectangle. |
| 47 | Es besitzt alle Eigenschaften eines Rechtecks.                  | It has all the properties of a rectangle.               |

The corresponding transcript (in gloss) for the ÖGS videos is the following:

**Transcript for the ÖGS videos in gloss**

**Translation in English**

|    |   |  |
|----|---|--|
| 42 | IX RECHTECK NICHT INHALT QUADRAT                                | This rectangle is not a square   |
| 43 | NICHT ALLE EIGENSCHAFT  | because it does not have all the properties.   |
| 44 | IX QUADRAT INHALT JA RECHTECK                                   | This square is indeed a rectangle.   |
| 45 | AUFPASSEN QUADRAT IX BESONDERS INHALT ALLE EIGENSCHAFT DA DA DA | It should be noted that this square is a specific one (a special rectangle), since all properties are fulfilled. |
| 46 | ABER RECHTECK NICHT IMMER QUADRAT                               | But a rectangle is not always a square.  |
| 47 | BEISPIEL IX RECHTECK INHALT KEIN QUADRAT                        | For example, this rectangle is not a square.   |

In the section above, there is a shift to constitutive verbal action. First, in lines 42 and 43, it states: “So, this rectangle is not a square. It does not possess all the properties of the square.” This refers to the specific rectangle, which is attached to the bulletin board in the background and was used to speak about reflections, rotations, parallel sides and right angles. After this, at line 44, there is a transfer to constitutive verbal action, since all squares are now being talked about in a general way: “But, what about the other way around? Is a square a rectangle? – Yes!” Instead of speaking about “this square”, it is now called “a square”. Thus, the verbal action changes from speaking about the concrete forms to then speaking about forms of one type in general.

In contrast to this, the verbal action in the corresponding part of the ÖGS video is, with the exception of line 5, ancillary. The ÖGS sign IX is an indexical sign, with which the pointer finger signs. It can be translated with “this”. It is noticeable that only in line 46 there is constitutive verbal

action. This means that only one shift from ancillary to constitutive verbal action is between lines 46 and 47. The expression “a rectangle is not always a square” can be described in no other manner than generally. All other sentences concretely reference the rectangle on the bulletin board.

A second thing is noticeable in this section, when comparing the ÖGS video with the script text: In the script, a concrete rectangle is at first discussed and then the discussion moves to only generally talk about squares. In contrast to this, “IX RECHTECK” (this rectangle) seems to act as an anchor in the ÖGS video. This occurs at both the beginning and the end and acts as the central point to which the rest then references. Such concurs with the observations, noted above, stating that sign language narratives often place something singular in the focus of attention, which is then referenced back to. In this section it’s “this rectangle”. In the context of the whole ÖGS video, however, “IX QUADRAT” (this square) takes on this role.

For example, such can be seen in the following excerpt (line 63 to 65), concerning the relationship between the rhombus and the square. In the script, solely constitutive verbal action can be observed:

**Script Text in German**

- 63 Ist eine Raute ein Quadrat? – Nein! Sie besitzt nicht alle Eigenschaften des Quadrats.
- 64 Ist ein Quadrat eine Raute? – Ja! Jedes Quadrat ist eine besondere Raute.
- 65 Alle Eigenschaften der Raute sind erfüllt.

**English Translation**

- Is a rhombus a square? - No! It doesn't possess all the properties of a square.
- Is a square a rhombus? - Yes! Each square is a special rhombus.
- All properties of a rhombus are fulfilled.

Whereas one can observe the following in the ÖGS video segment:

**Transcript for the ÖGS videos in gloss**

- 63 IX RAUTE INHALT QUADRAT? NEIN
- 64 ABER AUFFASSEN IX QUADRAT INHALT RAUTE? JA
- 65 WARUM? IX ALLE EIGENSCHAFT RAUTE DA DA DA
- a IX QUADRAT BESONDERS RAUTE

**Translation in English**

- Is this rhombus a square? No!
- But watch out! Is this square a rhombus? Yes.
- Why is that so? All properties – are fulfilled – those discussed earlier
- This square is a special rhombus.

The script’s linear-constructed text in the ÖGS video is once again changed. In the last part, “IX QUADRAT” (this square) is referred to multiple times.

In addition, it seems as if this is all ancillary verbal action. Nevertheless, there is something here, which cannot be equally expressed in spoken language. The ÖGS sign “IX” in line 64, is signed toward the bulletin board (see Figure 2, left-hand side). Here, a reference is being made to the concrete square. Though, later in line <sup>a</sup>3, “IX” is signed to the front, toward the audience (see Figure 2, right-hand side). This can be understood as an intermediate step for generalization, respectively as an intermediate shift between ancillary and constitutive verbal action. In this case the second

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<sup>3</sup> This line in the transcript does not correspond to a line in the script. Therefore, this additional line is marked with “a”.

phrase “IX QUADRAT” (this square) in line *a* not only refers to the concrete square, but also to the square as a paradigmatic example, which stands for the general case.

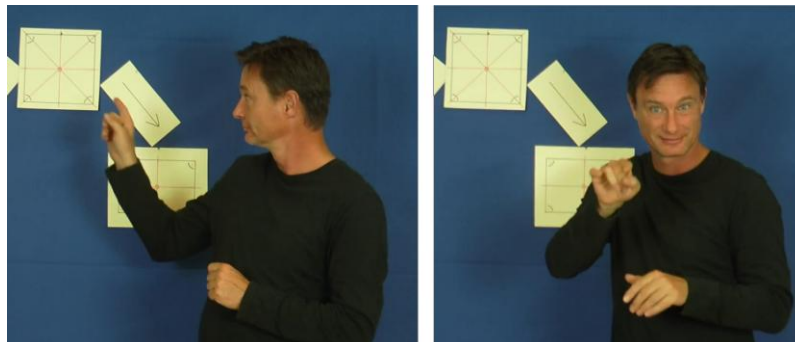


Figure 2: The ÖGS sign “IX” in line 64 (left-hand side) and line *a* (right-hand side)

### In comparison: ancillary and constitutive verbal action of spoken language in video and audio

Due to a lack of space for this study, both the spoken language video and the audio recording can only be briefly outlined, in order to substantiate the indications. More detailed examples can be found in Schreiber & Wille (submitted).

The explanations in the video, in spoken language, are categorized as verbal action, while the material action takes place through either pointing at the constituting properties of the forms, drawing in properties or by drawing in arrows to indicate relationships between the quadrilaterals. For example, this is apparent in the excerpt of the script, example *a*:

| <i>Scene</i> | <i>Action</i>                                  | <i>Spoken Text</i>   | <i>Depiction</i> |
|--------------|--|--|------------------|
| 2            | The person points to the angles and the sides. | It’s directly noticeable that there are four equally long sides. The sides are in a certain position in respect to each other. Adjacent sides build a right angle and opposite sides are parallel. |                  |

In this video, concrete forms are referred to in the sense of ancillary verbal action (see example *a*). Forms are recurrently referred to in a general sense, utilizing constitutive verbal action to illustrate general relationships, like in example *b*:

| <i>Scene</i> | <i>Action</i>   | <i>Spoken Text</i>   | <i>Depiction</i> |
|--------------|---|--|------------------|
| 4            | The camera zooms out so that all figures are visible.<br><br>The person draws an arrow between the square and | It’s also noticeable that this rhombus has properties similar to the square. Both have four equally long sides but different angles and, because of this, different symmetries. We are drawing an arrow from the square to this rhombus. The square is a special form of the rhombus, actually a rhombus, which has four right angles. |                  |

Initially, the concrete rhombus is referred to, which is located on the bulletin board. It is directly compared to “the square”, though this does not necessarily refer to the concrete square, which is located on the board. “The square is a specific kind of rhombus” is already formulated in a general sense, which is emphasized by referring to the properties, a “rhombus, which has four right angles”.

In the audio, exclusively general cases are described. One can only speak of forms in general, due to the absence of the visual mode of presentation. Thus, constitutive verbal action is utilized throughout this special setting. Such can be seen with student 1’s initially posed question and student 2’s corresponding answer:

- s1 Honestly, what is actually so special about a square? It’s really just a very basic quadrilateral.
- s2 The square combines the properties of all the other quadrilateral.

No specific square is being referred to, even though *the* square is noted. In particular, this square can readily be compared to all quadrilaterals. Such is also the case at a later point, when student 2 compares the general rhombus with the general square and when student 1 explains the general rhombus more in-depth:

- s2: That’s right. For the rhombus, the difference to a square is that the four angles mustn’t be right angles, right?
- s1: Yes, the angles don’t have to be right angles for the rhombus.

The utilization of material action is omitted and, in the here-discussed example, ancillary verbal action is not present.

## Conclusion

A comparison of the three medially different explanations shows that the prevalence of ancillary to constitutive verbal actions differ. In the ÖGS video, concrete forms are recurrently referred to. As previously discussed, when the ÖGS sign “IX” (this) is signed toward the front, this can be interpreted as a reference to a paradigmatic example.

Hasan (2002) explains that, in terms of constitutive verbal action, habits of mind can be mediated. She calls this invisible mediation. A typical mathematical “habit”, in the sense of empirical generalization, is that of considering concrete mathematical forms, equations, structures, etc., to reach generalizations. It can be observed that the modality of a visual language changes an explanation, in relation to spoken language.

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