



## Teaching practice regarding grade 3 pupils' use of representations

Isabel Velez, João Pedro da Ponte, Lurdes Serrazina

### ► To cite this version:

Isabel Velez, João Pedro da Ponte, Lurdes Serrazina. Teaching practice regarding grade 3 pupils' use of representations. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02435334

HAL Id: hal-02435334

<https://hal.science/hal-02435334>

Submitted on 10 Jan 2020

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Teaching practice regarding grade 3 pupils' use of representations

Isabel Velez<sup>1</sup>, João Pedro da Ponte<sup>2</sup> and Lurdes Serrazina<sup>3</sup>

<sup>1</sup> Instituto de Educação da Universidade de Lisboa, Portugal; velez@campus.ul.pt

<sup>2</sup> Instituto de Educação da Universidade de Lisboa, Portugal; jpponte@ie.ulisboa.pt

<sup>3</sup> Escola Superior de Educação, Instituto Politécnico de Lisboa, Portugal; lurdess@eselx.ip.pt

*In this paper we analyze the practice of a grade 3 teacher. We focus our analysis on teacher-pupils interaction in the classroom, aiming to understand how she strives to promote her pupils' use of representations. Data were collected through video recording of lessons and were analyzed in the introduction of a task, during pupils' autonomous work, and in a whole class discussion. The results show that the teacher's actions change according to her pupils' activity and difficulties and also vary depending on the moment of the classroom work. To promote her pupils' use of representations, the teacher adapts the type of questioning and her actions to the difficulties of her pupils.*

**Keywords:** Representations, elementary school teachers, teacher's practice, questioning.

## Introduction

Pupils' understanding of representations constitutes a fundamental basis for their mathematics learning, making it very important to know the way teachers deal with representations in their practice (Stylianou, 2010). A representation may be defined as a mental or physical construct that stands for a concept and enables to relate it to other concepts (Goldin, 2008). The fact that mathematical representations are related to each other in different ways creates difficulties for pupils' understanding and learning of representations (Goldin, 2008). Tripathi (2008) indicates that, in order to facilitate pupils' understanding of a given concept, teachers must use different kinds of representation. Some researchers, like Acevedo Nistal, Doreen, Clarebout and Verchaffel (2009), suggest that, as a starting point for learning symbolic representations, teachers must encourage pupils to create their own informal representations. In this study we aim to understand how an elementary school teacher explores a task with her pupils in the classroom, with special attention to the way she strives to promote the use of representations.

## Teachers' practice, representations and questioning

An important aspect of teaching practice is the way teachers explore tasks in the classroom (Ponte & Chapman, 2006). Pupils' activity on a task is determined by the actions of teachers, the role that teachers assume, how they introduce the task, the questions that they ask, and the way how they lead whole class discussions (Swan, 2007). Ponte (2005) indicates that the classroom work on a task may involve three main moments: (i) introduction of the task which may involve negotiations of meaning (Bishop & Goffree, 1986), (ii) pupils' autonomous work, (individually, in pairs or groups), and (iii) whole class discussion.

Representations play an important role in mathematics. Their understanding is a complex process because a representation may have different meanings and in turn, a meaning may have several representations (Goldin, 2008). For example, the representation "5" may mean the 5<sup>th</sup> floor, 5 pm,

or 5 as a quantity and, in turn the meaning of 5 as a quantity, can be represented as “||||”, “5” or “V”. For that matter, Duval (2006) indicates that, to understand the features of a mathematical object, we need to be able to make changes within a representation (treatment) or to change a representation in another representation (conversion).

To support pupils' learning of concepts, procedures and problem solving processes, the teacher may introduce new representations, linking them to pupils' previous knowledge (Stylianou, 2010). As Bishop and Goffree (1986) indicate, teachers must facilitate the interpretation of representations and encourage the establishment of connections among representations.

As pupils work on a task, the teacher's actions can be analyzed regarding how they promote the understanding of representations (Table 1). We defined four categories for pupils' activity that are related to teachers' actions: (i) support the pupils' design or selection of a representation; (ii) promote the use of a given representation; (iii) promote the transformation of a given representation; and (iv) promote pupils' reflection about representations. In table 1 we assume that there is a mutual influence between pupil's activity and teachers' actions. This way, pupils' activity can affect teachers' actions and teachers' actions promote pupils' activity. This framework about teacher actions is a specification of the general framework indicated in Ponte and Quaresma (2016).

<b>Pupils' activity regarding representations</b>	<b>Teachers' actions</b>
<b>Choosing/ Designing</b>	Promoting the free choice of a representation
	Challenging to choose a different representation
	Guiding about an adequate representation
	Providing explicit suggestions or examples
<b>Using</b>	Challenging to use a representation
	Asking to interpret a representation
	Guiding about the use or interpretation of a representation
<b>Transforming</b>	Informing pupils about how to interpret or how to use a representation
	(In)validating a representation chosen by pupils
	Challenging to establish treatments, conversions and connections
<b>Reflecting</b>	Guiding to establish connections
	Guiding to identify possible treatments and conversions
	Inform about treatments and conversions
	Challenging to systematizations
	Leading to systematizations
	Informing about systematizations

**Table 1: Teachers' actions in different moments of the pupils' activity**

Each teacher communicates in a different way with his/her pupils and how and when they do it. To Purdum et al. (2015), pupils' knowledge is influenced by teachers' questioning. Mason (2000) indicates three different aims in teachers' questioning: (i) focusing, that is when the teacher question pupils through a funneling effect in order to focus them in a certain aspect; (ii) testing, in which the teacher analyses pupils' comprehension, and how they articulate ideas and establish connections and (iii) inquiring, in which the teacher questions pupils to understand what they are thinking. Regarding questioning, Blosser (1975) identifies four main categories or question types: (i) managerial, to give operating instructions; (ii) rhetorical, used to emphasize an idea; (iii) closed, with a limited number of possible answers (iv) open, with a large variety of possible answers, a type of questions used to promote a class discussion or pupils' interactions. In this way, we considered three different types of questions, with some subtypes (Table 2).

Type	Subtype	Examples
Focusing	Rhetorical	We saw this already, didn't we?
	Processual	Could you open your books on page 58?
		What if we look back into the task?
Orienting		What if you sum it all?
Confirmation	Closed	How many will we have if you add 10?
Inquiring	Open	Do you agree with your colleagues' answer? Why?

**Table 2: Different types of teachers' questions**

## Research methodology

This paper is a part of a wider research about teachers' practices regarding mathematical representations. The participant of this study is a grade 3 teacher, Sónia, from a school cluster in the surroundings of Lisbon (where she has been for the last 10 years) and her 20 pupils (teacher and pupils' names are pseudonyms) who have been together since grade 1. During the research, Sónia was a member of a team of four teachers with whom she worked regularly, preparing and analyzing their teaching. The group indicated that the pupils were used to solve problems similar to the one reported on this paper, however, they chose this task taking into account their perception that it could be solved using a diversity of representations. Although the main research includes the analysis of pre and post classroom sessions, in this paper we only present and analyze teacher-pupils interactions during the classroom work, showing how Sónia promotes pupils' use of representation as they work on the following task: "In a theatre play performed by grade 3 pupils, João, Pedro and Ulisses wanted to be the King. On the other hand, Ana, Inês and Estrela wanted to play the Queen. How many pairs King/Queen may be formed?". Data was gathered by video recording during class observations and through collecting pupils' written work. It was analyzed through content analysis in the moments of introduction of the task, pupils' autonomous work and

whole class discussion. Teacher's actions were categorized according to teachers' actions indicated in Table 1 and to teachers' questioning presented in Table 2.

## Sónia's Class

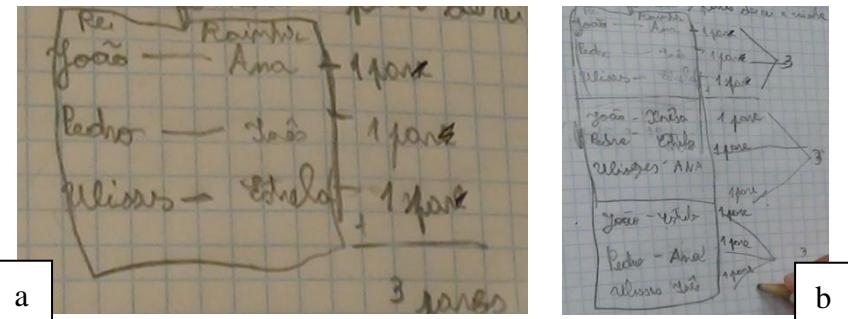
### Introduction of the task

To introduce the task, Sónia starts reading the statement of the problem and asks a pupil to go on reading it. Then, she guides pupils about the interpretation of the task (she focuses on number of boys and girls and the awareness that a problem may have more than one answer) and questions pupils through confirming questions about the conditions of the problem. At a certain point, a negotiation of meaning took place, since the pupils did not know what a "pair" was. Sónia challenges them to interpret this meaning through inquiring questioning ("Can I have two pairs and a half?", "What is a pair?"), but the pupils remain silent. Then she decides to question them through confirmation questions ("How many persons do I have in a pair?"), getting the interpretation from a pupil as "A group of two!"

### Pupils' autonomous work

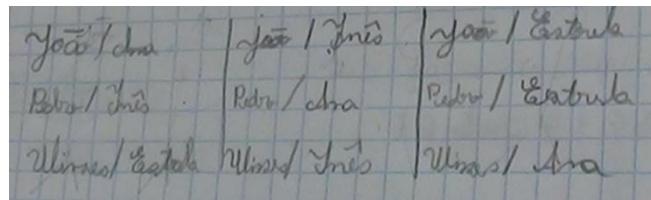
The pupils work autonomously for ten minutes, but Sónia notices that some of them struggle to understand what to do and she decides to discuss their difficulties collectively. She questions the pupils with confirming questions in order to guide them about the interpretation the statement of the problem ("Who can be the King and Queen?", "Only a boy can be the King", "Who are they?"). She suggests a specific representation to help the pupils to interpret the statement of the problem, by using an active representation, referring to the pupils' reality as if they were, at the time, involved in the school theatre play ("Imagine that . . . I am going to pick the King and Queen!... These three girls would raise their arms . . . And these three boys wanted to be the King... And now... Which are the possibilities?"). At the end of the discussion she challenges the pupils to use an adequate representation through inquiring questioning ("Let us discover!?").

A pupil, Angelo, says that it is possible to get three different pairs (Figure 1a). Sónia challenges him to interpret his representation through inquiring questioning ("Can you explain me what this is...?"). The pupil says that he made a "table" picking the boys and girls randomly. Sónia guides him about the use of his representation through inquiring questioning ("Why João does not like Inês or Estrela? Is he angry with them?"). As Angelo does not understand that his answer is incomplete, Sónia changes her actions again and informs Angelo that he did not consider that each boy could be paired with three different girls ("How many are the possibilities? It does not say: 'Tell me three [possibilities]...'"'). When the pupil acknowledges that he has an incomplete answer she lets him continue working. Later Sónia comes back to see his work (figure 1b) and she challenges him to interpret the chosen representation through inquiring questioning ("What are you doing?", "What are you repeating here?"). Angelo then explains why he considers nine pairs as he describes his representation ("If the first [group] is made... It has three [pairs]! Other [group]... It has one pair, another pair, another pair... They are three [groups]! (pointing to the third group) One pair, another pair, another pair... Three [more pairs]!").



**Figure 1 (a and b): Angelo's representations before and after Sónia actions**

Later, another pupil, Joaquim, begins to complain loudly, because he feels that he is spending too much time on his representation (he drew every Queen and King in detail). Assuming he had to draw, Joaquim questions Sónia. Noticing that more pupils are using similar representations, she decides to guide pupils with confirming questioning (“Did anyone told you: Spend a lot of time on drawings!? Or to draw all the Kings and Queens?). Another pupil, Fernando answers (“No! Why [should we draw]?! They have names!”) and fulfils the aim of Sónia. Then, she reinforces the pupils’ free choice of a proper representation (“If you think that you are taking too much time... Don’t do it...”). A few moments later, she returns to see his work) (figure 2).

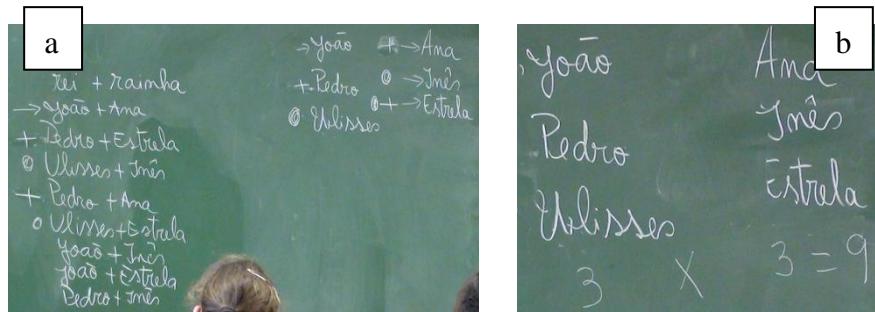


**Figure 2: Joaquim’s representation**

Sónia challenges Joaquim to interpret his representation using inquiring questioning (“What are you doing?”) and he responds correctly (“I made the first group! Then I draw a line and divided the first group from the second! João, Inês. Pedro, Ana. Ulisses, Inês... And João, Estrela. Pedro, Estrela. Ulisses and Ana! And there are no more [pairs]!”). At a certain point most of pupils had solved or tried to solve the task and Sónia decides to begin the whole class discussion.

### Whole class discussion

During the pupils’ autonomous work Sónia noticed that many of them had trouble in choosing a proper representation and in identifying the number of possible pairs. She decides to begin the whole class discussion by inviting Luís to present his solution (he has an incomplete answer). She asks him to interpret his representation using confirming questions (“Why did you not consider João and Estrela?”). Based on the representation of Luís she suggests another representation – a scheme with circles, arrows and crosses (Figure 3a), and the pupils acknowledge that it was an incomplete answer (Luís: “Ah! He can [also be paired] with Ana!”).



**Figure 3 (a and b): Sónia's representations**

During the remaining of the discussion, Sónia actions vary greatly. Sometimes she challenges pupils to systematize through inquiring questioning (“Why have not you done that?”, “Are there more possibilities?”) but when they do not respond, she leads the pupils to establish connections and to identify conversions and she informs the class about systematizations.

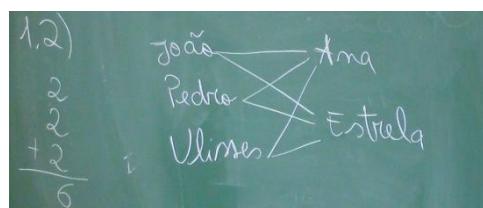
This task had a follow up question “During the rehearsals, Inês decided that she wanted to drop out of the play. How many pairs are now possible?” During the moment of autonomous work, only the fastest pupils got to solve this question. However, faced with the class difficulties in the whole class discussion, Sónia decides to solve it in whole class and she challenges the pupils:

Sónia: How many pairs are there right now? (some pupils answer “six” loudly) Why?

Laura: Because João can be [a pair with] Ana and Estrela. . . Pedro can be with Ana and Estrela . . . And Ulisses can make [a pair] with Ana and Estrela... It’s six!!

Sónia: So Laura says that João can be a pair with Ana or Inês (she writes the names on the board and she connects João with Ana and Estrela as she speaks)... So... Two possibilities for João (she writes the number “two” on the left of the first representation)... Pedro can be a pair with Ana and Estrela... [He has as well] two possibilities and Ulisses with Ana and Estrela (she continues both representations as she speaks) (figure 4)! So... All together (she transforms the “two” into a vertical calculus)...

Pupils: Six!!!! (the teacher writes “six” below the vertical column of 2s)



**Figure 4: Teacher's diagram and symbolic representations**

As Sónia challenges her pupils to interpret the question, Laura explains easily to the class how she thought. Sónia transforms Laura’s explanation into a written representation (figure 4), in order to lead her pupils to establish connections between representations. Afterwards, Sónia leads them to make connections between all representations (figures 2a, 2b and 3). She ends the discussion by suggesting the multiplication sign (“If we have... Three boys [she writes “3” below the boys’

names] and three girls ([she writes “3” below the girls’ names]… I have (she puts the × sign writing  $3 \times 3$ )… Nine! Nine possibilities!”).

## Conclusion

During the introduction of the task, most of Sónia’s actions were focused in promoting pupils’ understanding of the statement of the problem, by guiding them through confirming questions (Who? How? How many?). As several pupils had trouble with the meaning of the word “pair”, Sónia handled this problem leading a negotiation of meaning, challenging the class through inquiring and guiding questioning. In that way, Sónia’s actions begun by addressing the understanding of the statement of the problem, so that pupils could think about how to solve it and figure out what type of representation is more adequate.

During the moment of autonomous work, Sónia led the pupils to write their answers and representations and to justify them. She made the pupils to convert their mental representations into written ones. While interacting with the pupils, Sónia used challenging actions through inquiry questions (“Explain me that…”, “I am not understanding…”). When this did not work, she changed her actions and questioned pupils with confirming questions, leading them to explain their representation. Usually, she seemed to re-evaluate her pupils’ activity and shifted between actions in order to take them to use adequate representations. At the beginning of pupils autonomous’ work, Sónia promoted pupils’ free choice of representations and did not influence her pupils’ work. Later, while pupils were using and transforming their representations, she did not suggest alternatives nor guided them to find conversions or treatments, even when they were struggling. In that way, her actions (i) enabled the emergence of a large variety of representations to be considered during the whole class discussion, (ii) supported the establishment of connections; and (iii) promoted pupils’ reflective activity about their own representations.

In the whole class discussion, most actions of Sónia were informing about new representations and informing and guiding the pupils to figure out the connections and transformations that they could do. Due to her pupils’ difficulties during their autonomous work, she felt compelled to systematize all the information and to act with more guiding actions. As in McClain (2000), the pupils’ representations had less relevance. In fact, Sónia used pupils’ representations as a starting point for discussion, and then she introduced her own representations. At the end of the discussion, she suggested the sum and multiplication signs as adequate representations and connected them to her first representation (that she made from a pupil’s explanation). However, since there was no further discussion, it is unclear what understanding pupils made of that.

In summary, during the three phases of the work, Sónia’s actions tended to change according to her pupils’ answers and difficulties. At the introduction of the task, Sónia considered necessary to support the conversion of the statement of the problem into a different representation. Although pupils knew the multiplication sign, this did not mean that they knew how it can be used to model situations as in this problem (Acevedo Nistal et al., 2009). In that way, at the end of the discussion, the teacher felt the need to guide her pupils about the use and interpretation of the multiplication sign. Only in future classes, one may know if this led the pupils to understand the use of symbolic representation in this kind of situations. In this study, we see that Sónia changed her actions

according to pupils' activity in order to promote their use of representations. When pupils had an organized strategy and an adequate but incomplete representation, she questioned them, so they could review their strategies and representation and find out how they could finish the task. When pupils had a disorganized strategy and an adequate but incomplete representation she tried to help them to understand why the representation was incomplete, focusing in the need of using a more organized strategy. And when the pupils had an inadequate representation she questioned them, guiding the pupils to choose a different representation and strategy.

## References

- Acevedo Nistal, A., Dooren, W. V., Clarebout, G., Elen, J., & Verschaffel, L. (2009). Conceptualizing, investigating and stimulating representational flexibility in mathematical problem solving and learning: A critical review. *ZDM Mathematics Education*, 41(5), 627–636.
- Bishop, A., & Goffree, F. (1986). Classroom organization and dynamics. In B. Christiansen, A. G. Howson & M. Otte (Eds.). *Perspectives on mathematics education* (pp. 309–365). Dordrecht: D. Reidel.
- Blosser, P. E. (1975). *How to ask the right questions*. Washington, DC: National Science Teachers Association.
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 103–131.
- Goldin, G. (2008). Perspectives on representation in mathematical learning and problem solving. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 178–203). New York, NY: Routledge.
- Mason, J. (2000). Asking mathematical questions mathematically, *International Journal of Mathematical Education in Science and Technology*, 31(1), 97–111.
- McClain, K. (2000). An analysis of the teachers' role in supporting the emergence of symbolizations in one first-grade classroom. *Journal of Mathematical Behavior*, 19, 189–207.
- Ponte, J. P. (2005). Gestão curricular em Matemática. In GTI (Ed.). *O professor e o desenvolvimento curricular* (pp. 11–34). Lisboa: APM.
- Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutierrez & P. Boero (Eds.). *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 461–494). Roterdham: Sense.
- Ponte, J. P., & Quaresma, M. (2016). Teachers' professional practice conducting mathematical discussions. *Educational Studies in Mathematics*, 93(1), 51–66.
- Purdum-Cassidy, B., Nesmith, S., Meyer, R. D., & Cooper, S. (2015). What are they asking? An analysis of the questions planned by prospective teachers when integrating literature in mathematics. *Journal of Mathematics Teacher Education*, 18, 79–99.
- Stylianou, D. A. (2010). Teachers' conceptions of representation in middle school mathematics. *Journal of Mathematics Teacher Education*, 13, 325–343.
- Swan, M. (2007). The impact of task based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education*, 10, 217–237.
- Tripathi, P. N. (2008). Developing mathematical understanding through multiple representations. *Mathematics Teaching in the Middle School*, 13(8), 438–445.

