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The importance of a meaning-related language for understanding multiplication

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Multiplicative thinking is much more than recalling number facts. Furthermore, it is much more than repeated addition. It is about links and relationships and knowing how multiplicative ideas are connected and why multiplicative grouping processes work as they do. Supporting this conceptual understanding of multiplication among underachieving primary school children is the goal of a single-case intervention study presented in this paper. These children were fostered in a language sensitive way placing emphasis on the relationship of different multiplicative representations and a meaning-related vocabulary associated with this operation. The in-depth analysis of an individual learning trajectory of a low-achieving child shows that the progress may depend on whether and to what extent the meaning-related vocabulary becomes the vocabulary of thinking for these children.

Keywords: Multiplication, mathematical concepts, verbal learning, low achievers.

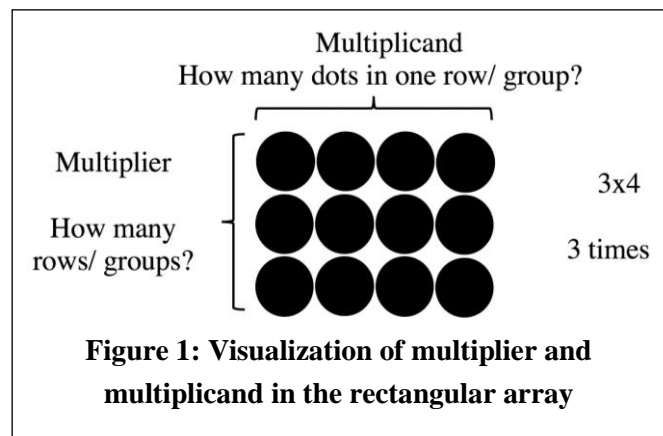
During their first years of school, children should not just be able to execute multiplication tasks, but more than anything else, should also understand them in terms of content as well. For primary school children who have not acquired a multiplication concept it is much more difficult to achieve higher learning targets (e.g. Downton & Sullivan, 2017; Moser Opitz, 2013). This is particularly the case because an understanding of multiplication represents a fundamental requirement for understanding further learning contents such as proportionality, algebra, fractions, decimal numbers, percentages... (Downton & Sullivan, 2017). In this respect, it is by now accepted that the development of a conceptual understanding for the multiplication is of far greater importance than learning the times tables off by heart (Gaidoschik, Deweis, & Guggenbichler, 2018; Moser Opitz, 2013).

Theoretical background: Fostering a conceptual understanding of multiplication

Kilpatrick, Swafford and Findel (2001) define the conceptual understanding as an understanding of mathematical concepts, operations and relationships. A mathematical concept is said to be understood when it constitutes a part of the mental knowledge and is said to reach a higher level if it is interconnected with existing knowledge and can be used to solve diverse mathematical tasks (Kilpatrick et al., 2001). What this means for the multiplication specifically is that children need to understand that cardinal quantities are combined with each other, wherein, formally speaking, the multiplier represents the frequency of the cardinal quantities and the multiplicand represents the size of the cardinal quantity (Figure 1). Both factors are therefore not equal (Downton & Sullivan, 2017). However, multiplication is often introduced in the form of repeated addition, which is the preferred strategy used to solve the multiplication tasks particularly in low-achieving children (Moser Opitz, 2013). But this equating of multiplication with repeated addition leads to an incorrect understanding of multiplication (Downton & Sullivan, 2017), in the case of addition, both numbers represent a respective quantity, which can be cardinally translated and merged. Multiplication, however, means to coordinate interrelated units.

If children tend to equate the multiplication with repeated addition or with count-by sequences (“four, eight, twelve”) – as another typical strategy of low-achieving children – this differentiation between multiplier and multiplicand does not become clear. Furthermore, the knowledge in relation to associative and distributive contexts is not accessible to them. Additionally, those solution strategies are very limited and cannot, for example, be used when it comes to fractions or decimal numbers (Downton & Sullivan, 2017).

In this respect, many researches have shown that a conceptual understanding of multiplication instead of automaticity needs to be fostered in the first place. Many of these studies take a strategy-training approach (e.g. Gaidoschik et al., 2018; Woodward, 2006). Particularly remarkable is that predominantly low-achieving children – even after a strategy-training – seldom use derived-fact strategies compared with normal-achieving children (Zhang, Xin, Harris, & Ding, 2014) or have difficulties in deriving (Gaidoschik et al., 2018). On the one hand, this can be traced back to the fact that the use of decomposition strategies is very working-memory demanding and low-achieving children are characterised by their limited working memory (Zhang et al., 2014). On the other hand, the cause can be seen in an insufficient conceptual understanding of multiplication (Downton & Sullivan, 2017). These children do not seem to have fully understood what phrases such as “three times four” or “four multiplied with three” actually mean. This means that perhaps the differentiation between multiplier and multiplicand needs to become more obvious for these children as a central basis for developing derived-fact strategies. However, current research projects have repeatedly shown that the transition from the repeated addition or count-by strategies to the understanding of multiplication is a – if not *the* – critical obstacle to the conceptual understanding of multiplication especially in the case of low-achieving children (e.g. Zhang et al., 2014). Therefore, in future research, this should be the starting point to identify which steps could help children overcome this critical obstacle. But if phrases like “three times four” or “four multiplied by three” are initially of no meaning to these children other phrases such as “three groups of four”, “the four three times”, “three rows of four columns each” may support the development of a conceptual understanding. It has still not been researched sufficiently whether and to what extent this “basic meaning-related vocabulary” (Prediger & Wessel, 2013; Pöhler & Prediger, 2015) supports a conceptual understanding of multiplication in low-achieving students. In terms of increasing a conceptual understanding of division among low-achieving children, it has been shown that the understanding of division can be fostered with the help of meaningful verbalisations such as, for example, “12 divided by 4 means how many fours fit into 12” (Götze, 2018). Whether or not students benefit from such support seems to primarily depend on whether this meaning-related vocabulary will become the students’ language of thinking (Pöhler & Prediger, 2015). This means that in the course of such fostering, it might primarily depend on if and to what extent the students are able to internalise these verbalisations and how they can contribute to the forming of mental models. In this



respect, the single-case research project presented in this paper focuses on the following research question:

How does a language sensitive support of multiplicative grouping promote a conceptual understanding of multiplication in low-achieving children?

Methodology of the study

The study presented in this paper pursues the goal of conducting a detailed in-depth analysis of the learning processes for the development of a conceptual understanding of multiplication among low-achieving children. Therefore, the study is structured as a single-case didactical design research (Pöhler & Prediger, 2015). The data presented are from the first circle.

A total of ten pairs, consisting of one normal- and one low-achieving child, were assisted by one trained preservice elementary teacher. The cooperative setting was selected because the discursive negotiation of meaning among heterogeneous children has proven itself to be very beneficial for learning in other studies as well (Götze, 2018; Zhang et al., 2014). The pairs of children were provided with fostering help on a total of four remedial lessons. Each of these remedial lessons lasted approximately 25 to 30 minutes. They were provided by three pre-service teachers who had been intensively trained beforehand.

At the time of the study, the participating children were in the third grade in a German primary school and had already learnt multiplication in their second year in school. As a side note, in Germany the rows of times tables are determined by the second factor. So, 1×2 , 2×2 , 3×2 , 4×2 ... are second-row tasks, 1×5 , 2×5 , 3×5 , 4×5 ... are tasks from the fifth row. For these children, the tasks 3×4 , 3×6 , 3×9 belong to different rows.

Based on the review of the current research, these children were fostered in a language sensitive way placing emphasis in the relationship of different multiplicative representations (concrete, iconic and symbolic representations) and a meaning-related multiplicative (not additive) vocabulary associated with this operation. For doing this, the following central learning and diagnosis steps were applied in order to foster a conceptual understanding (based on Pöhler & Prediger, 2015):

- Step 1: Informal thinking starting from students' resources.
- Step 2: Focusing meaning-related vocabulary for understanding multiplicative grouping and connecting this vocabulary with concrete, iconic and symbolic representations.
- Step 3: Independent use of the meaning-related vocabulary.
- Step 4: Use of meaning-related vocabulary in order to initiate decomposition strategies.

Especially in the second support unit (step 2) the children had to find reasons for the assignment of iconic rectangular arrays and symbolic terms. At the same time, the multiplicative understanding was addressed directly: "three times four" means that there are three fours or there are three groups, each of which has four in one group. The children should always explain this relationship: Where do you see the number of the groups? Where do you see the group size? Subsequently, during the third support unit (step 3), this understanding was deepened further through more game-based assignments (for example pairs card games). The children were furthermore required to name the matching terms for orally described rectangular arrays. In the case of difficulties, they were allowed

to lay the rectangle. Then, the children should lay rectangular arrays selected by themselves with manipulatives and describe these in a similar fashion. The children were repeatedly asked to interpret the group number (the multiplier) and the group size (multiplicand) in terms of content. The constant alteration between language reception and language production was intended to form mental models. During the transition to the decomposition strategies in the fourth support unit, the children needed to think about which arrays could be laid next to each other and which overall term would match. They were given some rectangular arrays, which they could distributively combine in sometimes different ways. What was important here once again was the fact that the children had to explain why, for example, 3×4 and 2×4 together make 5×4 , but also why 3×4 and 3×1 together make 3×5 . This required them to pay attention to the differentiation between multiplier and multiplicand, as well as to verbally describe their meaning and importance. This makes it absolutely necessary to have mentally understood the previous actions carried out on the rectangular arrays or to be able to fall back on these once again.

Therefore, the following section gives an insight into the learning pathway of the low-achieving child Ilay. The goal of the analysis is to ascertain under which conditions the use of meaning-related vocabulary fosters learning for the development of a conceptual understanding of multiplication and as a basis for developing derived-fact strategies. For a deeper insight and a comparison between different children's development see Götze (under review).

The individual learning trajectory of Ilay

Ilay (I) worked together with his normal-achieving classmate Nelli (N) during the fostering course. Ilay is a very communicative child who likes to contribute to class discussions and really likes to explain how he arrived at his solution. This is less the case with Nelli.

During the first support unit Ilay knew many multiplication tasks from the times tables off by heart. Otherwise he used the strategy of repeated addition or count-by sequences. While doing so he usually started to recite the times tables from one, something which was very time consuming. For instance, he needed a total of 45 seconds until he could provide the correct solution to the problem 6×7 . While reciting, he raised individual fingers one after another sometimes even two at once, which would indicate additive calculation. He explained how he arrived at the solution as follows.

- 56 Ilay: Because when you now, when you now... aahmmm... then that's 7 (points to his thumb) plus 7 (points to his index finger) are then 14. And then you work out these 2 (points to his middle finger and his ring finger) are also 14. There are 28 in total. And then you only need to add this 14 (holds up the next two fingers). In total around 42.
- 57 Teacher: And what comes out at the end of this problem? (points to the task 7×7)
- 58 Ilay: Around... 48 (he starts to count his fingers forward once again, whispers) 14, 28, 42. So... (after 25 seconds) 49.

Ilay worked out the result by availing of additive references (turn 56 and 58). For him it appeared that being asked to multiply was more a request to count up. In doing this, Ilay displayed typical one-sided additive ideas with respect to multiplication and little strategic knowledge (Gaidoschik et

al., 2018). Furthermore, he was not able to derive the solution for the new task 7×7 from the previous task 6×7 .

In order to emphasise the differentiation of multipliers and multiplicands for Ilay, during the second support unit the meaning-related vocabulary for the same groupings was introduced (step 2). In the following scene, approximately ten different rectangular arrays were laid out in front of the children, which also included the correct array for the task 3×5 (for Ilay and Nelli a task from the fifth row).

- 7 Teacher: I will now give you the problem. Do you know the game, “I spy with my little eye”? We will now play using the cards: I spy with my little eye lots of 3 groups of 5 each (places the note “3 groups of 5 each” in front of the children).
- 8 Nelli: 3 groups of 5 each?
- 9 Teacher: Do you have any idea which dot image I mean?
- 10 Ilay: (points uncertainly at the card containing the dot image relating to 3×5)
- 11 Teacher: Why could that match?
- 12 Ilay: Yes, because here are 5 (points to the uppermost row)
- 13 Nelli: ... because there are 5 in each row.
- 14 Ilay: ... because there are 5 in each row, so...
- 15 Nelli: ... 15.
- 16 Ilay: So when you now calculate (takes the card with the rectangular array 3×5), 1, 2, 3 (tips each of the respective first dots of the first column), there are 3 below and here, calculate 1, 2, 3, 4, 5 (tips each of the respective dots of the first line), then these are 15 and here are 3 groups of 5 each (points to the note with the same sentence).
- 17 Teacher: Okay, so where are the 3 groups of 5 each?
- 18 Ilay: 3?
- 19 Teacher: You have already said that these are 3 and 5. But where are the groups of 5?
- 20 Nelli: Here at the top (moves along the top line).
- 21 Ilay: Exactly, and we then have a group of 5 here (moves along the top row), two groups of 5 together, three groups of 5 together (moves along the other rows).

It can be seen that the children initially react in an uncertain manner to this unknown vocabulary (turn 8, turn 10). Even when Ilay directly found the requested dot image (turn 10), he was still not able to immediately explain why the description “three groups of five” matched this dot image. He pointed to the first column and the first row (turn 12, turn 16). This focus can be seen in many low-achieving children and indicates a limited conceptual understanding (Moser Opitz, 2013). Nelli, on the other hand, seemed to understand the content immediately, which can often be seen in the case of normal-achieving children (Götze, 2018). She emphasised that in each row there must be five dots (turn 13), which Ilay then repeated word-for-word (turn 14). Thus, Ilay had not yet understood the content of this formulation because he could not answer the follow-up question of the teacher (turn 17, turn 18). The teacher correctly recognised that although Ilay pointed to the numbers three

and five of the term in the rectangular array in his customary manner, he had not yet started to think of cardinal groups of always five dots. It was only when Nelli pointed out the group of five in the dot image to him that Ilay rethought (turn 21) and recognised this new perspective.

The overall evaluation of the data from the support units of all ten low-achieving fostered children demonstrated that this scene appeared to be a key scene in the learning process for low-achieving children such as Ilay. If they were to understand the grouping concept, they had to be given a new way of looking at the rectangular arrays by means of the new language being offered. In order to ensure that the children did in fact understand the content of the subject matter, substantial repetition was needed in order for the children to make this new way of looking at rectangular arrays a part of their own mental models.

In this respect, in the third support unit Ilay showed that he began to integrate the meaning-related vocabulary into his own mental thought patterns (step 3). It was astounding that already in the third support unit he seemed to give up on his additive viewpoint that he had so strongly been focussed upon up to that point. He explained why the term 6×6 matched to the corresponding rectangular array without having to be asked to do so by the teacher.

123 Ilay: So, I took this one here because that is 6, so 6 is here (moves along the left column with his finger) and there are 6 groups (moves his finger from left to right several times) ... And here 1, 2, 3, 4, 5, 6 (taps on each of the respective points in the first row), there are 6 in a group, and when you count down (moves the finger from top to bottom), then 1, 2, 3, 4, 5, 6 (taps on each of the respective points in the first points), that's actually really easy.

First, he addressed the groups number, in other words the multiplier, by pointing to the six groups in the right corner ("there are 6 groups"). Subsequently, he described the meaning of the second six in the term ("there are 6 in a group"). Drawing on the meaning-related vocabulary he had learnt, he completely independently explained the differentiation of both sixes in the term 6×6 : one indicates the number of groups, the other the group size.

In order to obtain first indications about a possible further development of derived-fact strategies, in the fourth support unit, the children were supposed to try to make deductions between the tasks. In the following scene, Ilay was asked to think about why 7×5 is always seven more than 7×6 .

47 Ilay: Because, that (points to the multiplier) stays the same and that (points to the multiplicand), that should, if you jump over one, then that should now also be 7s.

48 Teacher: I don't quite understand that. What do you mean, if you jump over one?

49 Ilay: If you count one to here (points to the multiplicands 5 and 6), then that will, then it is, if you, a 7, if you (points to both multiplicands) count up to 7.

50 Teacher: So, if we have 7 times 5 and 7 times 6. Why is it 7 more?

51 Ilay: Nelli, you count it.

52 Teacher: That's right. But I am not convinced yet. 5s become 6s.

53 Ilay: Oh! Because here, here, you have 6. And then one always comes in here, (points to both multiplicands), it only comes, if we do that with the dots, it only comes, so one, so one, so one row, so one dot always becomes more.

At the beginning of the scene, Ilay initially tried to explain the correlations with his own linguistical means and using lots of pointing gestures (turn 47). Indeed, he also occasionally used meaning-related vocabulary such as “sevens” but could not yet use it to explain the content-related correlation of 7×5 and 7×6 . Only the direct reference of the teacher in turn 52 (“fives turn into sixes”) seemed to activate the meaning-related vocabulary for him. He was then able to use it to explain the multiplicative (not additive!) correlations very independently (turn 53). In contrast to the first support unit, he did not use his finger to count the rows but thought purely in terms of content. This content-related thinking in multiplicative structures was shown several times in the fourth support unit.

It can be seen in the above analysis that Ilay completed the intended four learning steps. After the introduction of the basic meaning related vocabulary, he first needed some time for practice and networking with concrete representations (step 2), before this vocabulary slowly became Ilay’s own vocabulary of multiplicative thinking (Pöhler & Prediger, 2015; step 3). Perhaps this newly formed content related multiplicative thinking provided the basis for using derived-fact strategies. Repeated addition was a dead end for him in multiplicative understanding. With this respect, the meaning related vocabulary fulfilled a cognitive and epistemic function for him. Expressions such as “3 times 5” were no longer a “secret language”, and the multiplication in rectangular arrays was really perceived in a multiplicative way via these expressions.

Discussion and limitation

Ilay’s learning trajectory stands representative for other fruitful learning trajectories of other children. All these learning trajectories have in common that the children start to *think in groups of the same size*. The fostered vocabulary seems to support the development of a mental model in all these children. The meaning-related verbalizations become the language of thinking and therefore, the children can understand how to decompose or derive multiplication tasks. This multiplicative point of view has apparently not been obvious to the children in their previous learning biography through formal expressions such as “3 times 4” or “3 multiplied with 4” or “4 multiplied by 3”. However, basic meaning-related phrases such as “3 groups always with 4 (dots) in a group” or “3 groups of 4” illustrate thinking in multiplied groups of equal sizes. This vocabulary then fulfils *epistemic functions* in the learning process and contributes to conceptual understanding. The problem with this is repeatedly having to detach low-achieving children from using their old, supposedly safer strategy of repeated addition. Throughout, the children use both additive and multiplicative ideas at the same time. The in-depth analysis of children such as Ilay therefore shows how central it is to continue to promote this new multiplicative point of view. Furthermore, the in-depth analysis demonstrates that the multiplicative thinking seems to be a good basis for the development of decomposition strategies. If children can think in a content-related way they can also derive tasks from each other and recognise correlations between tasks.

Despite that, not all supported children show these learning developments (for details see Götze, under review). The most striking factor however is that these children do not start to think

multiplicatively. The meaning-related vocabulary does not become the vocabulary of thinking for these children. This is mostly caused by the teacher who starts to retrieve facts and vocabulary (for example “Where are the groups in this term? Where are the number of groups in the rectangular array?”) instead of supporting multiplicative thinking. Especially in this paper only one single-case study can be presented. For a comparative analysis it would be helpful to contrast Ilay’s individual learning trajectory with those of other children (see Götze, submitted). But that would go far beyond the scope of this paper.

This study is laid out as qualitative Didactical Design Research (Pöhler & Prediger, 2015). First of all, the aim was to derive hypotheses about the effectiveness of the support and not to operationalize them. Therefore, the duration of the fostering was very short, with just four support units. Yet, the study provides first indications that the conceptual understanding of multiplication can be promoted in a language sensitive manner when we can achieve that the children use the meaning-related vocabulary as their vocabulary of thinking. How long the fostering should actually last for the supported children to be able to independently think in a content-related way, remains unanswered. Future research must therefore check the hypotheses made in this paper.

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