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# Centripetal and centrifugal forces in teacher-class dialogues in inquiry-based mathematics

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*In this report I examine classroom dialogue in the practice of a teacher who is explicitly committed to a dialogical, inquiry-based approach to teaching mathematics. Of primary interest are the ways in which the dialogues on the mathematical tasks are in continual tension, alternately being pulled towards and pushed away from a proper mathematical discourse, as seen from my perspective as a mathematics educator and an observer of the lessons in the classroom.*

*Keywords: Classroom communication, discourse analysis, inquiry-based learning, dialogism.*

## Introduction

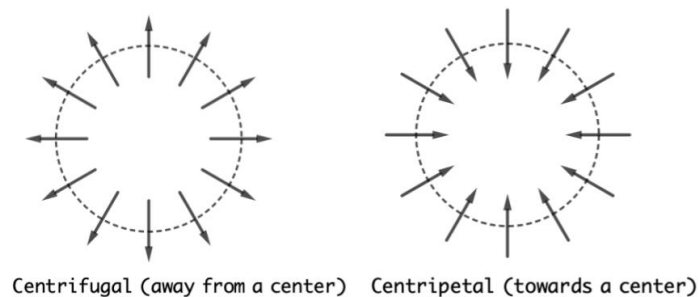
If students are to gain a command over mathematical discourse, they need to practice talking mathematics. It is well documented that in traditional classrooms it is predominantly the teacher that talks mathematics, and the IRE (initiation- (short) response-evaluation) format of teacher-student interaction is ubiquitous. Many studies show the meaning making potential of inquiry-based teaching-learning, where the students do more of the mathematical talk (e.g., Boaler, 2002; Goos, 2004; Lampert, 1990) but there is little research on mathematical discourse in inquiry-focused classrooms on the upper-secondary level. I report here on my longitudinal case study of inquiry-based teaching-learning in the practice of an upper-secondary level mathematics teacher working in a challenging classroom in Iceland. I analyse teacher-class dialogue on realistic tasks from a dialogical perspective, showing how the concepts of the centripetal and centrifugal forces (Bakhtin, 1981) may be used to describe and understand the dynamics of the dialogue.

## Theoretical perspectives and prior research

In line with discursively focused research, I consider mathematics learning as the increased command of mathematical discourse, a progress from discourse confined to informal everyday language towards the command of a more advanced and formal mathematical discourse – the culturally specific, tool-mediated, historically established ways of communicating that competent users of mathematics use (e.g., Sfard, 2008; Chapman, 1997; Barwell, 2016; Roth & Radford, 2011). Dialogues are the primary objects and units of analysis, and I consider dialogue, following Linell (1998, p. 13) as “interaction through language (or other symbolic means) between two or several individuals who are mutually co-present”. A dialogue is a chain of utterances, which are the complete meaningful contributions persons make to the dialogue. An utterance is always a response to what has gone before, addressed to someone(s) in anticipation and expectation of future responses. Utterances contain intentions, emotions, evaluations, which are dependent on context, and sometimes conveyed through means such as tone and gestures. From a dialogical viewpoint, utterances do not have fixed meaning but they do have partly open meaning potentials rooted in culture, history and ideology and the specific speech genres and social languages they belong to.

Communication does not presuppose or produce total shared-ness of meaning; rather it consists in people's continued and on-going attempts to expose and test their understandings (Linell, 1998).

In the inevitable difference of two or more perspectives inherent in each utterance Bakhtin identifies a tension between a centripetal force towards uniformity, a shared and common meaning, and a centrifugal force towards diversity of discourse, the meanings made in particular situations by particular persons (Bakhtin, 1981). These forces are depicted in Figure 1 as opposite directed arrows, as we imagine the dialogue happening on a boundary surrounding a shared meaning.



**Figure 1: Forces in dialogue**

From this Bakhtinian viewpoint I consider the mathematics teacher as working towards the (relative) uniformity of mathematical discourse, while the students make and express more diverse meanings, as they are imbued with language, values and views from their social worlds. When people attend mathematics classes and sense it as a painful, or even meaningless experience, this is because the words of the teacher and the textbooks do not resonate with them and do not move them. Following Bakhtin (1981) we say that the discourse lacks inner persuasiveness. In this all too familiar case, students do not make the discourse their own, it is alien to them. They behave as factory workers although they are not producing things of value for capitalist owners. Rather, they are producing strings of symbols, that are as such of no market value, in order to prove their own value (as good students and future productive citizens). Research published on mathematics in secondary school in Iceland points to a dominance of this kind of alienated form of mathematics work: an emphasis on symbolic manipulations but little emphasis on meaning and no, or very little, exploratory work; the textbook dominates and the class sessions are characterized by teacher transmission and students sitting at desks solving exercises individually (Bjarnadóttir, 2011; Gunnarsdóttir and Pálsdóttir, 2015; Jónsdóttir et al. 2014).

## **Research method**

The setting of the study is an upper-secondary school classroom in which most of the students have a history of very low achievement in mathematics. In order for me to make sense of the classroom dialogues I was present in the classroom the whole semester-long course as a mostly passive observer, concentrating on capturing audio and video of the speaker(s) who have the floor at a given time. I attended the class with a videorecorder in 32 of the 39 classrooms sessions of the course which lasted from late August to December 2012. Although my role was mostly passive, I did interact with students, discussed the situation of me being present in the classroom, asking for permissions to record a pair or small group conversation and sometimes responding (usually rather

minimally) to questions about the mathematics tasks. For the purposes of this paper I focus only on the three lessons where there was a public whole-class discussion about realistic mathematics tasks. The tasks are all teacher translations of tasks from Swan, Pitts, Eraser, and Burkhardt (1985). A video analysis tool, Transana, (Fassnacht & Woods, 2012) was used to transcribe video segments synchronously.

I coded the teacher-student dialogues using open coding, drawing on a grounded theory approach (Charmaz, 2006) aided by the dialogical principle that it is the participants' responses that provide evidence for the meaning of a prior utterance. Still, it should be borne in mind, as Bakhtin argues, that “understanding a dialogue as a researcher implies participating in that dialogue as a ‘third voice’.” (Wegerif, 2007, p. 21). This means that I, as the researcher, make sense as the other participants, anticipating, responding, and creating tentative sense in my own way, based on my cultural background and personal history. After a few iterations, recurrent themes emerged that seemed to capture the different ways the participants responded to the tasks presented, thus contributing to different types of dialogue. In the following I present purposefully selected short sequences of dialogue to illustrate the types. These fragments are then analysed in more detail in order to bring further attention to the centripetal and centrifugal effects of the contributions to the dialogue.

In order to make the reading of the fragments feel natural, the students have all been given unique pseudonyms.

## Results and discussion

Inspired by Bakhtin’s ideas of learning as adopting words (and gestures, phrases, drawings and all means of communication) of others and using them with one’s own intentions with varying degrees of “our-own-ness”, and his notions of centripetal and centrifugal forces operating in a dialogue, I classified four types of dialogue. Two of the types refer to distancing in dialogue: *the alien word* (11) and *irony* (29) and two are types of accepting dialogue: *authentic real world* (123) and *emerging mathematical* dialogue (48). The numbers in brackets indicate a count of utterances contributing to a types’ appearance to give some indication of their relative frequencies, although an utterance does not always contribute clearly to only a single type of dialogue. The first three types function more centrifugally than centripetally.

To illustrate the most frequent type, *authentic real-world* dialogue, I present a fragment of a dialogue on the question of the nature of a general relationship between the weight of a person and her high-jump ability. The task text states: “Suppose you were to choose, at random, 100 people and measure how heavy they are. You then ask them to perform in 3 sports; High Jumping, Weight Lifting and Darts. Sketch scattergraphs to show how you would expect the results to appear, and explain each graph, underneath. Clearly state any assumptions you make.” The following sequence of utterances starts after the teacher has drawn positive coordinate axes on the whiteboard and has exchanged a few turns with students about the nature of the task.

- Ari: Like, like, the body weight shouldn't matter crucially, like in high jump. [Eagerly]  
Teacher: What.

Ari: Like the guy who is one ninety and a guy who is one sixty. A guy who is one ninety could be heavier than the guy who is one sixty, isn't that right?

Teacher: But if you have two guys that are one ninety and one is heavier and the other is lighter, who

Ari: The lighter guy.

Teacher: What do you think about that? [Addressing the class]

Bjarni: Not necessarily.

Ari: It depends on the technique and where he is muscular.

The request for a relationship immediately provokes a comment from Ari that a functional relationship between body weight and height jumped is not appropriate, body weight is not the most important thing to consider in this context. He also introduces the theme of the height of the imagined people and the relations between those and their body weights. This is not at all mentioned in the task but may be related to the general knowledge that taller people usually are better at the sport of high jump and because the heights of people were a theme of a task these students worked on earlier. In their apparently sincere discussion about the context, they pull the theme of the dialogue from mathematical representations to sports and body types which I understand as a centrifugal force away from the core of mathematical discourse. Ari and the other students are perhaps understandably confused by the realism of the task. They do not know that they are expected to focus on an abstract (and arbitrary) relationship and that for the mathematics teacher it is not important what is important for success in high-jumping. The teacher attempts to pull the dialogue back, trying a what-if question where the real-world factors are controlled, trying to bring the attention to the variables of mathematical interest. The students seem unaffected by the centripetal force of the teacher's utterance and go off on a further tangent. There followed an animated discussion about important factors relating to the sport of high jump, while the intended task of representing a statistical functional relationship by a scatter graph receded from the dialogue. It is open to question whether some amount of this type of dialogue could actually be a necessary precursor to emerging mathematical dialogue. It can be seen as building emotional connections between teacher and students, showing that their thinking and perspectives are valued and not brushed aside.

The *alien word* was infrequent as a type of sustained dialogue. This is because usually when students experience mathematics as alien to them, they are simply silent, or show their alienation indirectly. They might for example choose to look at irrelevant computer programs, play with their phones, or talk with peers about social matters. Only rarely does the theme of the alien word become explicit in dialogue.

Teacher: Are you all following what we are doing? (...) Do you think this is mathematics?

Einar: I think this is unnecessary bother.

Teacher: Unnecessary bother?

Einar: Yeah.

Gunnar: It's philosophical mathematics.

Anna: If you think about it in that way, see, that this is the weight plus the weight, but like in weightlifting, I don't feel we are talking about mathematics, rather talking about sports than mathematics.

The first utterance in this sequence was a response of the teacher to the situation that the majority of the student group seemed to be confused by the discussion and had ceased to participate in the dialogue. He tries to pull the class into a meta-discussion about the nature of mathematics and mathematics class, but Einar rejects the question on the grounds that it is unnecessary and a bother. Mathematics to him does not include students' own consideration of real-world complexities. His contribution also indicates that what is important is the amount of bother expected of him. He evaluates bother negatively. Gunnar partially supports Einar in that this is a special type of mathematics. These contributions are centrifugal. Anna makes an interesting comment that resonated with me, as she expressed what I was thinking about the dialogue up till then. It was about sports and not about mathematics. In a way this contribution is centripetal in two ways. She gives a reasonable answer to the teachers' question (she is affected by his pull) and criticizes the discussion on grounds that (in my theoretical terms) it is an authentic real-world dialogue, which she considers (again, in my theoretical terms) to be centrifugal and not conducive to mathematical discourse.

*Irony* means saying one thing and meaning another. To illustrate irony as a type of dialogue, there follows a dialogue on a task the point of which is to investigate the relationship between the number of workers and the time it would take them to finish a task. The task text asks the student to sketch a graph describing the time it would take to harvest a field of potatoes, depending on the number of potato pickers. The task is presented with unmarked axes in the positive direction, with the label "Total time it will take to finish the job" on the y-axis, and "Number of people picking potatoes" on the x-axis. The students were asked to work on this in their seats, the task being in their text. One student soon exclaims: "I don't understand" and the teacher goes to the whiteboard in front of the class.

Teacher: We are going to think about ... can we maybe assume. What are we going to say that one person would take long to harvest this field?

Siggi: How big is this field?

Teacher: Well, you just decide.

Siggi: Okay, it's just one meter.

Teacher: One meter. [Disbelief in the voice]

Siggi does not answer the teacher's initial question but asks a different one, and I assume he thinks this is an important question to answer in order to finish the task (in fact in the abstract mathematical model this is completely irrelevant and does not need to be answered). The teacher does completely shut down this line of inquiry but indicates a choice available for the student even if it does not really matter what the choice is. Siggi then responds with an extreme example that does not make sense if interpreted in the context of the real-world. Double-voiced discourse is a

discourse that has “a twofold direction” (Bakhtin, 1984, p.184); it is directed toward someone else’s discourse or toward someone else’s position. One interpretation of the student’s response is that the task does not make sense to him or that he is not committed to make sense of it, that the question (and therefore the teacher) is silly and his answer is going to be silly; that the question is too open, and he will test its boundary. It contains an evaluation of the whole situation; it is an ironic rejection to engage authentically in dialogue. The teacher’s final utterance in this sequence is an echo, a repetition of the prior utterance, but in a falling tone, with a different pattern of stress on the syllables. I interpret this as an expression of disbelief, and as if to say, “that is ridiculous, you know it, reconsider your suggestion”. An alternative interpretation could have been possible: in mathematics it can be informative to specialize (try out specific values in a general expression) with extreme values (not infrequently 1 or 0). The student’s ironic assumption could have been taken as an attempt to specialize to see what happens in an extreme instance. However, in light of the effect of the utterance on the teacher and the dialogue closing up at this point, it seems clear that the teacher takes Siggi’s contribution to be ironic, and some form of rejection to take the task seriously.

To illustrate the *emerging mathematical* type of dialogue I present a fragment where students are working further on the potato picking task. They have been asked to draw a graph of the relationship between the number of workers (x-axis) and time it takes to harvest a potato field. The teacher has circulated and comes to the whiteboard and says that he has seen “a lot of these” and draws a straight line segment with y-intercept at 100 and a negative slope, but not touching the x-axis. The students’ drawings are consistent with the fact that they have not had much (or any) experience with other kinds of graphs in their mathematics in school. Their repertoire of graphs consists of straight lines, as this is what a graph means to them, and they use it to express increasing or decreasing functions. After some students confirm that this is how it is and the teacher acknowledging “in a way, approximately” expecting something more, there follows:

Katie: You need to know how many people you want at the end.

Teacher: Do we?

Katie: Yes.

Teacher: Can’t we just...

Katie: If you have the time, you know, the maximum time ... how many people are you going to have in the end.

Siggi: We’re going to have thirty people.

Katie: Okay, if you are going to have thirty people you just do a hundred divided in thirty, then you have the time a single person has.

Teacher: Okay, you gave us some, what, four hundred eighty minutes? But can we do this like ... in mathematics we can add people infinitely. Although there might not be enough room in the field for everyone.

First, Katie expresses that for the task to make sense and be solvable we need to know the maximum number of people. This is reasonable from a real-world perspective, as it would be

impossible to have an infinite number of workers, or even a too large number of people. After a contribution from Sigg, Katie tries to express the relationship time divided by number of workers, getting the “time a single person has” where it would be correct to say that this is the total time it takes, but indeed this is also the time each single worker would have to work. My interpretation is that Katie’s answer confused the teacher or that the teacher interpreted her as thinking that the workers would work in succession, one at a time. Therefore, he does not build directly on her contribution but returns to a particular number (480) of minutes mentioned long before in the dialogue, and to the question of whether we need to know the maximum number of workers. He indicates that they could be infinitely many, because such is mathematics, even if not realistic.

After the sequence above, there follow over a hundred turns where the teacher and the students slowly clarify the meaning of the graphical representation, the role of the axes, and what the points refer to, they discuss the assumptions needed to make a model, and then gradually move away from a linear model towards a reciprocal one, and at last some students articulated this by themselves, incorporating the model into their own discourse. Here then, the teacher and the student group continually exposed and tested their understandings and perspectives against each other, and the centripetal force was strong enough to keep the dialogue mathematical, bringing students closer and closer to shared mathematical discourse.

### **Concluding remarks**

The students seem to orbit around mathematical concepts while the teacher tries to pull them into a more mathematical discourse. This is the dominating feature of the first three types of dialogue above. The authentic real-world type of dialogue is of particular importance. The students show sincere interest and enthusiasm for the real-world situations in which they recognise important features of the situation. Unfortunately, these features are tangential to the mathematical discourse the teacher tried to draw them into and may result in the dialogue drifting away from mathematical discourse.

To make sense of a mathematical task, the students need to attend to the context and not suspend sense making, while at the same time it also demands that the context is downplayed and not taken too seriously. In that way, the official mathematical discourse itself is double voiced and ironic. It does not really care about the high-jumping ability of people and what factors are most important to consider when estimating or predicting these abilities. It only cares about using graphs and functions to represent idealized phenomena. The teacher needs to convey this and then slowly lead students to productive ways to make mathematical sense, building on the sense made previously. For example, he suggests particular numbers (specializing) to think about or gently challenges the students’ answers while often recognizing that they are onto something. In this way the mathematical discourse becomes internally persuasive, at least for some students. We saw how the teacher and the students slowly achieve a more shared understanding in the dialogue in the last example, as it slowly approaches a more mathematical discourse, through constant exposure and testing of understandings.

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