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Fibre mathematics: exploring topological forms through material practices

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Through its focus on “fibre mathematics,” this paper contributes to discussions of how mathematical representations entail complex mixtures of matter and form. Drawing on inclusive materialism (de Freitas & Sinclair, 2014) as a way of rethinking the material labour of mathematics, we analyze data collected in a two-week weaving workshop, focusing on how participants worked with the mathematical concepts of dimension and connectivity in problems that were at once topological and material. Focusing on one student’s work, “Akari’s Problem,” we explore how the loom operates as a rich experimental field for the (re)creation of mathematical concepts. This paper takes up issues associated with mathematical representations by focusing on the haptic, visual, and material way that mathematical forms come to life through making processes.

Keywords: Textile art, dimension, topology, materiality.

Introduction

The term fibre mathematics refers to the mathematical activity entailed in textile arts. Studies of fibre mathematics have been pursued within ethnomathematics – for instance, research on the weaving practices in the Bedouin culture of the Negev desert (Katsap & Silverman, 2016) and Quechua speaking communities in Peru and Bolivia (Ascher & Ascher, 1981; Urton & Llanos, 1997). These studies point to how these practices are not (only) representations of Western mathematical concepts, but entail innovative ways of doing mathematics through various maker processes. The challenge is then to study material practices of various kinds, no matter their location, for how they engender distinctive mathematical concepts. We argue that different modalities and representations entail distinctive meaning, precisely because they involve different materialities. Our shared earthbound status means that humans are likely to work with similar mathematical concepts and materials across the planet, although these will be bound up with the particular corporeal habits valued in different localities. Our approach thus questions theories of representation which treat the concept-matter relationship as one which assumes concept as form-shaper and matter as passive form-taker.

Our research indicates that students learning textile arts are often also learning complex mathematical concepts and exploring challenging mathematical problems. We show how one student’s work (“Akari’s problem”) entailed creative interventions that were both technical (associated with the loom and other materials) and mathematical (associated with formal and aesthetic goals). Through analysis of video footage and interview data, we show how problem solving in such cases involves the loom as an “open machine” with a distinctive “technicity” – two terms defined by the philosopher of technology, Gilbert Simondon (1958/2017). We explore how a
technical “hack” of a traditional craft tool opened up new conceptual space, allowing research participants to invent new fibrous forms and explore complex geometric shapes that broke with classically Euclidean representations of space.

The research questions we explore in this paper are: (1) How do problems in the textile arts offer opportunities to explore rich mathematical thinking? (2) How does weaving as a material practice entail the mathematical concepts of dimension and connectivity? (3) In studying maker processes as mathematical, does representation best describe the relationship between concept and materiality?

**Theoretical Framework**

Simondon (1958/2017) defines the “technicity” of a technical object as that which concerns its fundamental structure rather than its use. Attending to technicity is a way of better understanding the mode of being of the technical object, across various situations. This requires a genetic history of the technical object (in this case the loom), so that we can compare different manifestations and identify key elements of its structure. We do this to better understand how technology and mathematical practices are linked, and how material media are essential in mathematics, be they writing surfaces, touch screens, or paper and pencil. The work of Simondon offers us new materialist ways of thinking about how mathematics is made and performed outside of mainstream mathematical representations.

This philosophical framing of our approach sets the stage for a study of mathematical practices in contemporary textile arts. Following Simondon, who advocates for historical and philosophical studies of technology, we see technology as more than a tool that enhances human interests, more than a device for representing concepts conceived in the mind. Our relationship with technics is more complicated and we benefit from considering more nuanced ways of understanding the force of the technical object in our cultural practices (including mathematics). Writing just at dawn of cybernetics and the “information age,” Simondon argues we must examine our technologies to understand how our concepts are implicated in them, not as representations, but in a more direct assemblage.

We consider mathematical behavior as an ontogenerative practice (a making practice) that brings new concepts into the world, and in so doing reshapes the world. Learning mathematics becomes “an indeterminate act of assembling various kinds of agencies rather than a trajectory that ends in the acquiring of fixed objects of knowledge” (de Freitas & Sinclair, 2014, p. 52). This implies an understanding that mathematics takes no definite or final form, but rather it continually (re)emerges within the material practices of the workshop or the classroom. Our non-dualist approach argues that representations of mathematical ideas cannot be isolated from the phenomenon to which they are wedded. In particular, this paper focuses on technological interventions/actions that enable new formal and aesthetic choices that mobilize new conceptions of dimension and connectivity.

Weaving is performed with an ancient but ever-changing technology, called a loom. The loom creates a problem space for exploring pattern, texture, dimensionality, connectivity and number. The loom is both an “open machine” and an “associated milieu” for the technical-aesthetic activity of weaving (Simondon, 1958/2017). As an open machine, the loom itself can be undone and remade, maintaining its structural integrity while allowing for inventive alterations that enable
students to build complex topological shapes. Indeed, weaving of all kinds, be it planar cloth or a three-dimensional objects, operates through the relationship between the discrete and the continuous, using thread, yarn or other continuous matter to create patterns that often achieve their aesthetic impact through discretization of and contrast between individuated units of color or texture. Many weaving practices operate through this pixelated effect, with links to the history of computing and digital image making (Plant, 1995; Bachmann, 1998). Given this aesthetic aspect, and the pivotal role of the relationship between discrete and continuous magnitude in fueling mathematical invention (de Freitas, 2017), we consider weaving an excellent case study for examining the material labour of mathematics, and for rethinking the role of representation in mathematical behaviour. This perspective supplements research that has explored the aesthetic aspect of mathematics (Coles & Sinclair, 2019), the sensory capacity of the body (de Freitas, 2016), and the more-than-human and ontological nature of mathematics (Gutiérrez, 2017).

Methods and Data

Research data was generated through participant observation and informal interviews conducted in the weaving studio of a craft school in the eastern United States, during a two-week weaving workshop called: “Weaving Origami and Other Dimensional Possibilities.” Twelve students (including the first author), motivated by personal interest in the proposed course, applied and were accepted into the class by the craft school selection committee. Most of the twelve participants – who spent at least eight hours/day weaving during the workshop – had more than five years of weaving experience. Data consisted of field observation writing and sketching, video recording during studio sessions, and interviews of nine participants. Cameras were both fixed (to walls and looms) and worn on persons, to obtain diverse perspectives on the material and ritual practices entailed in weaving. Interviews focused on artifacts and instruments from the studio, and questions targeted participants’ manner of describing the various problems they encountered during the two-week course.

The workshop focused on a distinctive challenge – how to weave “dimensional” cloth. The workshop’s title, “Weaving Origami,” describes a specific technique developed by the workshop instructor, Sue Taylor, which involves weaving short bands of cloth that jut out from the classically two-dimensional woven plane. These bands can fold into various origami forms (Figure 1). The technique for generating these three-dimensional bands derives from a common weaving trick used to repair broken warp strings. The instructor showed students how to use the free weights employed in this technique to alter how the loom distributes tension. Students began by directly imitating the instructor’s work. After two days, many in the classroom quickly moved to innovate on this thinking.

In this paper, we describe the work of one participant (Akari), by looking at the way that “Akari’s Problem” involved complex topological forms and became a gathering point for workshop participants. To illustrate, we discuss two woven objects Akari produced during the workshop (Figures 2 & 3). These woven forms offer a snapshot of a generative material-mathematical challenge. In our analysis, we draw on video from a chest-mounted GoPro video camera Akari wore
while she worked on her first weaving and a loom-side interview with Akari, as she made her second sample.

Figure 1: Susie Taylor  
Figure 2: Akari’s first sample  
Figure 3: Akari’s second sample

Discussion

Dimension is a hugely important mathematical concept on which our ideas about measurement and space depend. Mathematicians in different specialized fields define dimension in many different ways (Pearse, 2003). The concept, however, proved problematic in the late 19th century, when various mathematicians (Cantor, Peano, Brouwer) paradoxically demonstrated how to map the interval to the plane (Pearse, 2003). New theories of dimensionality emerged in the 20th century from Lebesgue, Hausdorff, and others, alongside the study of fractals and fractional dimensions. Plugging into our intuitions of space as it crisscrosses mathematical domains, the concept of dimension is multiple. Freudenthal (1983) suggests we consider dimension within three frameworks: Euclidean (an integer property of objects and spaces), Analytic (determined by the number of variables needed to describe an object), and Topological (pertaining to freedom of movement and boundary relationships).

Panorkou and Pratt (2016) contend that mathematics classrooms rarely address dimension directly. Skordoulis, Vitsas, Dafermos, and Koleza (2009) argue that a Euclidean notion of dimension is typically assumed and sometimes in conflict with an Analytic framework. Most mathematics students will not interrogate the concept of dimension, despite its incredibly opaque and open-ended nature. In the workshop, however, dimension was an incredibly common buzzword. Participants struggling to invent dimensional cloth on the loom engaged with the concept directly. Moreover, they each had distinctive aesthetic problems to solve, which allowed for an open and diverse group inquiry.

Due to the technical requirements of the loom and soft pliable nature of cloth, the working of woven form into three dimensions already involves the kind of contortions conceptualized mathematically by topology. Topology is a field of mathematics that investigates how objects remain unchanged after stretching, bending and twisting. It is often referred to as “rubber sheet geometry” (O’Shea, 2007) because objects studied by topologists can be pulled and stretched without changing their
topological properties. Topology lends itself to the study of non-rigid forms, as in the case of weaving. Although weaving requires the heavy use of counting and anticipatory calculation in planning projects, the production of cloth is an essentially connective process in which lines of fibre fuse according to a systematic relationship to produce planar cloth. Weaving techniques developed to double the capacities of the human arm span further enhance the complex connective possibilities of the loom. These techniques allow one to interconnect several layers of cloth in a manner limited only by the scope of the loom itself and the amount of time a weaver wishes to commit to her project.

To explore how fibre mathematics and topological concepts of dimension can be productively entangled, we begin by articulating Akari’s original problem and analyzing her two weavings from a topological perspective. After experimenting at length with paper models, Akari planned a complex folded weave involving five parallel bands of cloth that emerge a ground cloth (Figure 4). Akari hoped to fold and twist these bands to reveal and conceal the colorful underside of each band. Because, in weaving, color forms through interlacing warp and weft, Akari planned a complex structure in which she would weave a cylindrical form made up of two intersecting planes (Figure 5, where these planes are colored pink and purple). This would allow two distinctive colors to alternate in prominence, depending on the pattern of folding. Looking to analyze the weaving topologically, we began by imagining the weaving as made up of a ground cloth with five pinched tori. (The pinched torus (Figure 6a) is called a pseudomanifold because there is a singularity at the pinch, creating an obstacle that cannot be traversed continuously. Pseudomanifolds are, however, still orientable.) However, further examination of the weaving revealed that there was not a singular pinch but rather a *crease* at the end of each tube (Figure 6b). Akari’s cloth had five protrusions, which were neither manifold nor pseudomanifold, but something else altogether (Figure 6c).

![Figure 4: Paper models of the cloth](image1)

![Figure 5: A diagram of Akari’s layers](image2)

These moments of creasing, where the weaving branches to form five bands, make this artifact complex in both the making and the mathematics. The creases represent a fundamental break in the object’s basic Euclidean texture. If we select one such crease, and zoom in far enough, to see what is going on, we find four planar cloths all intersecting in the line (Figure 6d). If we keep zooming in, further and further, in search of the basic Euclidean plane, we will never be able to find it. No matter how close we get, every point on that line will belong to neighborhoods in four planes, breaking Euclidean rules, and making the crease singularity quite problematic. Akari’s problem, in other words, involved making a mathematical form with yarn that exceeded conventional Euclidean geometry, and indeed exceeded conventional rules of behavior for manifolds and pseudomanifolds.
Although ostensibly her weaving is made of two-dimensional surfaces, topologically it cannot be modelled as a two-dimensional manifold. This is because the cloth is not “locally Euclidean” precisely at these creases. Hacking the loom allowed Akari to break with the planar grid in regulated but inventive ways. At these intersections, no matter how closely you stretch and examine the material, Akari’s cloth will never behave like a flat two dimensional plane.

Figure 6: a) Pinched torus, b) Akari’s tube, c) Rendering Akari’s cloth, d) Four planes of cloth

Akari came to know the properties of this shape not through this mathematical vocabulary or instruction but through an elaborate pattern of passes, which she developed to construct each of the five tubes. She noted in her interview how difficult this was: “My head was like wait! Wait! Wait!” Watching her activity on video, we see that Akari taps into a rhythm measured out by gestures made with and towards the loom. To keep her place in a 22-step progression of movements, she neatly organizes her threads to mark out visually her place in the pattern.

Despite her best efforts to plan her work, the materiality of Akari’s concept broke away from the representation she had generated in her paper model. About 20cm into her project, Akari discovered that her double-layered tubes of cloth are structurally too dense to fold according to her plans. Video footage from the chest-mounted GoPro shows Akari pausing to examine her work. Sensing this break, six workshop participants gather around to observe and comment on her work. In the video footage, Akari shares her thinking aloud, while manipulating the cloth. A key conversation ensues about how to best explore the new freedoms opened up by the technological transformations that all participants have performed on their looms. At a climactic moment, three participants point to the loom’s new tensioning system and exclaim in chorus “’Cause they're weighted! They’re weighted separately!” They point out that the new distribution of tension, now controlled by water bottles swinging off the back of the loom, works as a third dimensional coordinate in Akari’s cloth. This marks a collective realization about the dimensional possibilities opened up by the redistribution of tension on the loom. Analysis of this episode reveals how the coupling of tension with dimension was not simply the representing of an immaterial concept (dimension) in a material form (a thread under tension), but was instead a matter of delving deeper into the potentiality of mathematical concepts as they inhere and mutate within matter.

Concluding Comments

This paper builds on previous work on topological thinking in mathematics education, and the challenges of representing (or enlivening) complex manifolds in material form (de Freitas &
McCarthy, 2014; Strohecker, 1991). It seeks to contribute to a growing body of literature on the role of technology in developing mathematical aspects of spatial sense (de Freitas, 2017; Ferrara & Mammana, 2014; Panorkou & Pratt, 2016) and it offers new possibilities for exploring mathematics in informal learning settings (Nemirovsky, Kelton, & Rhodehamel, 2013), pointing to the importance of aesthetically structured activities in mathematics teaching and learning (Coles & Sinclair, 2019).

Our study shows how maker and craft practices are an important site for innovative and unconventional representations of mathematical concepts. Although the usual question posed to those studying informal mathematics is “where is the mathematics?,” we suggest that mathematics is found in the material practices embodied by the weavers. These weavers are not mathematicians (and they might be the first to admit that!) and although there was no direct instruction as to how to use topological concepts to build the dimensional cloth, the problem space created an opportunity for doing mathematics through direct engagement with technical objects (the loom, the threads, etc.). These problem spaces are engendered by the loom, but the solutions paths are not scripted by the loom, especially in this case, where the loom is an open machine and literally modified by individual participants (hacked with weighting) as they each explored their own problem space, driven by their aesthetic goal. The loom thus operates as a kind of generative problematics. The various goals of the weavers entailed different mathematical relationships, which had to be mobilized in order to create the desired patterns and asymmetries. The specific technicity of the loom has particular structural features that create opportunities to work with particular mathematical concepts.

When educators speak of mathematical representations, they typically point to an object or diagram or symbol as that which stands in for (or refers to) an immaterial mathematical concept. Does it make sense to speak of Akari’s cloth as a representation of a mathematical concept? Our case study shows how a weaving problem can engender a very complex mathematical form, indeed a form so complex it might be deemed the sort of “monster” that Lakatos wrote about when he discussed the intellectual labour of mathematical invention. Mathematical monsters are initially barred, but then you realize you want to play with them, and the rules need to change so that you can. Akari’s cloth is the result of a weaving process that played with the conventional tension of the loom, in order to dive into the indeterminacy of the concept of dimension. Once we attend more carefully to the way material media play a big role in mathematical labour, and perhaps rethink the nature of representation in terms of making processes, the question “where is the mathematics?” becomes even more interesting. Of course, one needs to attend to the differences between the material labour of a mathematician working with chalk or software and the material labour of a weaver at the loom. Our claim is that more attention to these material media will open up our understanding of mathematics in all its diversity, demonstrating how representation is entangled in making processes.

References


