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A lot of research is being done on the interplay of mathematical learning and language skills. However, graphic aspects have been met with little response so far. What role does writing play in the process of improving language proficiency? This paper empirically reconstructs how language use develops within one lesson that provides manifold opportunities for students to negotiate and discuss. Special attention is paid to the role pupil's written notes play within that development.

Keywords: Language skills, writing processes, mathematical learning.

Introduction

We know that language plays an important role in mathematical learning. Consequently, mathematics education is not only about teaching and learning mathematical ideas and concepts, but also about the development of a language that reaches the demands of describing mathematical activities and expressing fundamental ideas. Mathematical and language learning accompany each other. One approach to improve mathematical and language skills in an interconnected way is to initiate interactive learning situations. Such discursive situations are meant to encourage language use *and* mathematical learning. Why should that work? First, language serves a communicative function. So it appears to be a platitude that learning language requires using language and interacting with others. Second, language fulfills a cognitive function. The challenge to develop mental mathematical objects is strongly linked to the availability of appropriate means of expression and language skills. Not only theories on language support the assumption that interactive learning situations are beneficial for cross-linked mathematical and language learning. However, the conceptualization of learning as a social process backs that idea. Learning takes place in the process of participating in interactive situations (Miller, 1986; Fetzer, 2007a). Referring to social frameworks of learning, all participating students should benefit from such a proceeding. Those who actively contribute to a mathematical discourse improve their language skills by developing an appropriate language that has the potential to convey their idea. Those who rather receptively attend the situation might profit by being part of a mathematical discourse. They might not only catch up mathematical ideas, but also ways to express them. But how does language use develop in classroom interaction? Graphic aspects seem to play an important role in settings that challenge mathematical language development; taking notes on solving processes, recording explanations and accounting for answers in a written way are constituent elements of cooperative classroom settings. Consequently, when studying processes of learning to *speak* mathematically, the role of *writing* should be taken into account. What role do graphic aspects play within that development? This paper provides a closer look at the two essential situations concerning these research questions: the phase of writing, and the phase of presenting on the basis of graphic notes.

Development in language use

In order to describe developments in language use, we need to specify. Koch and Oesterreicher (1985; Oesterreicher, 1997) offer a distinction in two dimensions of language: the medial and the conceptual. The medial dimension is dichotomic: language can be either specified as phonic, when something is said and can be heard, or, it is assigned to graphic forms like writing. In contrast, the conceptual dimension is gradual. Language can be conceptually rather oral or rather written. It stretches between the poles of communicative nearness (oral) and communicative distance (written). This specification applies for both, medially graphic and medially phonic, forms of language. Talking with a good friend for example is closer to the conceptually oral pole than a presentation at a conference. A newspaper article is 'more' written than a twitter post. If we talk to our child when having breakfast, we might use short and incomplete sentences and combine them with gestures. As we are in a face-to-face situation, we can react directly to our counterpart and spontaneously negotiate our roles. Koch and Oesterreicher call this a language of nearness. The strategies of communication belong to the pole of orality. In contrast, conceptually written language refers to the pole of literacy and language of distance. It is used when the interlocutors are not necessarily in direct relation and the processes of speech production and speech reception might be separated from each other. Thus, aspects of the situational and cultural context have to be made explicit. Consequently, sentences are more complex, main clauses and subordinate clauses are formulated. Specific terms come into play to be precise and explicit. Koch and Oesterreicher call this a language of distance. Examples are a text of law (graphic) or a scientific lecture (phonic).

Koch and Oesterreicher help to classify the registers of everyday language and academic language (Cummins, 2008). Halliday describes a register as "a variety of language, corresponding to a variety of situation" (1985, p. 29). According to him, the term register points to the detection that individuals adapt their use of language to a given situation. Thus, the register of everyday language is rather conceptually oral, no matter which medium is realized. It has to fulfil the function of a fast and unproblematic communication in our everyday life. As such, the oral language may be supported by gestures or by reference to a context. Words do not have to be clearly defined and sentences may be short or even incomplete. In contrast, the register of academic language is conceptually written, again irrespective of its medium. In academic contexts, language should be as explicit and precise as possible and intelligible without any reference to a specific situation. For that reason, words have to be well defined. Sentences might be complex in order to reflect relations.

Eventually, there is the register of mathematical language. It is an academic register related to a certain field; mathematics. Technical terms like probability, multiplication or subtracting, as well as specific sentence structures, are constituent elements of mathematical language. Especially on the medially graphic level, very short and decontextualized sentences or graphic representations are characteristic for that register.

In mathematics classes, we face the challenge to develop individual language use from orality towards literacy, from everyday language in the direction of learning to speak and write mathematically. The learning and teaching situation remains the same: students and teacher know each other and interact face-to-face. Nevertheless, a development towards precise and decontextualized academic, respectively mathematic, language has to be initiated. Fetzer (2007a; b) proves empirically that writing in mathematical classes and mathematical discourse on the basis of students' written works fundamentally support *mathematical* learning. In this paper, special focus is put on *language* development, and the way in which the implementation of medially graphic elements helps to provoke a move in the direction of literacy is reconstructed.

Data and methodological aspects

The examples analyzed in this paper are taken from the empirical study MitMaL (**Mit**einander **Ma**the **L**ernen) (Co-operative Mathematical Learning). In this study, cooperative and individual learning settings are compared to each other. Two math classes were observed and video- recorded for a couple of weeks. Both groups dealt with the same mathematical subject, but implemented different methodical approaches. In one class (group i), **i**ndividual learning situations dominated. In contrast, the other teacher predominantly initiated **c**ooperative group work and provided the opportunity for mathematical whole class discussion (group c). Pre-test, post-test and follow-up-test constitute the quantitative part of the study. Analyses on the transcripts of the videos account for the qualitative part of the study. They are conducted in a reconstructive manner applying analyses of interaction (Cobb & Bauersfeld, 1995). This method refers to the interactional theory of learning and is based on the ethnomethodological conversation analysis. In contrast to conversation analysis, it focuses on the thematic development of a given face-to-face interaction rather than on its structural development.

The videos for this paper are taken from group c, who worked in a cooperative manner. In order to reconstruct the development of language use, the paper concentrates on one single math lesson on probability in grade four. Debora is accompanied throughout the whole lesson. In this way, her individual performance and development can be reconstructed from the beginning until the end of the lesson. Special attention is paid to the role the notes taken in the group work play in the presentation. How do they influence the verbal language skills?

Empirical example

The empirical example is taken from a math lesson on probability. 24 students of grade four try to find out which of the four given rules is the best to win when turning a colored wheel of fortune. The wheel of fortune is equally divided into eight sections, numbered 1 to 8. Two fields are red, two are blue, three are green, and one is white. The rules are: *(1) You win with 1, 2 or 3*; *(2) You win with red*; *(3) You win with white or blue*; *(4) You win with 2, 4, 6 or 8*. Card four is the most probable to win.

The lesson starts with a thematic introduction in a whole-class situation. The tutor presents the wheel of fortune, and some students go for a first attempt. Afterwards, the children work in groups of four. They are equipped with a wheel of fortune, one set of cards with rules, and with a task sheet: *Work together and find out which rule is likely to be the best to win. Why do you think so?* The challenge is to work on the tasks, take notes and give reasons in written form. At the end of the lesson, children and tutor come together. They discuss their findings with their written notes at hand.

Language use in the context of probability: Talking about probability requires mathematical terms like probable, improbable and equiprobable. In German, ‘probable’ sounds familiar to children. The expression is used in everyday language and is applied in the sense of ‘perhaps’ or ‘might be’. In contrast, ‘improbable’ and ‘equiprobable’ sound rather strange to young students. These expressions belong to the academic, respectively mathematical, register. However, talking and writing mathematically requires more than academic expressions and mathematical terms. Describing, explaining and giving reasons require academic language proficiency on the sentence level. In this particular task, the students first have to describe their selection in written form. This is possible using a main clause. Conceptually spoken language in everyday register meets the demands. In addition, an explanatory statement on their selection is required. This is much more challenging, as German language demands subordinate clauses in order to formulate causal connections. In contrast to main clauses that are structured subject – predicate – object, subordinate clauses are put together in an inverted structure: subject – object – predicate. This inversion does not necessarily belong to the everyday register. The outstanding challenge in the context of probability is the fact that we do not only need a causal conjunction followed by a subordinate clause with inverted sentence structure, but also an infinitive clause as a third component. This last aspect clearly belongs to academic language. To put it in other words: in order to compliment the decision with reasons and justification, not only mathematical terms, but academic registers and complex sentence structures are especially necessary.

Material for the group work: The rules rely on conceptually spoken language. They address the students directly and refer to the concrete given context of turning the wheel here and now. Mathematically spoken, it rather aims at a situational or context-bound understanding of probability and representativeness of personal attempts rather than at abstraction. The rules provide for a strong link to the students’ experience on the one hand, and to their everyday language on the other hand.

The tasks (see above) apply conceptually rather spoken language as well. Again, the students are addressed directly. None of the mathematical terms of probability is used. Instead, the children are asked to choose one of the cards. A second sentence aims at an explanation. Interestingly enough, these colloquial questions can hardly be answered on a conceptually spoken level: if you work on the task in the order of appearance, you first have to determine the selection. That is easy. Afterwards, you need a causal subordinate clause and the infinitive clause, which is grammatically complex. In principle, conditional connections could be a solution: “If you choose…, you are likely to win. That is because…” This structure is much easier in German than the causal connection. Nevertheless, it clearly belongs to the academic register. However, such an answer is only possible if the students have already reached a rather high level of abstraction at this time. Condensed: if students work on the tasks and provide answers, they either have to fulfil a move on the conceptual level from spoken to written language, or they have to develop an abstract understanding that enables them to choose a slightly less complex language use.

Beginning of the lesson (phonic): Children and tutor are sitting in a circle around a colored wheel of fortune ready to be spun. Debora does not contribute to this opening. The tutor starts the lesson referring to the students’ experiences with wheels of fortune: “Perhaps you know something like this. Such a wheel of fortune has something to do with winning or not. This is why it’s called wheel of fortune, cause if you win, you are lucky”. (Wheel of fortune is ‘*Glück*srad’ in German, and being lucky is ‘*Glück*haben’). In the first introduction to the theme, the tutor links not only the context of own experiences with the mathematical context of probability, but also different language registers.

She plays on the fact that *Glück* bridges both registers. Pointing at the paper wheel of fortune, she moves her finger around in circles and says: “Today we have a closer look at the wheel of fortune.” Repeatedly in this introduction, she moves her finger in this manner as if she spun the wheel. By doing this, she might strengthen an activity-orientated representative understanding of probability: Rule X wins because my own (little) random sample ‘proves’ it. This understanding does not claim for decontextualized academic language.

The tutor’s language changes within the introduction. First, she seems to pick the students up and see them through by applying the everyday register. She distinguishes “good rules” you are likely to win with, rules you might “not necessarily” win with and rules that make it “quite impossible” to win. When explaining the task, she even grades further down and speaks of “a good way to win” and “stupid cards”. This colloquial language is accompanied by the turning gesture. Acting and speaking like this, the tutor marks the concrete situation as the starting point of the experiment on probability. Not before her last sentences, she uses the mathematical expressions: “For example, with this card, you will most probably win; with these two cards, it is perhaps equiprobable to win; and with this one, it is rather improbable.” This is the only time these terms are quoted in the introduction scene.

Group work (phonic and graphic): For a closer look at the group work, Debora’s group is chosen exemplarily. Three phases of language use can be reconstructed within the working process: (1) conceptually spoken language, (2) everyday language with scattered mathematical terms, (3) everyday language with specifications and conjunctions.

(1): Debora is the girl to take the initiative in the group. She suggests cutting the wheel out. While doing this, she lays the foundations for an activity-orientated approach and context-specific experiences. This first phase is dominated by the register of everyday and conceptually spoken language. Sentences remain incomplete; gestures play an important role for understanding. The challenge of writing is no issue of negotiating yet. The following scene emerges as soon as the girls have read out the rules. It illustrates the register of everyday language the girls apply.

Debora:	*Points at Dalya* That’s best
Sara:	Yes
Debora:	*to Dalya* say it again *reaches for the wheel and takes it*
Dalya:	I got *reads out her card* you win with 2/ 4/ 6/ or 8\
Debora:	*puts her fingers on the wheel* it’s the majority- all fields but three-
Sara:	*points at one field*
Debora:	I see- *moves her fingers on the wheel* five as well/
Sara:	no
Debora:	*looking at the wheel, fingers still fixed on fields* makes no difference- it’s equal\

However, they work straightforward on the task and find a first solution that is mathematically correct. The choice is formulated using conceptually spoken, deictic and context-bound language: “That’s best.” No mathematical term is applied. A first approach to explanation might be the physical detection (by touching) of four (out of eight) ‘lucky’ fields.

(2): The second phase can be characterized by conceptually spoken language with scattered mathematical terms. Graphic aspects still do not come into play. In this scene, one can reconstruct one example of how mathematical language works its way into the discourse: Debora picks up a comment from a neighbouring group “Red is the most improbable.” She spontaneously integrates the term in her active language use and confirms with a look on the wheel: “Yes, red is the most improbable.” In the following, this term remains a constituent part of the interactive process. All four girls stick to conceptually spoken language using the register of everyday language. Their sentences remain incomplete with scattered mathematical expressions. For example, Debora sums their findings up as follows: “Red is the most improbable. One, two or three is same, and Dalya’s is best.”

(3): As soon as it comes to *writing*, there is no way of supporting words by pointing or showing. Written language has to carry the whole meaning. Accordingly, writing mathematically arises the necessity to develop language use. Debora’s group works sequentially. First, they answer the first part of the task: *Work together and find out which rule is likely to be the best to win.* They choose the conceptually spoken wording of their verbal formulation from the beginning “That’s best” and elaborate on it: “Best is card four which says you win with 2, 4, 6 or 8.” The deictic component of the face-to-face interaction is replaced by naming the card “card four”. The necessity to write results in specifying the concrete situation. The rule is taken from the card and copied word by word. This appears to be an appropriate strategy to create an answer that is socially accepted as ‘good mathematical language’. At the same time, the sentence structure remains conceptually oral. The elements ‘specification by concreteness’ and ‘copying phrases’ are put together one after the other. Though the subordinate clause is introduced with the appropriate conjunction, it is not conducted in the right grammatical arrangement.

Later in the group work, the tutor reminds them to provide reasons for their decision. Subsequently, Dalya picks up the pencil.

Julia:	Because they are well spread\
Dalya:	*holding the pen* Yes-
Julia:	*points her finger to the paper* Well- because they are well spread on the turntable\
Dalya:	yes *writes*
Julia:	Wheel of fortune – write wheel of fortune\
Dalya:	Well- we’ve found a because- uhm I’ve written *reads out* best is card four which says you win with 2 4 6 or 8 because they are well spread on the wheel of fortune.
Debora:	spread *thumb up*

In this short scene, the justification is graphically recorded. The aspect of the winning fields being evenly or well spread on the wheel of fortune was mentioned earlier only once by Julia. At that time, her idea would not come through in the interactional process. Why is she successful now? The tutor directly addresses the group and asks them to find a justification. The girls are running out of time. In this situation, Julia starts her suggestion with “because”. Her first utterance is rather vague. Perhaps feeling supported by Dalya’s “yes”, she then elaborates “the turntable”. As a third move towards improved language, she implements a mathematical term “wheel of fortune.” Summing up, her suggestion is put forward at the right time within the solving process. Julia is working with the elements of *specification* and *integrating mathematical terms*. The *conjunction* ‘because’ indicates a justification. That was the task. The sentence structure itself still reminds of spoken language. This form of speaking mathematically appears to be convincing to the group members. Debora seems to try that wording on her own by repeating it. Her thumb up can be understood as acceptance. Again, she picks up an expression and integrates it spontaneously in her active vocabulary.

End of the lesson (phonic and graphic): Gathering for the end of the lesson, the students have their sheets with their written works at hand. Debora is the first child to contribute in the presentation. As she hesitates and starts over twice, it seems as if she has to orientate herself first. In this situation, her group’s notes appear to be helpful. After a quick glance at the paper, she performs much more fluently: “*Best* we found out is card four.” In German, this is an example of conceptually spoken language: her word order is irritating. Precisely, ‘best’ is used as an adverb referring to ‘to find out’. Taking into account the given context, it appears to be more reasonable that she uses ‘best’ to characterize card four. Thus, her sentence structure is grammatically incorrect. Instead, it might reveal her order of remembering: we had to select (“best”), we had to work (“we found out”), we made a decision (“card four”). Here, the notes appear to support the memory: what did we do? In the following, her language develops. “Cause best is card four”. This is, in fact, only half a sentence. But, in contrast to the first attempt, it is in the right German word order. Then, she reads from the notes, probably in order to make sure to get the correct wording: “which says you win with 2 4 6 or 8”. Turning her eyes away from the notes and looking to the tutor, she proceeds: “Cause they are quite well spread on the wheel of fortune.” On the notes, the subordinate clause is in the wrong word order. Speaking aloud, she corrects the sentence structure. This can be interpreted as a step towards abstraction and language of distance. Her language develops towards academic register. Summing up, the notes warrant three aspects: orientation, reminder and thematic reassurance that helps to develop language skills.

Analysing the other contributions in this presentation confirms these findings. Every single contribution is thematically on point. Obviously, graphic notes help to *orientate*. Moreover, the students supply their comments with phrases that structure the whole presentation process: “We found out the same” or “Not as the others”. That provides for orientation and structure not only within individual understanding processes, but also within the interactional discourse. The aspect of *reminding* of the own working process appears to take affect strongly. Every single student who contributes to the whole class presentation thematically closely links his or her comment to the notes. The quick glance at the paper is to be reconstructed in each sequence. Especially the observation that medially verbal language use improves compared to the graphic version in the notes confirms as an empirical result. For example, in one group, the notes say: “Weil Gewinnkarte vier das halbe Glücksrad einnimmt.” (Cause winning card four takes half the wheel of fortune.). Contributing in the class, the presenting boy turns ‘half’ into a noun and combines it with a genitive construction (“nimmt die Hälfte des Rades ein”). Using substantives and genitive constructions is a German speciality, which is especially characteristic for academic register. Accordingly, this is understood as *development in language skills*.

Conclusions

How does language use develop in cooperative classroom settings? What role do graphic aspects play? This paper provides a closer look at the two essential situations concerning these research questions: the phase of wiring, and the phase of presenting on the basis of graphic notes.

The necessity to *write* mathematically activates a development in language move at the conceptual level. Mathematical terms are embedded in rather spoken sentence structures; a first move towards mathematical language. Specification on the concrete context serves for more precision in descriptions. Even at the level of sentence structure, first developments can be reconstructed. On the one hand, students implement phrases that were socially accepted earlier as mathematically elaborated. On the other hand, they implement conjunctions. If-clauses and causal-clauses are characteristic elements of academic and especially mathematical register.

Presenting is eventually a medially verbal action. But, in the given case, graphic aspects work their way into interaction. They contribute fundamentally to orientation, structuring and reminding both individual learning processes, and the interactional process. Particularly the graphic notes thematically reassure students and thus provide capacity to develop language skills.

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