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Explorative study on language means for talking about enlarging figures in group work

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This paper presents a first step for designing language support for Grade 9 students working collaboratively on the topic of similarity, more precisely an adaptation of Brousseau's tangram task. The qualitative study identifies (topic specific) language means which students use for talking about their ideas on how to enlarge figures, combining geometrical and arithmetical ideas. Furthermore, a textbook analysis shows the target language that students should use and understand at the end of the teaching unit. The identified difference between both repertoires of identified language means calls for developing language-responsive teaching-learning arrangements in the future in which both language repertoires can be bridged systematically by macro-scaffolding trajectories.

Keywords: Language means, language repertoire of the learner, macro scaffolding, design research.

Background: Need for supporting language means for rich discursive practices

Students' participation is pointed out as crucial for learning mathematics by many researchers (e.g., Sfard, 2008) and is seen as "a process of enculturation into mathematical practices, including discursive practices (e.g., ways of explaining, proving, or defining mathematical concepts)" (Barwell, 2014, p. 332). Qualitative studies identify sequences of explaining and arguing as especially important for students' conceptual learning, in whole class discussions as well as in small group work (e.g., Mercer, Wegerif, & Dawes, 1999; Erath, 2017). However, especially for students with low language proficiency, participating in discursive practices that allow talking about conceptual knowledge is challenging (Erath, Prediger, Quasthoff, & Heller, 2018). Since language acts as a medium of thinking (Vygotsky, 1987) and following Interactional Discourse Analysis (Erath et al., 2018), this calls for supporting students in accomplishing demanding discursive practices in order to provide access to mathematical learning opportunities. Whereas in whole-class discussions, overcoming language-related obstacles can be spontaneously supported by the teacher, group work requires a prepared support when the teacher is absent. One dimension of prepared support refers to the establishment of norms about the ways students talk to each other, such as the "ground rules" proposed by Mercer (e.g., Mercer et al., 1999), emphasizing activities such as sharing relevant information and discussing alternatives. Students with low language proficiency often need additional support for accomplishing these norms, when lexical means or syntactical structures are required for explaining and arguing in the mathematical topic. Thus, referring to Gibbons' (2002) approach of macro scaffolding, these language means can be provided in prepared ways.

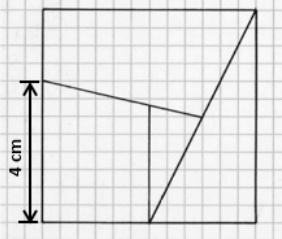
As Prediger & Zindel (2017) emphasize, the identification of mathematically relevant lexical and syntactical means for a specific mathematical topic is an *empirical task*; it can be conducted by analyzing textbooks and teaching-learning processes. Following the research approach of

empirically identifying relevant language means, this study investigates a textbook and empirical video data for identifying the language means students initially use, the target language means, and for exploring possibilities of bridging between them.

Research context: Exploring similarity in the Tangram Task

Especially teaching-learning arrangements that activate students' individual ideas and allow processes of solving problems and discovering mathematics have the potential to elicit vivid discussions about mathematics (e.g., Brousseau, 1997) and make demanding discursive practices necessary. For identifying students' initially used language means, the presented study focuses on the first task in a teaching unit on similarity using an adaption for four students of Brousseau's (1997, p. 177ff.) tangram task (see Figure 1; translated from German by the author). The task follows a more dynamic perspective on similarity since it leads to an understanding of similarity as the result of the process of enlarging figures true to scale: Hölzl (2018) terms approaches to similarity as *dynamic* if they relate to transformation geometry and as *static* if they refer to Euklid and define similar figures by identical angles and aspect ratios.

a) We would like to have a larger version of this figure.
In doing so, a length of 4 cm should become a length of 7 cm.



(1) Each chooses a part of the figure.
(2) Execute on the following steps in individual work:

- Draw the enlarged version on squared paper
- Cut the figure out

b) Check in your group if the enlarged pieces fit together again.
What could be the reason?

Figure 1: Task 1 of the teaching-learning arrangement of enlarging figures

Thus, this paper empirically investigates students' initial language means in group work related to a task on enlarging figures true to scale and compares this repertoire to the target language means as identified in a widely used textbook by following these two research questions:

RQ1: Which language means can be identified in students' talk and in textbooks?

RQ2: Do some of these means have the potential to bridge between the identified repertoires?

Methods: Qualitative analysis of students' interactions solving the tangram task

The presented analysis is part of the larger project MAGENTA that is conducted in the tradition of design research (Gravemeijer & Cobb, 2006). It aims at developing a language-responsive teaching-learning arrangement for the topic of similarity in German Grade 9 classrooms as well as building local theories on the students' learning processes.

Methods of data collection

This paper reports research from the second cycle of collecting video data. The design experiments were conducted by two preservice mathematics teachers for their master thesis. Altogether, 6 groups of two or three students each (at the end of Grade 8 or 9; aged 14 to 16; all from the same lower secondary school in an urban quarter in the Ruhr area) worked for 3 lessons on similarity. For all students, the design experiment was the first lesson on the mathematical concept of similarity. Following the idea of establishing ground rules, the importance of explaining and arguing and the way of working together was discussed with the students. For this paper, video data and written documents of the first task of the teaching-learning arrangement (Figure 1) were analyzed for investigating students' initial language means by making collections of the original expressions that were later on group by the used language means. The textbook analysis was conducted along the most sold textbook for comprehensive schools in North Rhine-Westphalia "mathe live" focusing on the pages introducing the topic of similarity until the summary (Böer et al., 2008, p. 20f.).

Methods of data analysis

First, all sequences were collected in which students were working independently on Task 1, until students proceeded with another task or until the teacher started to intervene. Second, these sequences were transcribed and enriched by students' written products. The third step followed inductive category formation (Mayring, 2015) and coded all utterances by generalized versions of the language means that were used (i.e. language means using variables instead of numbers) in order to gain a repertoire of students' lexical and syntactical means. For the presented analysis, only categories related to talking about changing the sides of figures are considered due to space restrictions. The textbook analysis collected all language means in tasks related to enlarging figures and in the summary.

Empirical results: language means for talking about enlarging figures

First, students' initially used language means and the target language means in textbooks are identified. Afterwards, the idea of bridging between these identified repertoires is discussed.

Students' language means for talking about enlarging figures

Task 1 proved highly suited for eliciting discursive group work, especially if students had different strategies for enlarging their parts, like in the group of Julian, Lara, and Nitika. Julian is the only student in the data of the presented research that utters the mathematically sound idea of *multiplying* each side with a constant factor that he calculates by using the rule of three (in German "Dreisatz" is a term for the rule of proportion) and that he furthermore connects with the idea of scale: "I don't know if this is correct now, but this 4 cm (...) this is like such a scale thing, right? So if 4 cm are 7 cm and 4 cm it is asked for two, the half, and the half of 7 is 3.5. Do you know what I mean?" (F1_V1_T1_G1, line 53). All other students with multiplicative ideas think of enlarging a figure as *doubling* the lengths of the sides (which is only correct in special cases). Most students have *additive* ideas for enlarging figures. Hamsa, Jussuf, and Younis serve as an example for students talking about additive ideas, especially representing students struggling with expressing their

mathematical ideas. The extract (F1_F2_V1_T1_G6; translated from German by the author) starts after the boys discover that their enlarged pieces do not fit together as a square:

- 72 Hamsa: *[holds up Jussuf's enlarged piece]* What's that? *[all laughing]*
 73 Younis: This is much too small!
 ...
 78 Younis: Eh, mine is also a bit too large after all...
 79 Hamsa: *[towards Jussuf]* Make new again!
 80 Jussuf: Why? This is 7 cm, I think.
 81 Hamsa well you have to- look, if 4 cm become 7 cm, you have to make larger by 3
 82 Jussuf: There's from 4 to 7 after all...
 83 Hamsa: You should do this one, right? *[points to the drawing in the task]* That are 6 cm here. They become 9. *[unclear]*
 84 Jussuf: *[also points to the drawing]* I see, we should use from THESE data. I see. Well then. *[starts to draw]* [...]

Like all groups, the boys first describe quite broadly that somehow the size of the figures or some sides of them are too small or large (lines 73 to 78). Afterwards, Hamsa explains Jussuf how to additively enlarge the sides of the figure using the language means “if... (then)...” (line 81), “x cm become y cm” (lines 81 and 83), and “making larger by x” (line 81). The mathematically inadequate idea of enlarging the sides by adding 3 cm to all or some sides occurs in nearly all groups of the study. An overview on students' language means for talking about enlarging figures is printed in Table 1, sorted by the related mathematical ideas.

Language means	Examples from students' talk
<i>Students' language means for talking about additive ideas</i>	
to make x cm	Did you also make everything 3 cm, Leonie?
(to calculate) x plus y / plus x	Yes, I did everything plus 3. / So then you calculated 6 plus 3?
x become y	That are 6 cm here. They become 9.
x longer / larger / higher / more / ...	Yes, I though 3 cm longer. / Everything 3 cm larger. / Um, maybe it's just always 3 cm higher [...] to be 3 cm more?
x more / longer than	You have to be 3 cm more than true to scale. / You have to write 3 cm more than normal.
x on top	There was written that there are already 4 cm that you should enlarge on 7 cm in that I just calculate 3 cm on top.
to make larger by	look, if 4 cm become 7 cm, you have to make larger by 3
to enlarge on	There was written that there are already 4 cm that you should enlarge on 7 cm in that I just calculate 3 cm on top.
difference between x and y	I think 10.5, because 9 is only calculated plus 3. And that's just the difference between 4 and 7.
<i>Students' language means for talking about the idea of doubling</i>	
to take / make the double / to double	And eh it says 4 cm. And then I just took the double. Thus 8. And then this is enlarged. And quasi doubled.
<i>Students' language means for talking about multiplicative ideas</i>	
x corresponds to y / x are y	So, 4.5 are 7.875. So, this side is 4 cm long eh 4.5. But to begin with let's put away the 0.5. So, if we then calculate 0.5 corresponds to 0.875.
x times y	Just take the 6 cm and then 1.75 times 6. Simply just calculate times.
to calculate with the rule of three	Ehm, I did calculate with the rule of three for 1 cm 1.75 times 6, because it was 6 cm, that makes 10.5
on x cm	I have the rule of three, thus on 1 cm. To 3 cm because my side, well two were 3 cm, I think at least [...]

from x to ...

| From 100 to .. And then from 1 times 3 and 1.75 times 3 are 5.25.

Table 1: Students' language means for talking about enlarging figures identified in 6 groups

Like Hamsa in line 81, students with additive ideas often initially use language means related to visual impressions of larger, more, higher, huger and related verbs. Furthermore, they often describe processes (instead of e.g. talking about characteristics or relations). Some groups also use the phrase “x cm become y cm” (like Hamsa in lines 81 and 83) that is also offered in the text of the task.

Another interesting inside refers to the line “x more/longer than” in Table 1 (not printed in the transcript but from this group): Jussuf's utterance “You have to be 3 cm more than true to scale” (F1_F2_V1_T1_G6, line 249) combines two ideas in one sentence: the inadequate idea of additively enlarging the sides and the concept of scale which usually refers to the mathematically sound multiplicative idea of scaling with a magnification factor. Here, Jussuf seems to use “true to scale” for referring to the length in the original figure and/or for expressing that the shape is conserved. The phrase “true to scale” is taken up by the group and later on even used in their written general instruction for enlarging figures (Figure 2). But, since it is not discussed regarding its mathematical meaning, the term stays vague and is still used in connection with additive ideas.

~~Man muss 3cm länger als wie das Maßstab
sein. Dann zeichnet man es auf
dem Blatt, wenn man
alles Maßstabgetreu ist schneidet
man es sorgfältig aus. Zum
Schluss legt man die Figuren
zusammen.~~

You have to be 3 cm longer than the scale. Then you draw it on the sheet. When you is all true to scale you cut it out thoroughly. At the end you put together the figures.

Figure 2: General instruction on enlarging figures from Hamsa, Jussuf, and Younis

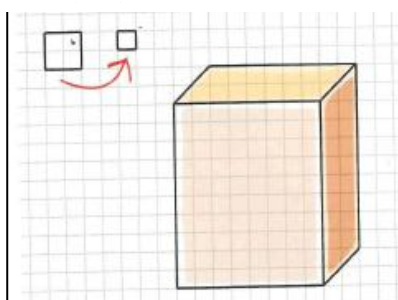
The analysis of further language means related to multiplicative ideas reveals two important points. First, talking about the rule of three makes it necessary to talk about corresponding numbers, for example by the means “x corresponds to y” or “x are y” (see Table 1). These language means are relatable to a static perspective on similarity since they do not only describe a process but also relations. For example, the static perspective also talks about ‘constant length ratios of corresponding sides’. The expression “x corresponds to y” could also work with additive ideas but is not observed in this way in the analyzed data. Instead, only the expression “x becomes y” is identified which is very close to the formulation of the task and also has another meaning than “x corresponds to y”. Particularly, “x becomes y” (as a lot of the language means related to additive ideas) describes a process and not a relation and is thus more related to a dynamic perspective on similarity. Second, calculating with the rule of three makes it necessary to talk about calculating “on 1”. Mathematically this parallels the idea of calculating the magnification factor which is introduced in most German textbooks as the constant result of dividing the lengths of corresponding sides (see next Section).

Language means identified in a textbook

The analyzed textbook starts with a page headed “Enlarging and Reducing” and introduces the new topic by referring to the dynamic perspective on similarity: several tasks ask students to draw enlarged/reduced pictures without giving any instruction. Task 5 is the only one talking about the process of reducing by “merging several boxes” (Böer et al., 2008, p. 20; see Figure 3 on the left

side). The second page has the caption “Similarity” and starts with the questions “Which data does the drawer need to draw an enlarged version of the house? How do the line segments change, how do the angles change during enlarging?” next to a small and a larger drawing of a house, directly followed by the summary printed on the right side of Figure 3. Whereas the first question refers to a dynamic perspective on similarity, the second question is unclear but might be intended as a transition to the static perspective. In the summary, the first sentence refers to a dynamic perspective without talking about the corresponding processes, the remaining text to a static perspective.

5 Drawings in squared grid can be reduced by merging several small boxes. Merge four boxes each for reducing in your



Übrigens

Der Maßstab k wird auch **Ähnlichkeitsfaktor** genannt. Er kann verschieden angegeben werden:
 $3:4 = \frac{3}{4} = 0,75 = 75\%$

Wird eine Originalfigur maßstäblich vergrößert oder verkleinert, so sind Bildfigur und Originalfigur zueinander **ähnlich**. Das Längenverhältnis zwischen den Strecken der Bild- und der Originalfigur wird durch den **Maßstab k** festgelegt.

$$k = \frac{\text{Länge der Bildstrecke}}{\text{Länge der Originalstrecke}}$$

Jeder Winkel der Bildfigur ist genauso groß wie der entsprechende Winkel der Originalfigur.

Is an original figure enlarged or reduced true to scale, the picture figure and the original figure are similar to each other. The length ratio between the line segments of the picture and original figure is determined by the scale k . [...]
 Each angle of the picture figure is just as big as the corresponding angle of the original figure.

Figure 3: Extracts from the textbook (Böer et al., 2008, p. 20f.), translated by the author

On the one hand, it was to be expected that the book does not offer a general instruction for enlarging figures on the first pages, since it follows the tradition of taking up students’ ideas. Thus, except for “merging boxes”, there are no language means for the dynamic perspective identified. On the other hand, the summary focuses on the static perspective on similarity with a strong emphasis on the concept of scale (most textbooks use the term ‘magnification factor’ which is here only mentioned as marginalia). The more mathematical perspective of this lack of transition between the perspectives will be discussed elsewhere. The analysis of language means shows, that “enlarging true to scale”, “being similar to each other”, “length ration” and “scale” (not as the concept scale but as the name for the quotient of length of the line segment in the picture and in the original) are language means that are of importance but not used in the tasks or texts before. The expression “corresponding” is used in this example only in connection to the angles. Other textbooks use this mean more prominent and especially for corresponding sides. A short review of more textbooks shows that starting with a dynamic perspective on similarity is quite common. After these first tasks of investigating how to enlarge figures, there is often a cut towards the summaries which are written from a static perspective on similarity, i.e. about “length ratios”, “magnification factors”, etc. which coincides with a change in the used language means from describing processes towards talking about relations. The comparison of students’ initially used language means and the target language identified in textbooks shows that there is a gap: On the one side students talk about processes and visual impressions, especially if they refer to additive ideas. On the other side, textbooks initiate thinking and talking about this dynamic perspective on similarity (without offering related target language) but focus often exclusively on the static perspective in the printed summaries which is

connected to talking about relations. Thus, even though using different tasks, the challenge of leading over from a more dynamic to a more static perspective on similarity can be identified in the textbook itself as well as by comparing the task from the project and the textbook.

Bridges from students' language means to the language means of textbooks

Of course, it is not possible to compare students' oral language in the process of understanding with the written language of textbooks that display the already understood version of mathematics. Nevertheless, students need to be able to relate their language and the corresponding mathematical ideas from the phase of discovering mathematics with the counterpart from the textbook that is often the basis for all other lesson phases (e.g., also as an idea in Gibbons 2002). Exploring possible bridges between students' initial language means and textbooks, three points appeared central: First, the textbook analysis revealed a cut between the use of the dynamic perspective in the first tasks and the use of the static perspective in the summaries which is paralleled in the used language means. Since the textbook does not offer language means for talking about the process of enlarging, it can be assumed that students in classrooms speak quite similar to the students in this study. This emphasizes the need of developing tasks that explicitly work on this transition of mathematical ideas along with supporting the corresponding language means in small groups or teacher guided whole-class discussion. Second, the notion of scale is used by Jussuf and Julian as well as in the presented textbook. But, this notion seems problematic since it is tied to a specific mathematical concept that needs to be understood in order to be used properly. Third and most important, the identification of language means used for talking about similarity in a dynamic perspective shows that students with additive ideas often use language means related to the visual change of sides and their language is very tied to describing processes. This also holds for the most cases in which students talk about their idea of doubling. In contrast, if students talk about the multiplicative idea, they tend not to use language means related to visual change. Especially Julian uses the language mean "x corresponds to y" which is close to "corresponding angles/sides" used in textbooks and also displays a more static perspective. Thus, the language means around "corresponding" can be identified as possible bridges between students' language and the language in textbooks since it can be used to describe parts of the process of enlarging as well as to talk about relations between features of figures. This could also hold for the expression "calculate on 1" which refers to the rule of three used in the process of enlarging but can be also interpreted as talking about the magnification factor which describes a relation.

Conclusion and outlook

The study is limited in the small size of student groups that were observed. Thus, it is planned to extend the analysis to more groups and to enrich the data with more textbooks. Nevertheless, the presented research points to important directions: On the one hand, the study shows that students mainly talk about processes and refer to visually observable change, often staying a bit diffuse and not using mathematical wording (like adding, plus, etc.). On the other hand, textbooks often talk about similarity by means of characteristics and relations. This gap between students' initial language means and the target language parallels the difference between talking about similarity from a dynamic or static perspective. This highlights the need for developing a task that explicitly

picks out this transition as a central theme and supports students by offering language related support. This especially holds for students with low language proficiency that are still acquiring language and discourse practices that allow talking about characteristics and relations of figures. The analysis points to two language means that have the potential to bridge between students initial language means and the target language: “x corresponds to y” and “calculate on 1” can be used for talking about similarity from a dynamic perspective and static perspective. Nevertheless, there is a need of further research for developing language-responsive teaching-learning arrangements in the future in which both language repertoires can be bridged systematically by macro-scaffolding trajectories as well as for teacher moves for supporting the group work and following whole class discussions.

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References

- Barwell, R. (2014). Language background in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 331-336). Dordrecht, the Netherlands: Springer.
- Böer, H., Kliemann, S., Puscher, R., Segelken, S., Schmidt, W., Trapp, M., & Vernay, R. (2008). *mathe live Erweiterungskurs 9. Mathematik für die Sekundarstufe I*. Stuttgart, Germany: Klett
- Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht, the Netherlands: Kluwer Academic Publishers.
- Erath, K. (2017). Talking about conceptual knowledge. Case study on challenges for students with low language proficiency. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 321-328). Singapore: PME.
- Erath, K., Prediger, S., Quasthoff U., & Heller, V. (2018). Discourse competence as important part of academic language proficiency in mathematics classrooms. *Educational Studies in Mathematics*, 99(2), 161-179. doi: 10.1007/s10649-018-9830-7
- Gibbons, P. (2002). *Scaffolding language, scaffolding learning. Teaching second language learners in the mainstream classroom*. Portsmouth, NH: Heinemann.
- Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 17-51). London, England: Routledge.
- Hölzl, R. (2108). Ähnlichkeit. In H.-G. Weigand et al., *Didaktik der Geometrie für die Sekundarstufe I* (pp. 203–225). Berlin, Germany: Springer Spektrum.

- Mayring, P. (2015). Qualitative content analysis. Theoretical background and procedures. In A. Bikner-Ahsbals, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (S. 365–380). Dordrecht, the Netherlands: Springer.
- Mercer, N., Wegerif, R., & Dawes, L. (1999). Children's talk and the development of reasoning in the classroom. *British Educational Research Journal*, 25(1), 95-111.
- Prediger, S. & Zindel, C. (2017). School academic language demands for understanding functional relationships. A design research project on the role of language in reading and learning. *Eurasia Journal of Mathematics Science and Technology Education*, 13(7b), 4157-4188. doi: 10.12973/eurasia.2017.00804a
- Sfard, A. (2008). *Thinking as communicating*. Cambridge, England: University Press.
- Vygotsky, L. S. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.