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Relations between school achievement and language abilities in mathematics classrooms

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My research aims at studying how school achievement is related to social background in mathematics classrooms. I am comparing two sessions held in two contrasted schools, by the same teacher on the same subject. The analyses show that the fact that some elements related to the language of mathematics are “transparent” to the teacher might explain the differentiation in students’ achievement. The discussion will show how this research, conducted in a French context, meets preoccupations of the international community of researchers, in particular about the characterization of mathematical language and the question of multilingualism.

Keywords: Teaching practices, learning inequalities, multilingualism.

Introduction

My research focuses on the social dimension of language in order to investigate how learning inequalities are produced inside mathematics classrooms. This meets some preoccupations of CERME researchers, but with a point of view built in the French didactic research context. Building on the seminal work of Bourdieu and Passeron in the sixties that drew attention to the correlation between school achievement and sociocultural background, sociologists in France assume a “relational hypothesis”, stating that “the production of learning inequalities results from a confrontation between the socio language and the socio cognitive dispositions of the students on the one hand, and the obscurity and the implicit character of school requisites on the other hand” (Bautier & Goigoux, 2004). This implicit character results from the fact that these requisites are “transparent” to teachers which renders them “invisible” to students (Rochex & Crinon, 2011). This implicitness, which is nevertheless a condition of learning (Erath & Prediger, 2015; Planas, Morgan, & Schütte, 2018) sometimes creates ‘misunderstandings’ (Rochex & Crinon, 2011) for students who don’t have the ‘keys’ to decode it.

Moreover, sociolinguists like Bernstein, or Lahire and Bautier in France established that among these requisites, the ones concerning linguistic competencies play a fundamental role in the production of learning inequalities. This concerns not only linguistic competencies, but also competencies related to uses of language (Bautier, 1995; Planas et al., 2018): in everyday life, language is mostly used to support action in a given situation, whereas in learning activities, language is used to reflect on the world, speculate on it and categorize it using concepts. Students are more or less familiar with these latter forms, according to their sociocultural background.

An open question that interests researchers in mathematics education is trying to understand how the learning and teaching process within the classroom produces these learning inequalities, when specific knowledge is at stake (here, mathematics). Investigating this question supposes to identify, among the features of the language of mathematics and mathematics education (Pimm, 2004), linguistic competencies that might be unequally mastered by students, and the way they interfere in

the teaching and learning process in classrooms. The issue here is also to be able to consider alternatives especially to provide resources to support new teaching practices, in order to contribute to reducing learning inequalities by fostering every student's learning.

In this study, I considered the way language issues are handled in mathematics classrooms and the potential effects on learning outcomes in relation to students' sociocultural background. In this paper, I compare lessons held in two contrasted schools, one of which was situated in a 'disadvantaged' area, while the other school was not. The lessons were given by the same teacher on the same mathematics topic. Our analyses of the lessons aim at pointing out how differences in what occurs might induce differentiated learning for the students in the two classes; in particular, I try to understand the role played by linguistic competencies in these differences.

After presenting the theoretical framework and my methodology, I am presenting the preliminary analysis of the knowledge at hand, and of the tasks students are working on, during the observed sessions. I shall particularly stress linguistic issues. In the third part, I will present the results of the analyses.

Theoretical framework and methodological implications

My theoretical framework is based on Activity theory adapted to mathematics teaching and learning in a school context (Robert & Rogalski, 2005; Vandebrouck, 2013). The main hypothesis is that learning results from students' activity which results (mainly) from the tasks the teacher chooses for students and the way s/he implements them in the class. Learning is defined as conceptualising. Conceptualization (as a product) of a specific piece of knowledge is characterized by three aspects, namely: the 'availability' of the knowledge to solve tasks in which it is relevant, its integration in the network of prior knowledge, and the use of associated 'signs' (in particular linguistic ones). Conceptualisation results from opportunities to use this piece of knowledge in various tasks, with various "adaptations" in the activity to solve the tasks¹. From a Vygotskian perspective, managing specific linguistic signs - in particular verbal language - is an integral part of conceptualising, and the use of signs and the availability of a piece of knowledge to solve tasks develop dialectically. Our hypothesis on the role of the teacher in this process, resulting from the combination of Vygotskian and Piagetian ideas (Rogalski, in Vandebrouck, 2013), is that the teacher has to foster the activity of students to solve mathematical tasks, but s/he should also ensure that the solving of tasks actively supports opportunities for conceptualising. Interactions in classroom are then considered as potential opportunities for students' conceptualising but also as traces of how students interpret the mathematical and classroom norms (Planas & al., 2018).

The collected data consists of videos of the sessions about the notion of angles in two different classrooms of the first year of secondary school (6th grade) in two different schools: one which is situated in a disadvantaged area and one which is not. I shall respectively call them class 1 and class 2. In the two classrooms, the teacher, 'Mathew', is the same one and he is the regular teacher of

¹ Robert, in Vandebrouck 2013, identified seven types of adaptations including the fact of mixing different pieces of knowledge or having to establish a procedure with several steps.

both classes. Mathew explained in an interview that he sees no reason to make differences between the tasks he chooses for students of the two schools because his choice is made based on the coherence between the task and the aimed knowledge.

In the following section, I first explain the learning issues related to the mathematical content at stake (including official instructions), particularly stressing linguistic issues. I then describe the tasks set by Mathew to his students, predicting the possible activities that students might develop when completing them. Following this, I analyse the implementation of the tasks in the classroom. Using video-recordings, transcripts and in-situ observations (especially of the tracks of students' activity - what they do or say), I characterize students' activities as precisely as possible. Analysis of the videos also includes identifying what the teacher does or says and how it impacts directly on students' activity, but also the transformation of students' activity into learning.

Analysis of the mathematics at stake in the classroom sessions

Preliminary study of mathematical contents

In France, official instructions recommend to introduce the notion of right angle in grades 1 to 3 when pupils learn to distinguish between different geometric shapes (triangle, square ...), but the general angle concept is introduced in grades 4-5. In grades 4-5, pupils learn how to compare angles (by superimposing one on the other) and how to reproduce a given angle using a template of it on tracing paper. They discover the different types of angles (acute, right or obtuse angles), and they use a set square to validate visual estimations.

In 6th grade, students are still supposed to deepen their understanding of angle as an attribute, but the main teaching objective is learning how to use a protractor to measure angles and to construct an angle with a given measure. Moreover, the objects of geometry are supposed to change progressively, from drawings to theoretical objects, from the end of primary school to the end of lower secondary school. At the end of primary school, conceptualising of elementary geometrical figures is then still in progress: students are expected to know, in particular, about squares and right, isosceles and equilateral triangles and they can use instruments to confirm that a given figure is one of them, checking right angles using a set square and checking the equality of sides' lengths using a ruler. However, this does not guarantee that these figures are already fully conceptualised. In particular, students may know about squares and equilateral triangles, but do not necessarily conceptualise "the" square or "the" equilateral triangle as a figural concept. In particular, considering elementary geometrical figures as concepts eliminates the ideas of position and orientation in space, thickness and particularly size: the concept of square represents any square whichever dimensions it has. Indeed, what discriminates forms is angles, which is the element which is invariant under similitudes.

Thus, a crucial linguistic issue is to be able to discriminate between "a" square, "the" square, "all" squares or "any" square². These terms may be used correctly unconsciously by experts, but carefully considering these variations appears to be fundamental when it comes to mathematics

² The same distinctions also exist in French: "un carré", "le carré", "tous les carrés", "n'importe quel carré".

learning and teaching. Following Vygotsky, it may be assumed that a reasoned use of these determiners constitutes a tool to foster the conceptualising of elementary geometrical figures. It also constitutes an indication of this conceptualisation when it is used by students. However, discriminating between these uses is complicated by the fact that the determiner “the” could either be used to refer to the concept, in its generic sense (“the square is a figure which angles are all right ones”), but also to refer to a specific object, in its deictic use (“look at *the* square we draw on the board”). Note that in French, unlike in English, the determiner used to signify singular or plural is not the same and the adjective also agrees with the noun (“the equilateral triangle” translates as “le triangle équilatéral”, whereas “the equilateral triangles” translates as “les triangles équilatéraux”).

Analysis of the tasks

During the two sessions I observed, Mathew uses two tasks he found in a paper in a specialized journal³. The paper contained the description of a whole scenario about the teaching of angles which had been designed by a group of teachers. The idea of the project was to use a polyhedral box with faces that are squares, equilateral triangles and right isosceles ones. The first task of the scenario consisted of cutting models of the box with different scales to separate the faces and then group them to make ‘families’. Students are then asked to name and describe the families.

The second session was devoted to finding the measures of each angle of the faces, knowing that a right angle measures 90° (this value being announced by the teacher). I will particularly focus on the goal of establishing that the angles of the equilateral triangle(s) measure 60° . The expected activity of students is to put together three equilateral triangles (in adjacent positions) and observe that they obtain a “flat angle” (they are not supposed to know this expression) and that it corresponds to two right angles put together. They are then supposed to compute the double of 90 and divide the result by 3. This constitutes a complex reasoning in 6th grade, because there are a lot of subtasks and adaptations: for example, a lot of different knowledge is supposed to be used (about geometry, measure and numbers). Moreover, it relies (more or less implicitly) on the use of the property of additivity of adjacent angles’ measures. It also supposes to use the fact that angles of any equilateral triangle are equal and/or that all the angles of all equilateral triangles are equal. These properties are trivial for mathematicians (one can even consider that they define the equilateral triangle as a figure), but they are not necessarily known by 6th graders. At this age, a triangle still refers to a specific drawing and not necessarily to a figure as a concept; moreover, the notion of angle is not yet conceptualized either. In particular, students may not master the fact that angles are invariant under similitudes, i.e. that the measure of an angle does not depend on the length of its sides. Finally, equilateral triangles are generally characterized for students by the equality of their sides. Mathematically speaking, these properties concern THE equilateral triangle as a concept. The options offered to students then are either to first observe the fact that all three angles of a given equilateral triangle are equal, then apply the procedure and then generalize to all the equilateral triangles of any size, or to observe the fact that all the angles of all the equilateral

³ A journal that includes papers that present propositions for teaching (or for teacher training), coming either from researchers in mathematics education or from teachers.

triangles that they have are equal, whatever their size might be, and then apply the procedure. Both options seem to be difficult for 6th graders. Completing the task and understanding what knowledge is at stake also implies considering the pieces of paper as representing geometrical figures and that some of them are the same figure, whichever size they are or even if they are not exactly superimposable because of the imprecision of cutting. Given all these considerations, these tasks might contribute to the conceptualizing of elementary geometrical figures as concepts, but it is not an explicit objective in the initial scenario, which is centered on angles.

The two classroom sessions

At the end of the first session, families have been characterized in both classrooms by properties stated by the students: right angles and equal sides for the squares, one right angle and two equal sides for the right isosceles triangles; finally, about the equilateral triangles, students in both classes mentioned the equality of the sides, but only in class 2 did they mention the equality of angles.

Differences in students' activities

Analyses show that in class 2, almost all students suggest solutions after 10 minutes. In class 1, Mathew observes that after 6 minutes students seem lost and he suggests putting some equilateral triangles together and observing if anything special happens. Only 5 students suggest answers (which remain partial) when the teacher stops individual work after 12 minutes. These differences suggest that students in the different classes were not equally able to complete the task: whereas students from class 2 do not seem to experience major difficulties completing it, the task seems very difficult for students from class 1. Analyzing the concluding phase in class 1 provides some clues to understand these differences. After showing that putting together three equilateral triangles gives a straight line (which is suggested by the most advanced students) Mathew tries to have students understand how this helps finding the measures of the angles. The addition of two right angles and the resulting 180° is mentioned, as well as the idea of dividing by 3 and the resulting value of 60° . After this, Mathew suggests writing down the whole solution on the board, but what appears is that some students' activity remains very far from what is expected, making me doubt their ability to seize the task as a real opportunity for learning. The following excerpt illustrates where the misunderstanding lies:

- T: If I put the angles together, the angles of, of, which angles?
- S1: of 60° .
- T: each one measures 60° , but they are angles from which piece(s)⁴?
- S2: equilateral triangle.
- T: [writing on the board] if we put together three angles.
- S1: the three angles of each triangle, one angle from each triangle.
- S3: from each triangle.

⁴ French language doesn't allow discriminating between singular and plural in the oral discourse.

- T: from which triangle(s)? From any triangle?
- S1: the three over there [pointing at the board, where pieces are magnetically hung]
- T: yes, but from which- Are they all three the same or not the same?
- S1: they are identical.
- S2: equilateral
- T: [writing on the board] yes. If we put together three angles from equilateral triangles [...]

The question the teacher asks about “which piece(s)” and later “which triangle(s)” refers to the type of triangle and waits for “equilateral triangle(s)” as an answer but S1 refers to the material pieces: the discourse is not about the same objects. One might reasonably doubt that the students previously identified that all equilateral triangles have all their three angles that are equal (whereas, in class 2, students knew it from the first task). Note that, at the end, Mathew resolves the (theoretical) problem by mentioning the fact that all these triangles are “the same”, but it is probably not sufficient to make sure that all the students understand the reasoning.

The question that remains and is of great importance (in order to understand how why school achievement in mathematics is differentiated according to the sociocultural background of students) is to understand if these differences in students’ achievement could result from teacher’s choices, either because he does things differently or because some things he does similarly don’t have the same effect on all the students.

Differences in the teacher’s discourse in the two classes

This excerpt suggests that the teacher has not understood what enables students, in class 1 to understand the reasoning. Moreover, the role of the property that all angles of all equilateral triangles are equal seems “transparent” to him: not only is it not explicitly stated when exposing the proof, but Mathew has students check that all the angles of all the equilateral triangles (all the pieces they have in their hands) are equal *only afterwards*.

I also compared the discourse of the teacher during the whole sessions. What appears as common between the two sessions is that Mathew alternates between saying “the equilateral triangles” and “the equilateral triangle”, attesting that these two expressions are equivalent for him. A difference that appears and seems important is that in class 2, he will earlier and more systematically use geometrical language: when presenting the task, he talks about “finding the measure of the angles of each figure”, whereas in class 1, the question is firstly formulated as “if I take this piece, the question is, this point, can I know how much it measures?” which is rapidly rephrased as “the aim is to say, for each piece, each angle, what is its measure?”. Vocabulary is more mathematical in the last formulation (for example he uses “angle”), and I conjecture that talking about the pieces instead of the figures does not help students to focus on the relevant objects. Moreover, it might explain why, at the end of the session (see excerpt above), some students still have not identified that what is at stake is the measure of the angles of the figure represented by the pieces of paper of the same family, and not the measure of the corner of each piece of paper.

Discussion and conclusion

Differences between what happens in the two classrooms might explain some of the variability of students' activity and hence, learning. These differences seem to be related to elements introduced by either the teacher or the students, but without the teacher being aware of the possible impact of these differences (i.e., transparent elements). For example, mentioning "the" equilateral triangle might have implicitly helped some students to identify what was at stake but was not sufficient for others, especially in class 1. Moreover, the fact that the teacher mentions the pieces of paper in class 1 might have helped the students to start working, but jeopardized their identification of the learning issue.

Our study illustrates how the combination of socio linguistic and sociocultural characteristics of students on one hand and the transparency of some elements for the teacher on the other hand, appears to result in differentiating students' opportunities for learning in the two classes. The question that remains concerns the causes of the transparency for the teacher of elements that seem crucial to explain differentiation in students' achievement. I hypothesize that the issue of conceptualizing elementary geometrical figures is hidden behind language elements: the fact that "the equilateral triangles" becomes "the equilateral triangle" is very subtle. This is probably aggravated by the fact that elementary geometrical figures (and associated verbal language) are elements that are "naturalized" for teachers, like most of elementary mathematical knowledge, especially concerning mathematical logic (see Durand-Guerrier, 2013, or Chesnais, 2018).

Even if this research appears as closely related to the French context (by the fact that observations were conducted in French classrooms and concern French language but also that the research was conducted using French theories), it meets some crucial CERME's preoccupations for three reasons. First, following Pimm (1987), some features of mathematical language are common to diverse languages. Thus, considering the issue of the use of determiners in relation to quantification and conceptualization could constitute a relevant common object of research. Second, the transparency of issues related to mathematical language in classrooms is shared by teachers in all countries, and this hypothesis would be worth investigating. Third, the impact of the sociocultural background of students in school achievement could meet some preoccupations related to multilingualism. When multilingualism is related to poorer school achievement, it is often related to social backgrounds (in France and countries like South Africa as Phakeng, 2018, shows). Mixing the two preoccupations could help, in research about multilingualism, not to focus essentially on the mastery of language in terms of lexical and syntactical elements, but also on uses of language, which might be of greater importance to explain difficulties in school achievement (Bautier, 1995). This could explain why multilingualism seems to be an obstacle for students with lower sociocultural background whereas it appears as an advantage for students with upper sociocultural background. Even if the French educational system deliberately ignores multilingualism (except for new arrival students), French schools situated in disadvantaged areas often have a large number of students for whom French is not the first language. Considering the impact of multilingualism in mathematics achievement may allow enrich the approach of these students' difficulties and resources (Planas et al., 2018).

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