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Diagnosis tools of dyscalculia – contribution of didactics of mathematics to numerical cognition

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Research about dyscalculia must still be developed. The dominant perspective focuses on the individual's cognitive characteristics. We seek to know the place of mathematics education in this research and how to reconcile approaches to reach a better understanding of the disorder. We present here a methodology of analysis for tests which are designed to evaluate basic mathematics skills with various theoretical frameworks (cognitive sciences, psychology or didactics). We highlight some biases in tests from numerical cognition thanks to didactic frameworks and open perspectives to build a mathematical difficulties detection tool to facilitate the exchanges between teachers and speech therapists by providing a common inventory of the child's difficulties.

Keywords: Dyscalculia, mathematical learning disabilities, tests, numerical cognition.

Context

These last decades are clearly marked by an increase of the research and a better understanding of learning disabilities. While some disorders are now well identified and managed (dyslexia for example), others remain more complex and less studied (Lewis & Fisher, 2016). That is the case for the *mathematical learning disabilities* (MLD) which would affect 5 to 8% of students (Geary, 2011). There is currently no consensus about the definition of this trouble (Lewis & Fisher, 2016). MLD are often reduced to difficulties in processing numerical quantities and arithmetic calculation (thus the use of the term of dyscalculia). But an increasing number of studies indicate that MLD are heterogeneous (Fias, Menon, & Szűcs, 2013) and affect several aspects of mathematical skills (Kaufmann & al., 2013). These definition problems make the diagnosis and its methodological validity debatable (Lewis & Fisher, 2016). From the educational point of view, politics are concerned by the difficulties in the learning of mathematics and the processes of “inclusion” (e.g. “Loi pour la Refondation de l'École”, since 2013 in France). We claim that specific studies should be structured and developed in mathematics education regarding MLD in order to improve the identification and the remediation of MLD in an educational context. In particular, that implies a better knowledge of the existing research dealing with MLD. In fact, MLD are studied by different disciplinary fields and each of them develops its own models and hypothesis (Giroux, 2011). We can highlight two distinct approaches: the cognitive sciences approach is centered on the cognitive functioning and individual characteristics (Wilson & Dehaene, 2007), while the mathematics education approach focuses on knowledge specificities as well as the didactic characteristics of learning situations. The difficulties are not only due to an individual's dysfunction: it is also necessary to look for what prevents or stimulates the learning in the interaction between the student, the knowledge and the didactic situations. This compartmentalization of approaches is not without consequences. Indeed, many professionals who surround children with and are not leaning on the same approaches of the disorder: cognitive sciences for the paramedical professionals and

mathematics education for the teachers. The dialogue between these actors is therefore sometimes difficult, especially as medical confidentiality prevents the dissemination of some information. This is particularly problematic in the case of the exchanges between teachers and speech therapists.

Objectives

Research about MLD are mainly focused on the cognitive approach so we question the place of mathematics education and the way to reconcile the different points of view to a better understanding of MLD. We consider more precisely the reconciliation of approaches through the creation of a mathematical difficulties detection tool that can be used both by teachers and speech therapists. This device should facilitate the exchanges between these two types of professionals by proposing a common inventory of child's difficulties that can be used by each of them (for a diagnosis and for pedagogical adaptations). To develop this tool, we conducted an analysis of existing tests designed to evaluate mathematical basic skills at the end of kindergarten or at the entry to elementary school. To ensure the diversity of the theoretical foundations, we selected diagnostic tests used by medical professionals (for the most part from research in numerical cognition), but also tests used to evaluate the child in school (from mathematics education area). The final selection contains Francophone diagnostic tests (*MathEval*, *Examath*, *ECPN* and *UDN-II*) and school tests (grade 1) (a numerical skills test of basic school program, *ERMEL*, and the device "*Quatre étapes*"). We also selected an Anglophone diagnostic test (*Woodcock-Johnson*) and diagnostic tests translated into several languages used internationally (*Zareki-R* and *Tedi-Math*). Figure 1 points out the main backgrounds of these tests and their distribution into the research areas.

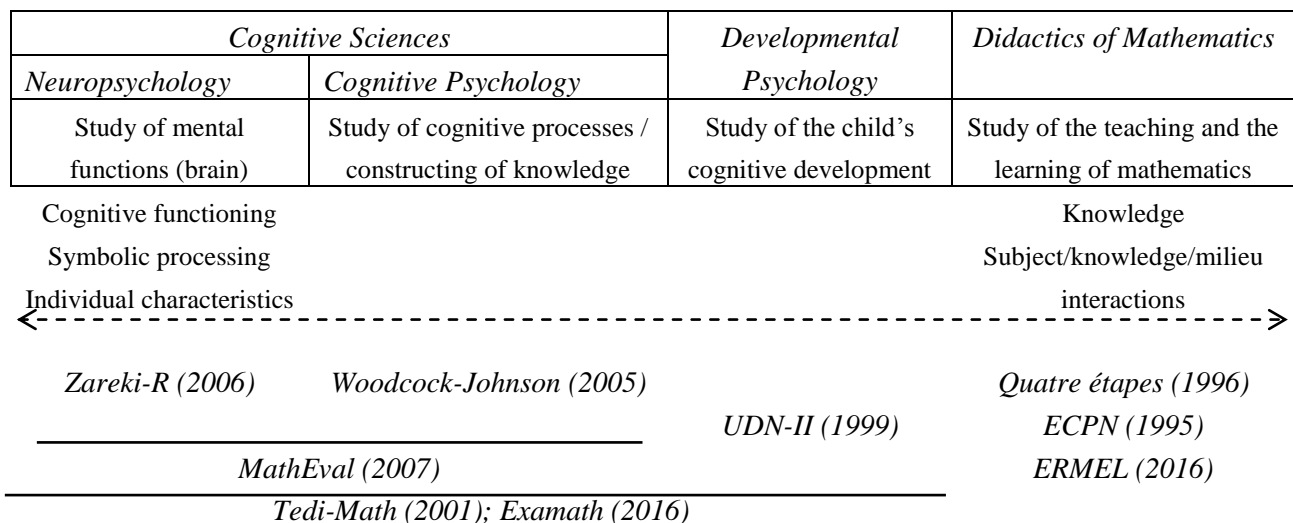


Figure 1: Analyzed tests in numerical cognition and didactics of mathematics (following the organization of research areas studying mathematical learning difficulties of Giroux (2011, p. 152))

Methodology

In order to define our analysis criteria, we made a state of the art about number construction and first number learning in didactics of mathematics and in numerical cognition. Thus, we highlight different number tasks and the variables that can affect the complexity of these tasks or the resolution strategies. These variables constitute our analysis criteria. In mathematics education, Brousseau's Theory of Didactical Situations (1997) allowed us to identify situations giving

meaning to number as well as their didactic variables. Moreover, thanks to Vergnaud's Theory of Conceptual Fields (1996), we could identify criteria for analyzing additive problems. In numerical cognition, we identified characteristics of numerical representations according to the different number processing models (especially the Dehaene's triple code model, 1992, the McCloskey's model, 1985, and the Von Aster's developmental model, 2007). We have also drawn analysis criteria for cognitive functioning of mathematical activities, based on the calculation model of Shrager and Siegler's strategy choice model (1998) or the transcoding's model of Barrouillet, Camos, Perruchet and Seron (2004). Finally, we studied the impact of some underlying cognitive functions like working memory, attentional functions, visuo-spatial abilities or digital gnosis in mathematical activities. Indeed, it has been shown that problems with these cognitive functions are generally associated with low mathematical performance (see Finnane (2006) for example). To articulate our criteria within a functional analysis table, we identified four categories of task based on our state of art (tasks where the number is used to express a quantity or a position, operation resolution tasks, digital code and transcoding tasks and tasks used to evaluate mental representation of numbers). Then we listed the tasks related to these four categories in the tests and used the criteria identified in state of art to analyze them. For this article, we choose to describe the analysis tables and to illustrate them with some examples. The whole analysis is available in Peteers (2018).

Tasks where the number is used to express a quantity or a position

Numbers allow us to express quantities (cardinal aspect) and positions (ordinal aspect). We therefore searched in each test if these two functions were present. For the use of numbers to express quantities, we analyzed in detail the four quantification procedures identified in our states of art (didactics of mathematics and numerical cognition): term-to-term correspondence, counting, estimation and subitizing. Figure 2 summarizes the criteria used to analyze these types of tasks.

For example, the Tedi-Math (Van Nieuwenhoven, Grégoire, & Noël, 2001) does not contain tasks in which numbers are used in their ordinal aspect. For the quantification procedures, only counting is evaluated through different items. Some evaluate the procedure itself (Gelman and Gallistel's principles) and others evaluate the spontaneous use of counting (construction of an equipotent collection to a given one). If we analyze more precisely these tasks, we can notice that the collections used in counting tasks are always composed of non-manipulable objects, which makes the "enumeration"¹ more difficult (Briand quoted by Ouvrier-Buffer, 2013). For the construction of an equipotent collection, the model collection remains accessible, which does not stimulate the use of counting procedures because the term-to-term correspondence remains possible. However, the task will be considered successful only if the child uses counting.

¹ Briand (1999) has underscored several preliminary steps in the learning of counting. In the series of actions required for counting, the pupils have to look over all the elements of a finite collection one time and only one. This task characterizes a non-taught knowledge, called by Briand "enumeration". Enumeration is clearly linked to the one-to-one principle (Gelman & Gallistel, 1978). The number word is a tag (and such a tag is a convention) but a tag is not necessarily a number word. Learning "enumeration" is not an explicit part of teaching. The research of Briand (1999) proves the necessity of activities involving "enumeration" in the pre-numerical domain.

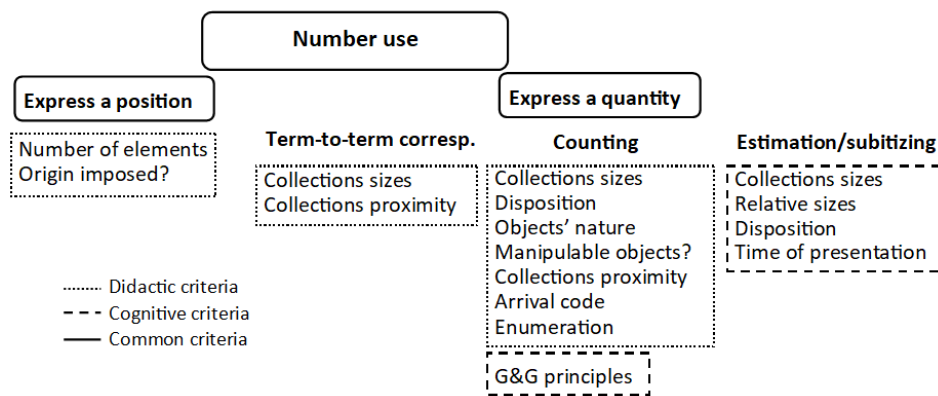


Figure 2: Analysis criteria for tasks regarding number use (quantity or position)

Tasks used to evaluate mental representation of number

There are different theories in numerical cognition about the nature of representations of numbers magnitude (Noël, 2005). In each test, we therefore looked for the two types of representation (decimal or analogical) evaluated and how these representations are evaluated. We identified three types of tasks that can be used to evaluate analogical representations: analogical comparison tasks, comparison tasks involving oral or arabic code and tasks consisting in positioning numbers on a number line. For the evaluation of decimal representations and the comprehension of our number system, we considered two types of tasks: tasks related to writing numbers and tasks involving material collections. Figure 3 shows the criteria used for the analysis of these types of tasks.

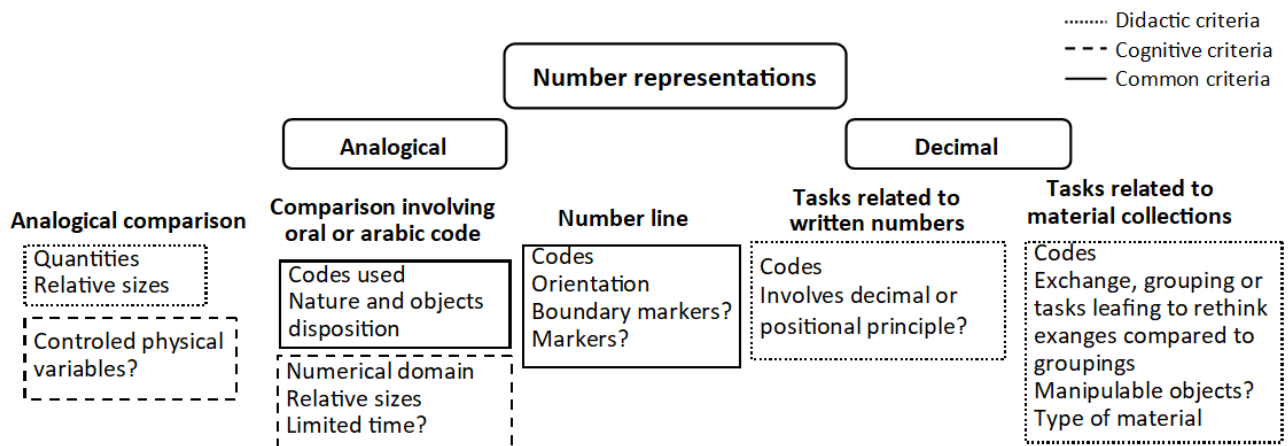


Figure 3: Analysis criteria for number representations tasks

Digital code and transcoding tasks

We identified in the tests the tasks used to determine the mastery of oral code (extension of the stable and conventional part of the verbal number sequence and its organization). We also made an inventory of other tasks designed to evaluate the mastery of oral code and we did the same for the written code. For the transition from one code to another, we chose the following criteria (Figure 4).

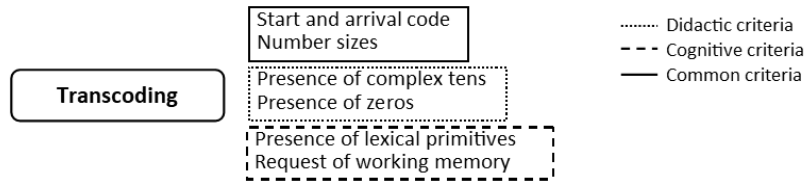


Figure 4: Analysis criteria for transcoding tasks

For example, here are two transcoding tasks (Table 1, from oral to arabic code), one from the Zareki-R (Dellatolas & Von Aster, 2006) and the other from the UDN-II (Meljac & Lemmel, 1999).

| Dictated numbers in Zareki-R | Dictated numbers in UDN-II |
|---|--|
| 14 – 38 – 1200 – 503 – 169 – 4658 – 756 – 689 | 3 – 5 – 9 – 15 – 18 – 27 – 54 – 76 – 89 – 93 – 138 – 272 – 605 – 1061 – 4396 – 7042 – 10001 |

Table 1: Transcoding tasks from the Zareki-R and UDN-II

We can see that, in both tests, the writing of numbers containing zeros is well evaluated. However, unlike the task proposed in UDN-II, we find in Zareki-R only few complex tens and lexical primitives. The numbers proposed in this test are relatively large and their transcoding strongly solicits working memory. This type of task does not allow us to precisely identify the source of errors. The task proposed in UDN-II makes a more precise characterization of the difficulties.

Operation resolution tasks

Finally, we identified among the tasks proposed in the tests, three categories of tasks requiring the resolution of an operation: analogical operations involving a material support, symbolic operations involving written or oral code and operations with verbal wording consisting in resolving a problem involving an operation. The criteria used to analyze these types of tasks are summarized in Figure 5.

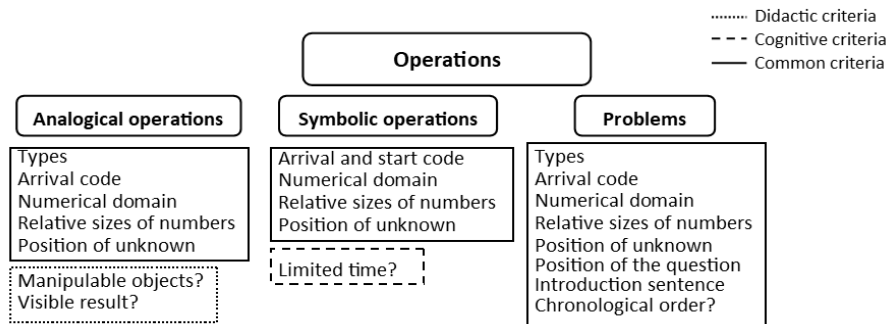


Figure 5: Analysis criteria for operation resolution tasks

Results

Based on our analysis table, we wrote a descriptive analysis of each test. We present here some general results (see Peteers (2018) for more details). We can note that all the tests are different in terms of the knowledge and skills evaluated. The differences are particularly marked for the numerical cognition tests, which is not surprising since these tests are based on different models. Then, we can notice that some types of tasks are specific to theoretical frameworks. In other words, there are tasks specific to the numerical cognition tests (estimation or subitizing tasks, analogical comparison tasks, ...) and others specific to the didactic tests (tasks in which the number is used to

express a position, tasks in which the child must extract a given number of elements from a collection, ...). We can also find some types of tasks in every test regardless of their theoretical foundation (numerical cognition or mathematics education). This is the case for the counting tasks, the evaluation of the numerical verbal sequence, the transcoding tasks, the analogical operation resolution tasks, ... However, the variables of these tasks change depending on the tests. In counting tasks for example, the number of objects in the collection and their disposition vary in each test. There are also differences depending on the theoretical foundations. In our example of counting, the tasks proposed in the cognition tests focused more on the counting procedure (with statements like “how many elements are there?”). In the didactic tests, the focus is on the types of situations in which the child can use a resolution procedure based on counting. The manipulability of the objects also differs according to the theoretical foundations of the tests considered. The cognition tests propose only counting tasks with non-manipulable objects unlike the didactic tests. From a didactic point of view, two elements can be underlined. We note that some knowledge and skills highlighted in our state of art in mathematics education are not evaluated in any of the tests (in both cognitive or didactic tests). This is the case of “enumeration” for example (Briand quoted by Ouvrier-Buffet, 2013). However, the choice of variables used in the counting tasks in the cognition tests does not always facilitate enumeration. Indeed, the objects used in this type of task are not manipulable (like our example with Tedi-Math), so, the separation between counted and not counted objects must be done mentally. Enumeration difficulties can therefore impact the resolution of counting tasks and are not identified because any task allows the evaluation of this specific skill. We can also notice that the choices of variables used in the tests are not always appropriate. Indeed, if we take again the example of the equipotent collection construction task in the Tedi-Math, the model collection is always accessible during the construction of the equipotent collection which does not stimulate the use of counting because the tasks can be resolved without using the number (using term-to-term correspondence for example).

Conclusion and perspectives – towards a detection tool

We have shown in this article a part of the results of our PhD thesis. We can point out that the mobilized didactic frameworks (Theory of Didactical Situations and Theory of Conceptual Fields) allow us to structure our analysis table, and also bring us another perspective about numerical cognition tests which are not used for school environment. They provide us a framework for critical analysis to highlight some biases of these tests in relation to what is taught and to mathematics education knowledge (non-evaluated elements such as enumeration or questioning of the choices of variables in some tasks). These elements confirm the interest of didactics of mathematics in research about MLD, in particular regarding the diagnosis. We have identified a set of tasks and variables that can be used by the different professionals involved (especially teachers and speech therapists). We now have bases (theoretical tools from didactics of mathematics and numerical cognition frameworks and analysis of the most frequently used tests) to build a mathematical difficulties detection tool (presented in Peteers, 2018). This device uses digital technologies environment but also concrete manipulations for tasks like equipotent collection construction in order to avoid visuo-spatial difficulties for instance. This tool is made of four units (Prerequisites; Cardinal and ordinal aspects; Representations of number; Operations). Each unit is composed by

tasks coming both from numerical cognition and didactics of mathematics. For each task, variables are chosen depending on the age of the pupils (three levels of difficulty are defined) and the well-known learners' difficulties identified in didactics of mathematics. The "Prerequisites" unit deals with: analogical comparison, subitizing and estimation, enumeration, term-to-term correspondence, verbal number sequence. The "Cardinal and ordinal aspects" unit explores the spontaneous use of counting, the Gelman & Gallistel's principles, the counting up process, and the identification of the position of a number (ordinal aspect). The "Representations of number" unit takes into account transcoding tasks, number line and numbers' comparison tasks, and situations requiring decimal positional principle. Finally, the "Operations" unit proposes tasks involving analogical operations, symbolic operations and problems. For each task, we define a coloring code (green, orange, red) to evaluate the success or the failure. A synthesis is then automatically generated for the teacher: it helps her/him build remedial interventions. After such interventions, the teacher can reuse some tasks of the test in order to evaluate the impact of them. In case there are no benefits, the whole synthesis of the test can help teacher to orientate the child to a paramedical professional and inform the speech therapist. We have conducted a first experiment to test our detection tool. Considering the results of this experiment, our tool is currently the subject of a new development (exclusion of some tasks and inclusion of new ones; improvement of the coding process and the synthesis etc.). We still have to normalize it. We will give more details about this device during the conference.

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