



HAL
open science

Teaching mathematics to students with intellectual disability: What support do teachers need?

Chun-Ip Fung, Dichen Wang

► To cite this version:

Chun-Ip Fung, Dichen Wang. Teaching mathematics to students with intellectual disability: What support do teachers need?. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. <hal-02431480>

HAL Id: hal-02431480

<https://hal.science/hal-02431480v1>

Submitted on 7 Jan 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Teaching mathematics to students with intellectual disability: What support do teachers need?

Chun-ip Fung¹ and Dichen Wang²

¹The Education University of Hong Kong, Hong Kong; cifung@eduhk.hk

²The Education University of Hong Kong, Hong Kong; dwang@eduhk.hk

Abstract: Mathematics teachers in Hong Kong find they have no confidence in designing effective instructional plans for students with intellectual disability (ID). Based on our four-year experience of dealing with mathematics teaching problems faced by teachers of students with ID, we claim that an understanding of mathematics knowledge structure is the pre-condition of generating hypothetical learning trajectories. To illustrate this point, a study on developing a progressive learning framework for teaching mental objects to students with moderate or severe ID is presented here.

Keywords: Design science, intellectual disability, mathematics teaching, teacher professional development.

Introduction

Over the last four years, we have been engaged in a professional development programme for supporting mathematics teachers of students with intellectual disability (ID). ID is a disability characterized by significant limitation in intellectual functioning and daily adaptive behaviors (AAIDD, 2010). Students with ID face serious obstacles in abstract thinking, reasoning, comprehension, and communication (Taylor, Richard, & Brady, 2005), which greatly influence their learning performance in mathematics. The cognitive limitation suggests that the mathematics learning trajectory for students with ID cannot possibly be the same as those for students with normal intelligence.

In the local community where we are working, the government provides teachers with a total of 250 hours of training for support students with special educational needs (SEN), including students with ID (Education Bureau, 2018). The majority of time is spent on developing teachers' skills and strategies in supporting students learning and instilling teachers' knowledge in handling students' cognition, behavioral, emotional, sensory, communication, and physical needs. Subject content (such as mathematics or science) is rarely covered. It seems that policymakers regard the enhancement of teachers' knowledge for teaching mathematics to students with SEN as just adding generic pedagogical knowledge to whatever mathematics specific preparation teachers have already acquired in previous training.

Similarly, in the research community, the importance of mathematical knowledge for teaching SEN students has not received much attention. Allsopp and Haley (2015) conducted a synthesis of research and found that, during 2004 to 2014, only 16 studies included the criteria of teacher education, mathematics, and students with learning disabilities. On the topic of intervention studies for ID students' mathematics education, only 16 papers were found to have been published between 1989 to 1998 (Butler, Miller, Lee, & Pierce, 2001), and 7 papers were found to have been published between 1999 to 2010 (Hord & Bouck, 2012). After a glance of the very limited literature on mathematics

education for students with SEN, it is easy to discover that studies on intervention based on a mathematics-specific perspective are rare.

The pedagogical content knowledge (cf. Shulman, 1986) for teaching mathematics to SEN students is an under-addressed area. Is mathematics for teaching SEN students the same, in both content and form, as mathematics for teaching students without SEN? Or how different are the two? To what extent do generic knowledge and skills about SEN students' learning enable teachers to carry out mathematics teaching that is effective to SEN students? We attempt to discuss these questions based on our experience with teacher development in recent years.

The paper describes data from a study in which we collaborate with school teachers to develop an instructional framework for students with moderate (IQ from 35 to 50) or severe ID (IQ below 35). We use this study to illustrate teachers of students with moderate and severe ID need support for designing appropriate mathematical experiences for their students, a point that is overlooked by most teacher training programmes of special education.

Fourteen teachers from five special schools in Hong Kong participated in this study, six of whom teach students with moderate ID, and eight teach students with severe ID. Qualitative data are collected, approved by participating schools, teachers, and students' parents, including written notes of the meetings, video recording of teaching experiments, and audio recording of interviews with three teachers who took part in the teaching experiment.

Generating a hypothetical learning trajectory

In each of the participating schools, teachers have been trying hard to find ways to expand their students' possibilities in mathematics learning. When teachers were asked about their difficulties in teaching, the following comment, made by Teacher 1, was common among the teachers:

When I prepare to teach a piece of mathematics content, I think that I learned that content by this way, but for children with ID, how do they learn?

It shows that teachers understand that the learning trajectories of ID students are different from what they experienced themselves in schools during their years as students, but they have no experience of what it is like to learn mathematics with the limitation that their students have. Given all this, teachers usually are left alone to make up the details of their teaching – extrapolating from whatever mathematics textbooks or other learning resources might offer. However, most of the available materials are designed for students without ID, and teachers would easily find themselves being left in the unknown area of teaching.

In what follows, we outline general steps by which we help teachers to arrive at a hypothetical learning trajectory for students with ID. The procedure resembles the French approach of didactical engineering (Artigue, 1994), in which a serious analysis of the teaching contents and constraints at epistemological, cognitive, and didactical levels constitutes an essential component.

Step 1

We listen to the teachers talk about the teaching problem they are confronted with and have an observation about the target student groups, during which, we try to answer the following questions as far as we can:

- What is the teaching problem?
- What is the goal that the target students need to achieve?
- Is there any constraint that need to be considered?

The answers to these questions provide us with a general picture of our mission.

Step 2

Before we start designing a hypothetical learning trajectory of a topic, we carry out an epistemological analysis of the content structure, which aims to portray a longitudinal picture of knowledge acquisition. During the process, we try to reconstruct a path of knowledge development that on the one hand links up well with prior learning experiences, and on the other hand, contributes to effective subsequent learning. The structures identified in this stage underpin a skeleton of the hypothetical learning trajectory, which ensures the resulting trajectory will be mathematically sound.

Step 3

The skeleton of the hypothetical learning trajectory provides teachers with a direction for generating an instructional plan. In the third step, we need to work with teachers to insert various intermediate stages, with attention paid to characteristics of students, classroom constraints, curriculum constraints etc. With teachers' input, we begin with a thought experiment (cf. Freudenthal, 1991), imagining how the teaching-learning processes will proceed and spotting out the gaps in the proposed learning trajectory that will impede students' knowledge construction.

Step 4

Lastly, teachers, with some help from the researchers perhaps, fill in the fine details to arrive at a detailed instructional action plan for their own students. Afterwards, it will be field-tested and open to amendment based on implementation evidence.

An illustrative example: developing a framework to teach mental objects to students with moderate or severe ID

The learning of mathematics involves creating mental objects (Freudenthal, 1991). Number, length, angle, circle, square, weight, straight line, ... are mental objects commonly encountered in mathematics lessons. They, being intuitively clear among children with normal IQ, are often found to be very difficult for students with moderate or severe ID. Unfortunately, these difficulties are usually unknown to their teachers who were once children with normal IQ in their younger days.

When teachers try to communicate the meaning of mental objects, how can they convey those messages to students with ID? And, how can they check if students indeed acquire what their teachers intend to communicate? The situation becomes worse for students with specific language impairment. When verbal communication fails, is there any possibility of conveying abstract mental

objects to students? In short, how can we draw students' attention to mental objects without relying on language? This is the situation we were confronted with in Step 1 after discussions with teachers. The main teaching problem is how to convey the meaning of a mathematical concept (which is, in essence, a mental object) to students with moderate or severe ID? As these students often do not have satisfactory verbal communication skills, teaching is constrained to using a minimal amount of verbal communication. The goal to achieve is that students can identify the mental object from among others in a multiple-choice question.

In Step 2, we proceeded to analyze the knowledge structure concerned. Indeed, a mathematical term sometimes refers to a relation, as in the case of 'being less', an operation, as in the case of 'subtract', or a class of objects, as in the case of 'straight line'. For example, in Figure 5, we will say "This is a straight line" when we point to each of the three different straight lines. In essence, the term 'straight line' refers to the class of figures in which no two of them are indeed identical. Teaching involves conveying a class of objects that shares certain properties without ever going explicitly into those properties in detail. When language is not the vehicle of communication, we are left with the scenario that students can only 'sense' the meaning through examining a large number of examples and non-examples. This understanding forms the basis of the Pick the Odd One (POO) framework.

POO is a framework for designing a series of multiple-choice activities under which a student could progressively develop a sense of a mental object without relying too much on verbal communication. Although the teacher is free to employ any verbal explanation during its execution, the main thrust that drives students to the target mental object comes from the teacher's confirmation of the correctness of a student's choice, which may well be done using body language or facial expression.

In Step 3, we had to portray various intermediate stages of the POO. These stages should encompass sufficient variation across examples and non-examples of the mental object to be conveyed. To begin with, there should be a key concept, or a focus mental object, that will appear in the series of multiple-choice activities. Going through the series of multiple-choice activities, the student will systematically visualize or experience these variations. Each multiple-choice item includes a total of four or five choices, with at least one choice corresponding to the focus mental object, and at least one choice corresponding to the other. The student should indicate (by pointing or other means) the odd one out. Through systematically varying these choices, the student's attention is drawn to the focus mental object. Timely feedback from the teacher serves to shape how the mental object is developed, and hence is an indispensable component of the activities.

There are four stages of the POO. In Stage 1, each item consists of exactly one choice corresponding to the focus mental object (straight line in this case), while the remaining are identical choices in identical orientation (Figure 1). In Stage 2, each item consists of exactly one choice not corresponding to the focus mental object, while the remaining are identical choices corresponding to the focus mental object, in identical orientation (Figure 2). Variations of the forms of the mental object and the otherwise across items should strive for comprehensive coverage as far as possible (see Figure 3 and Figure 4). In Stage 3, each item consists of exactly one choice not corresponding to the focus mental object, while the remaining choices correspond to the focus mental object in different forms (Figure 5). Up to this point, the student should have seen or experienced a large number of

examples and non-examples of the focus mental object. The teacher could now reveal relevant terminology in written or oral form. In Stage 4, no specific restriction is imposed on the choices and the student is required to pick out the unique choice corresponding to the focus mental object by its name (Figure 6). In other words, the student tackles multiple-choice questions commonly found in regular mathematics class only at Stage 4. All the previous activities are designed to fill the gap before the student is capable of handling this.



Figure 1: POO stage 1



Figure 2: POO stage 2

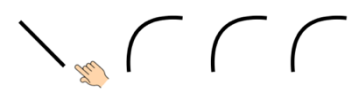


Figure 3: Variations of stage 1

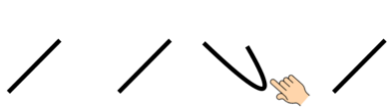


Figure 4 Variations of stage 2

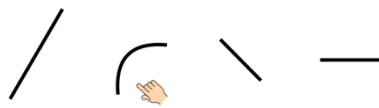


Figure 5: POO stage 3

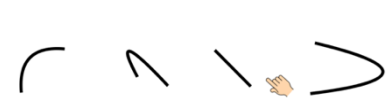


Figure 6: POO stage 4

In principle, this framework can be applied to teach a variety of mathematical mental objects or informal notions such as straight line, circle, emptiness, being long, inner part, being heavy, many, etc.

In Step 4, teachers proceed to design the details of the series of multiple-choice activities. Pilot tests carried out by teachers indicated that there should be a preparatory stage (see Figure 7 and 8) to confirm that the student is capable of matching object or picture at hand with a group of identical objects or pictures not necessarily in the same orientation.

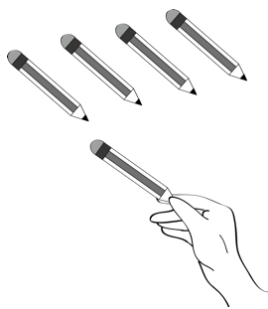


Figure 7: Exactly identical objects

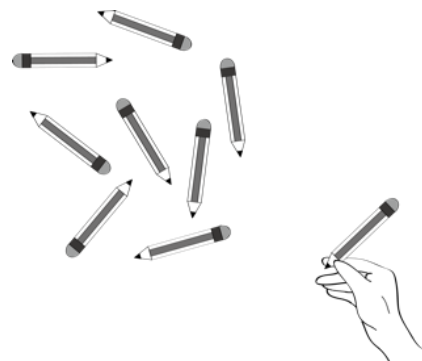


Figure 8: Identical objects in different orientations

After inserting the preparatory stage, the second round of teaching experiments was conducted. The progress in moderate schools was very obvious, in which most students could correctly choose the unique straight line among the four choices after two weeks' training, and some of them could classify straight lines and curves (in the form of physical objects and printed figures) in the third week. Teaching experiments were also conducted with four students (S1, S2, S3, and S4) with severe ID who have little ability of communication. One of the students, S4, is with autism spectrum disorder, and he always exhibits restricted and repetitive behaviours. The learning pace of the students was slow, but progress was still observable as shown in Table 1.

Student	1st video recorded lesson	2nd video recorded lesson (after 6 weeks)
S1	Able to choose the straight line from four choices (POO Stage 4)	Able to classify straight line and curves
S2	Cannot choose the straight line from four choices (POO Stage 1)	Can choose the curve from four choices (POO Stage 2)
S3	Cannot classify the straight line and curves	Able to classify straight line and curves
S4	Cannot give teacher an identical object	Able to put two identical objects into a group

Table 1: Severe ID students' learning performance in two video recorded lessons

Students' learning progress was also observed by the interviewed teachers. They all agreed that the framework helped their students learning mental objects. Teacher 1 who applied the framework to teaching her students with moderate ID the mental object 'being less', told us:

For example, I would like to teach them the concept of 'being less'. During the activity, 'being less' will be the different one among the choices. If you directly point to the group with less objects and tell the student "this is being less" [without drawing their attention to the difference in amount], they will mistakenly name the objects they see 'being less'. Actually, they do not get what you mean, and they haven't even compared them [the quantity]. However, in this framework, they need to identify the difference first and then, we teach them the difference is 'being less'. Before we applied this framework in teaching, our students' performance of 'being less' was not stable, but through the framework, I pointed to the difference and told them it is 'being less', they soon have a grasp of the concept.

Teacher 1 found the framework provided a way for teachers to communicate with moderate ID students. She had never thought that POO could be an effective starting point to learn a mental object.

Teacher 2, who was in her second year of teaching students with severe ID, found that POO not only assisted her students' learning but also enhanced her professional competence in teaching students with severe ID.

Interviewer: What changes have you had after participating in this teaching experiment?

Teacher 2: Before this teaching experiment, I didn't know what content I should teach [to students with severe ID]. Now, when I teach mathematics, I have something to follow. I can make a reference to the framework. There are many levels and steps, so I can evaluate what level a student has achieved and know what learning activities should be assigned to the child in the next level. This makes lesson planning easier because the framework enables me to plan my teaching for each individual student.

Interviewer: What changes do your students exhibit?

Teacher 2: One student could pick the odd one only when it differs from the others significantly, now she is able to do it with even a small difference. I found that when they are familiar with the learning activity, they can make progress much faster. It is good for them to learn different topics under one progressive learning framework. Before this teaching experiment, I included a variety of learning activities in my class because I believed it would make a fruitful lesson. However, from the perspective of students with severe ID, this is not the case. They cannot handle a frequent switch of contents and learning methods, and consequently make little progress.

Interviewer: Did your students perform up to your expectation?

Teacher 2: More than half of the students performed better than I expected.

Interviewer: What was your expectation before?

Teacher 2: Before? I didn't expect much. I just gave them some manipulatives related to mathematics and hoped they could explore by themselves. I was satisfied if they could touch and play with them. But now, I expect more. I hope they can make some mathematical sense out of the activity instead of just playing.

Teacher 3, who has more than twenty years of experience in teaching moderate ID students, also mentioned:

We have learned more about mathematics. This is because that part of mathematics falls onto the blind spot of our knowledge. We were not able to see them because we skipped them quickly when we learned. As our starting point [of learning] was not that low, it is natural that we missed all the steps that our students need to have. Therefore, we would not think of these steps in our teaching. Fortunately, the learning trajectory developed explicitly lists those steps out, reminding us of what we missed in our teaching.

Concluding remarks

Our experience with the framework POO reveals the following observation that deserves serious attention. Teachers need support on creating a knowledge structure for understanding and planning for students' progress. Such a structure should enable teachers to divide teaching into small steps that can possibly engage students with moderate or severe ID. This is especially important when teachers cannot possibly draw on their own previous learning experience where those intermediate learning stages were virtually absent. To produce such a knowledge structure and subsequent action plan constitutes the design science of mathematics education (Wittmann, 1995, 2001). It involves studying various phenomena and their corresponding mathematical meanings, which can only be done from within mathematics (Freudenthal, 1983). Judging from students' performance and teachers' feedback, this approach is seen to have a positive impact on improving the quality of learning of ID students. It re-confirms the importance of mathematics in mathematics education (Akinwunmi, Höveler, & Schnell, 2014), something that may sound too trivial to consider, yet often too easy to overlook.

References

- Akinwunmi, K., Höveler, K., & Schnell, S. (2014). On the importance of subject matter in mathematics education: A conversation with Erich Christian Wittmann. *EURASIA Journal of Mathematics, Science & Technology Education*, 10(4), 357–363.
- Allsopp, D. H., & Haley, K. C. (2015). A synthesis of research on teacher education, mathematics, and students with learning disabilities. *Learning Disabilities: A Contemporary Journal*, 13(2), 177–206.
- AAIDD – American Association on Intellectual and Developmental Disabilities. (2010). *Intellectual disability: definition, classification, and systems of supports*. Washington, DC: American Association on Intellectual and Developmental Disabilities.
- Artigue, M. (1994). Didactical engineering as a framework for the conception of teaching products. In R. Biehler, R. W. Scholz, R. Strässer, & B. Winkelmann (Eds.), *Didactics of Mathematics as a Scientific Discipline* (pp. 27–40). Dordrecht, The Netherlands: Kluwer.
- Butler, F. M., Miller, S. P., Lee, K., & Pierce, T. (2001) Teaching mathematics to students with mild-to-moderate mental retardation: A review of the literature. *Mental Retardation*, 39(1), 20–31.
- Education Bureau. (2018). *Teacher professional development on catering for students with special educational needs (SEN)*. Retrieved from <http://www.edb.gov.hk/en/edu-system/special/sen-training/index.html>. Accessed 10.04.2019.
- Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht, The Netherlands: Reidel Pub. Co.
- Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Hord, C., & Bouck, E. C. (2012). Review of academic mathematics instruction for students with mild intellectual disability. *Education and Training in Autism and Developmental Disabilities*, 47(3), 389–400.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.
- Taylor, R. L., Richards, S., & Brady, M. P. (2005). *Mental retardation: Historical perspectives, current practices, and future directions*. Boston, MA: Pearson.
- Wittmann, E. C. (1995). Mathematics education as a “Design Science”. *Educational Studies in Mathematics*, 29(4), 355–374.
- Wittmann, E. C. (2001). Developing mathematics education in a systemic process. *Educational Studies in Mathematics*, 48(1), 1–20.