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Centrality-based Opinion Modeling on Temporal Networks

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ABSTRACT While most of opinion formation models consider static networks, a dynamic opinion formation model is proposed in this work. The so-called Temporal Threshold Page Rank Opinion Formation model (TTPROF) integrates temporal evolution in two ways. First, the opinion of the agents evolve with time. Second, the network structure is also time varying. More precisely, the relations between agents evolve with time. In the TTPROF model, a node is affected by part of its neighbor's opinions weighted by their Page Rank values. A threshold is introduced in order to limit the neighbors that can share their opinion. In other words, a neighbor influences a node if the difference between their opinions is below the threshold. Finally, a fraction of top ranked nodes in the neighborhood are considered influential nodes irrespective of the threshold value. Experiments have been performed on random temporal networks in order to analyze how opinions propagate and converge to consensus or multiple clusters. Preliminary results have been presented [1]. In this paper, this work is extended in two directions. First, the impact of various centrality measures on the model behavior is investigated. Indeed, in earlier work, the influence of a node is measured using Page Rank. New results using Directed Degree centrality and Closeness Centrality are derived. They allow to compare global against local influence measures as well as distance-based centrality, and to better understand the impact of the weighting parameter on the model convergence. Second, the results of an extensive experimental investigation are reported and analyzed in order to characterize the model convergence in various situations.

INDEX TERMS Clusters, Consensus, Opinion Dynamics, Page Rank, In-Degree Centrality, Closeness Centrality, Temporal network

I. INTRODUCTION

PEOPLE share information and exchange their opinions in their day to day life through various offline and online interactions. For example, they may discuss about a brand in order to make an opinion about it. Spreading of fake news related to any issue or a product on social platforms can affect the opinions of the people [2]. Politicians can pick influential people in order to attract voters [3]. Through all these evolving interactions, opinions keep on changing. Many researchers use static networks to study opinion dynamics [4], [5]. The nodes of the network represent the people and the edges account for their interactions. However, this is far from the real-world scenario, where opinions, interactions and the population is continuously changing. Nodes and edges may appear as well as disappear at different time instances.

Deffuant et al. [4] introduced a model where opinions

are continuous variables and exchanges are limited to agents with similar opinion. In this model, two agents update their opinion only if the difference between their opinion is smaller in magnitude than a threshold. The rationale for the threshold that can be apprehended as openness to discussion is that agents interact only when their opinions are close enough. Otherwise, they do not even bother to discuss. Kandiah and Shepelyansky proposed The Page Rank Opinion Formation model (PROF). In this model, binary opinions shared among linked individuals are weighted by their Page Rank probability [6]. Thus the model allows to give a greater importance to the opinion of an influential neighbor (with high Page Rank probability) as compared to the opinion of a less influential neighbor (with low Page Rank probability).

These models have been introduced on static networks,

while many networks are time-varying. Their evolution occurs on a timescale which may have an impact on the dynamical processes occurring between the nodes. Some researchers [7]–[9] have studied the opinion dynamics on temporal networks. In their setting, the number of nodes is fixed and the links between the nodes are evolving with time. However, they do not consider the "Page Rank effect". This gives us the motivation to introduce and to extend the TTPROF model [1]. As this model considers dynamics on the network together with its temporal evolution, it represents a major improvement compared to previous works. The concept of threshold on difference in opinion values of adjacent nodes is introduced in the proposed model along with the sharing of weighted opinions. In order to take the proposed model one more step closer to reality, opinions of a fraction of top ranked nodes (according to a centrality measure) are also considered during the sharing of opinions process independently of the threshold value. Indeed, these nodes are the most influential in the network, and therefore they can impact the opinion of their neighbors even if their opinions are quite different.

The rest of the paper is organized as follows. In Section II, related works are discussed. Section III introduces the proposed opinion dynamics model formulation. In this model, centrality measures (In-Degree, Page Rank and Closeness centrality) are used to weight the opinion of the nodes. It allows to compare the impact of global, local and distance-based centrality measures on opinion dynamics. Experimental results are reported and discussed in Section V. Furthermore, different network size are considered in order to investigate how it affects the opinion convergence. Finally, the conclusion is presented in Section VI.

II. RELATED WORK

Interactions in social networks play a vital role for framing opinions among individuals about a product, brand or a topic. Indeed, people take advice from their acquaintances to make their opinion. After taking opinion from different people of different thinking, individual can make better decisions. This phenomenon is termed as the wisdom of crowd effect [10].

Starting from a random state, and using simple rules of opinion formation, the system self-organizes through local interactions. It can lead to the emergence of a global consensus, in which all agents share the same opinion. Alternatively, the system can reach a state of polarization, in which a finite number of groups with different opinions survive, or of fragmentation, with a final number of opinions scaling with the system size. Different rules of opinion sharing have been proposed including the following features: biased conformity, compromise, and stubbornness [11]. In conformity models, individuals adopt the opinion of their neighbors. Clifford and Sudbury [12] proposed the discrete conformity opinion model called voter model in which agents adopts the opinion of one of their neighbors chosen at random. This model always leads to a global consensus. A variant of the voter model known as LPA (Label Propagation Algorithm)

has been proposed [13]. In LPA, agents adopt the opinion of the majority of their neighbors. In this model, polarization is achieved instead of consensus. The system converges into two sets of clusters of opinions who disagree with each other.

Degroot proposed an averaging and compromise model [14] in which agents update their opinion with an average opinion of their neighbours and the average of their previous opinion as well. This model leads to consensus. In biased conformity models, more weight is given to the neighbors with similar opinions leading to the flocking behavior [15]. Considering the flocking model along with the averaging model, Hegselman and Krause [5] proposed a model in which a confidence region for an individual is used. It is made of the set of neighbors whose difference in opinions with the individual is in the limit of a given threshold. Opinion update is made with the average value of neighbour's opinions in the confidence region. Suppose an individual i has opinion y_i . Considering a fixed threshold value, μ , the confidence region of the individual k is denoted by the set: $S_i(y) = \{k : |y_k - y_i| \leq \mu\}$. Deffuant et al. [4] proposed a model for opinion sharing in which every pair of nodes randomly interacts relating it to random graph. Here, one to one interaction among the nodes is considered. Two nodes readjust their opinions if it lies within a threshold value i.e. if there are 2 nodes with opinion x and y such that $|x - y| < d$ (threshold), there is a readjustment among the opinions. Page Rank Opinion Formation (PROF) model has been proposed in [6]. In this model, individuals share binary opinions which are weighted by their Page Rank value.

All the studies discussed above consider static network. However, in real-world situations, the network structure evolve with time. Recently, analysis of various aspects of temporal network have been performed, giving an insight into the day to day changes on social networks [16]–[18]. Several studies on opinion formation on evolving/ temporal networks are reported in the literature. One can refer to the survey made by Vazquez and Federico [7] on the threshold model and the voter model [12] considering static as well as evolving network.

Kozma et al. [9] have implemented Deffuant model [4] on static and adaptive network. In their setting, nodes can break their connection and get linked to other nodes. Results show that in such adaptive networks, consensus is difficult to achieve compared to static network. Maity et al. [8] have studied opinion dynamics on time-varying datasets in which the interval of variation is different. These works on evolving networks consider one to one interaction between nodes. Weighted opinions are not taken into consideration while sharing the opinions.

Many researchers have focused on the clusters/community formation in networks based on two factors: similarity in opinion and similarity in structure. The work of Newman et al. [19] concentrates on finding the communities in the network based on its structure. Nodes which are densely connected comes in one community preserving the community structure. This direction of research is one of the

most popular in the literature for static networks [20]–[22]. Greene et al. [23] have discussed the community structure in evolving/temporal network. They apply the cosine similarity index for finding the common neighbors among the nodes to measure the similarity between the nodes. They focused on the structure of the network to cluster the nodes in a community instead of focusing on the characteristics of the nodes. Some authors [4], [13], [24] consider the characteristics of the nodes i.e. opinions are considered according to which nodes are grouped into clusters/communities as discussed earlier as well. Note that cluster formation in a network is very useful in various applications like politics, marketing [25]. It can be used to see the groups of people thinking in different directions related to a product. This can be of great help to design an efficient marketing policy. Research moves in the direction to find the important nodes among the communities. Centrality measures are used to find the important nodes in the network. Various centrality measures are discussed and have been divided into different groups according to the similar application in [26]. Some authors have worked on finding the most central nodes within a community [27]–[30] and evaluating the effect of central nodes within a network.

III. THE OPINION FORMATION MODEL

In [1], a model is introduced that integrates 1) confidence region, 2) weighted opinion update according to the influence of the neighbors, and 3) temporal aspects of the opinion formation process. In this model, the influence is directed. In other words, nodes can be influenced by their first degree neighbors if there is incoming link and that the neighbor belongs to their confidence region. A neighbor belongs to the confidence region of a given node, if the difference between their opinions is below a threshold or if this neighbor belongs to a defined fraction of the most influential nodes of the network. Weights are assigned to the opinion of each node in the network according to their centrality values (Page Rank, In-Degree or Closeness centrality). It allows to specify the importance of a node in the network for sharing its opinion with its neighbours. Note that it is the opinion of the nodes that are weighted and not the nodes or link of the network. Indeed, in this work we restrict our attention to directed but unweighted networks.

In real-world situations, networks grow with time. Indeed, new individuals are introduced in the network and new connections are formed among individuals. In this work for the sake of simplicity, we restrict our attention to networks with a fixed number of nodes. However, the interactions between the nodes change with time. It may happen that some of the old connections disappear, but generally the rate of forming new connections is greater than the disappearance of old connections. This phenomenon is also visible in infrastructure networks such as railway network, airport network [31]. Thus, taking inspiration from this phenomena, we consider that the network model is growing in terms of edges. In other words, at every new time stamp, the probability of

adding new edges is higher than the probability of deletion of existing edges. Variables used in the paper are listed in Table 1.

TABLE 1. Common used terms and their respective symbols

Terms	Symbol
i^{th} Node	v_i
Directed edge from node j to node i	e_{ji}
Network Size	N
Shortest distance from node j to node i	$d(j, i)$
Size of k^{th} connected component g_k	N_{g_k}
k^{th} connected component	g_k
Rank of i^{th} node	r_i
Opinion of i^{th} node	o_i
Weighted Opinion of i^{th} node	w_i
Opinion Difference Threshold	α
Fraction of Top Ranked Nodes	β
Single TimeStamp	t
TimeStamp at convergence	t_c
Number of Clusters at convergence	n_c
Opinion at convergence	O_c
Size of cluster at convergence	S_c

Initially, each node v_i is assigned a continuous random uniform opinion value (denoted by o_i) in the range $[0,1]$. This opinion value keeps on evolving with the effect of neighbor's opinion until it stabilizes, and there is no more change in the values of opinion. At each timestamp t , the opinion value of a node i is denoted by $o_i(t)$. The set of opinion values for all the nodes in a network of size N at timestamp t is denoted by $O(t) = (o_1(t), \dots, o_N(t))$. At timestamp t_c , opinions converge into certain number of clusters (n_c).

Update rules for opinion sharing among the nodes in temporal network

At every timestamp, the opinion of an individual might change as it gets influenced by its neighbors opinion. The proposed model for opinion dynamic is summarized by Algorithm 1 and discussed next.

Individuals in a network are considered to have their own opinion for some social issues, and that it is modeled as a continuous variable in the range $[0.0, 1.0]$. Each individual has also a set of confidence bound indicating the individuals with whom he can share its opinion. Nodes in the confidence bound are calculated on the basis of two parameters. First, if the difference in opinion between two neighbors is lower than a threshold (α) value, they can influence one another, incrementally changing opinions to become more similar to each other. In the proposed model, we assume that individuals are homogeneous and that they share the same threshold value α . This parameter allows to integrate the real-world scenario where an individual is not influenced by another node if their opinions are too different. In this case, he does not bother to exchange with this neighbor. Second, if his neighbor is an important individual at the overall network scale. Indeed, some individuals are very influential and it is somehow difficult to ignore their opinions even if the difference between their opinion and their neighbors opinion is higher than the threshold α . For example, in a working environment we have

Algorithm 1 Evolving_Opinions($G(V, E(t)), \alpha, \beta, o_i$)

- 1: Form the initial Erdos Renyi directed random Graph $G(V, E(t))$ with p value and a fixed number of nodes N at timestamp $t = 0$.
- 2: Assign opinion values $o_i(t)$ to the node v_i according to a uniform random distribution where $o_i(t) \in [0,1]$ and $v_i \in [1, N]$
- 3: Fix a confidence bound value α for the network where $\alpha \in [0,1]$.
- 4: Fix a fraction value of top ranked nodes β on the network where $\beta \in [0,1]$.
- 5: Do
- 6: Compute the centrality value of all the nodes $c_i(t), i \in [1, N]$.
- 7: Rank the nodes in the decreasing order of their centrality value $r_i a(t), i \in [1, N]$
- 8: Assign a score to the nodes according to their rank: $w_i(t) = \left[1 - \frac{r_i(t) - 1}{N}\right], i \in [1, N]$
- 9: Weight the opinions of the nodes $x_i(t) = o_i(t)w_i(t)$.
- 10: Add the β fraction of the top ranked nodes in the set T .
- 11: Find the nodes in the confidence bound. i.e belonging to the neighbour set of node $v_i, S(i)$ and satisfying one of the conditions $|o_i(t) - o_l(t)| < \alpha$ or $v_j \in T$ where $\alpha \in \mathbb{R}[0, 1]$ and $v_l \in S(i)$ and $v_j \in S(i)$
- 12: Average the opinion of the node i with the weighted opinions of all the nodes in the confidence bound $x_i(t+1) = x_i(t) + \sum_{l, v_l \in S(i)} x_l(t) + \sum_{j, v_j \in S(i), T} x_j(t)$.
- 13: Normalize the updated opinion $o_i(t+1) = \frac{x_i(t+1)}{w_i + \sum_{l, v_l \in S(i)} w_l(t) + \sum_{j, v_j \in S(i), T} w_j(t)}$.
- 14: Addition and deletion of the edges E in Graph $G(V, E(t))$ to make it $G(V, E(t+1))$ for the next time stamp.
- 15: Until the convergence i.e. $(o_i(t+1)) - o_i(t) = 0 \forall i$.
- 16: Return the convergence timestamp t_c and the number of clusters of opinions n_c .

to consider the opinions of superiors even if it is far from ours. Thus, there might be some influential individuals whose opinions have to be considered whatever the threshold value. These two sets of nodes constitute the confidence bound that can influence a node opinion.

Let's consider that the neighboring set of a node v_i is represented by $S(i)$ where $i \in [1, N]$ and the size of the set $S(i)$ is equal to the In-Degree k_i of the node v_i . Indeed, we consider as neighbors only nodes that have an edge directed to node i . These nodes can share their opinion with it. Node i can share its opinion with a node in its incoming neighbors if their difference of opinions is below the threshold value α or if the incoming neighbors belong to the set of the top influential nodes. In other words, if any of the following conditions are satisfied.

- **Case 1:** If the difference between the opinion of the node v_i and the neighboring node with an incoming link from $S(i)$ is under the threshold limit. $|O_i - O_l| < \alpha$ where $\alpha \in \mathbb{R}[0, 1]$ and the node v_l with opinion $O_l \in S(i)$
- **Case 2:** If the incoming neighboring node $v_j \in S(i)$ is in the set of the top influential nodes T

To share the weighted opinions from the nodes in the confidence region, importance of nodes are calculated using Page Rank, In-Degree and Closeness centrality measures.

The Page Rank [32] of a node i is given by,

$$PR(i) = \frac{1-d}{N} + d \sum_{j \in Ne(i)} \frac{PR(j)}{K_{out}(j)} \quad (1)$$

where, N is the total number of nodes in the network, d is the damping factor ranging $\in [0,1]$, $K_{out}(j)$ is the outgoing links from the node j , $Ne(i)$ is the set of the neighbouring

nodes from where there are incoming links towards i . To calculate the Page Rank in the given network, if there is an out-link from node i to node j then an in-link is created towards i from j i.e. edges are considered in reverse manner in which node j tries to request information from node i or try to follow node i . Therefore, the edges in the network are reversed and Page Rank of nodes is calculated.

The In-Degree of a node i is given by,

$$K_{in}(i) = \sum_j a_{j,i} : e_{ji} \in G \quad (2)$$

where, $a(j,i)$ is the value of adjacency matrix A of directed network G , if there is a directed edge from node j to node i then it will be 1 otherwise 0.

The Closeness centrality of a node is a measure of centrality in a network, calculated as the reciprocal of the sum of the length of the shortest paths between the node and all other nodes in the graph. The Closeness centrality of a node i is given by,

$$CC(i) = \frac{N_{g_k} - 1}{\sum_j d(j,i)}, i, j \in g_k \quad (3)$$

where, g_k is the k^{th} connected component in a network with size N_{g_k} , node j and i belongs to same connected component g_k , $d(j,i)$ is the length of the shortest distance from node j to node i considering the In-Degree towards the node i . If disconnected components are there in the network, then the closeness centrality values are computed for each connected component respectively as mentioned in Eq. 3.

All the computation for finding the important nodes are done by using incoming links. Nodes are ranked in the

decreasing order of their centrality values. If two nodes have same centrality values, then the nodes are selected as per their node ids from low to high value. Low node id is preferred over higher node id in such case. The rank of a node i is given by $r_i \in [1, N]$. The set of rank is given by,

$$R(t) = [r_1(t), r_2(t), \dots, r_N(t)] \quad (4)$$

A score w_i is assigned to each node according to its rank:

$$w_i = \left[1 - \frac{r_i - 1}{N} \right] \quad (5)$$

The set of Scores at time t is given by:

$$W(t) = [w_1(t), w_2(t), \dots, w_N(t)] \quad (6)$$

W and $R \in \mathbb{R}^N$.

The actual opinion at time t is given by: (7)

$$O(t) = [o_1(t), o_2(t), \dots, o_N(t)] \quad (7)$$

The weighted opinions $x_i(t)$ are calculated according to the score $w_i(t)$ as follows:

$$X(t) = \langle O(t)W(t) \rangle \quad (8)$$

$$x_i(t) = o_i(t)w_i(t) \quad (9)$$

x_i at next timestamp is calculated by finding the average of the weighted opinions of the nodes from the set $S(i)$ at previous time stamp which lies in the confidence bound of node i as:

$$x_i(t+1) = x_i(t) + \sum_{l, v_l \in S(i)} x_l(t) + \sum_{j, v_j \in S(i), T} x_j(t) \quad (10)$$

Finally, to scale down the values of o_i between 0 and 1, we normalize the average weighted opinions to find the o_i at next timestamp:

$$o_i(t+1) = \frac{x_i(t+1)}{w_i + \sum_{v_l \in S(i)} w_l(t) + \sum_{v_j \in S(i), T} w_j(t)} \quad (11)$$

Update of opinions is sequential with deterministic node selection.

This procedure of opinion update to find the opinion at next time stamp iterates until the opinions converge at timestamp t_c

A temporal Erdos Renyi Random graph $G(N, P)$ is generated with a probability p , 0.2 of having an edge among the N nodes [33]. At each timestamp t , new edges are added and existing are deleted. In order to mimic real-world situations the number of links keep growing. Indeed, 10% of the edges are added and the 5% of the edges are deleted at every timestamp.

IV. EXPERIMENTAL SETUP

We analyze the influence of the two input parameters i.e. opinion difference threshold (α) and the fraction of top ranked nodes (β) on the opinion convergence in the further sections. Indeed the opinion formation model is controlled by the opinion difference threshold α and the fraction of top ranked nodes β . Initially, the fraction of top ranked nodes is set to zero. The impact of the opinion difference threshold, α is studied on the convergence of opinions (number of clusters & convergence time). The threshold value varies in the range [0,1] with a non-linear variation. We set the opinion difference threshold, α to zero and let the fraction of top ranked nodes varies in the range [0,100%]. In order to get more insight into the influence of its size, we consider networks of size 100, 500,1000. In each case, results obtained with the opinion model based on Page Rank are compared with the opinion model that uses the In-Degree and Closeness centrality. Finally, the same type of experiments are performed combining both control parameters of the opinion model. The different cases that can be formed to analyze the results are reported in Table 2. We first report the results of experiments for networks of size $N = 500$ before investigating the network size influence on the opinion dynamics.

TABLE 2. Summary of the different combination of parameters used in the experiments. For each set of parameters the opinion formation model based on Page Rank is compared to the opinion formation model based on In-Degree and Closeness centrality.

Cases	Size of Network	α	β
case 1	100	[0,1]	0
case 2	500	[0,1]	0
case 3	1000	[0,1]	0
case 4	100	0	[0,100]
case 5	500	0	[0,100]
case 6	1000	0	[0,100]
case 7	100	[0,1]	[0,100]
case 8	500	[0,1]	[0,100]
case 9	1000	[0,1]	[0,100]

V. EXPERIMENTAL RESULTS AND ANALYSIS

A. INFLUENCE OF THE THRESHOLD ON THE CONVERGENCE OF OPINIONS

First, we investigate that how does the threshold influence the opinion dynamics independently of the other parameters (case 2 in Table 2). Hence, the fraction of top ranked nodes is fixed to 0%. In this case, the top rank nodes have no influence and the confidence region depends only on the value of the threshold $\alpha \in [0,1]$. Initially, there are as many opinions as nodes in the network of size $N=500$. The opinions of the nodes are weighted using the Page Rank, In-Degree centrality and Closeness centrality.

1) Relating the threshold to the number of clusters in which opinions converge

First we report the result using the Page Rank-based opinion formation model. Let's start with the trivial limiting case.

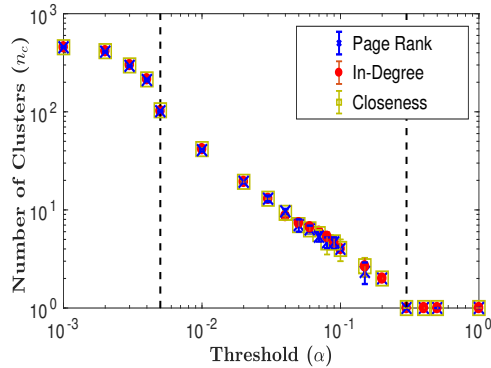


FIGURE 1. Plot representing the number of clusters of opinions at convergence (n_c) versus the threshold value (α). Page Rank, In-Degree and Closeness based opinion formation models are plotted in blue, red and green color respectively. Vertical dashed lines at point 0.005 and 0.3 shows the different threshold value ranges as low, medium and high. The fraction of top ranked nodes (β) is fixed at 0%. Five simulations are performed on networks of size 500. Mean values and Standard deviation are reported.

When the threshold value, α equal to 0, the nodes do not take into account any of their neighbors opinion. Consequently, none of them changes their opinion, and there are as many clusters as initial opinions i.e. the number of nodes. As the threshold value increases, the number of clusters in which opinions converge decreases exponentially. A log-log plot is used in Figure 1 with blue color. It reports the number of clusters versus the threshold values in order to highlight this behavior. First of all, we can notice that we can consider three range of threshold values. For small threshold values the number of clusters decreases slowly from the initial number of nodes. For medium threshold values we observe a linear decreasing slope. As we use a log-log scale, it means that the number of clusters decreases exponentially with the threshold value. Finally, for high threshold values, the consensus is reached. We observe 3 different ranges for the threshold values. In the range $[0, 0.005]$, the number of clusters decrease monotonically from 500 to 100. It shows that a linear relationship in the range $\alpha \in [0.005, 0.3]$ which is an exponential decrease. Finally, the consensus is always reached for $\alpha \in [0.3, 1]$. Indeed, in this range, all the opinions converge to a single cluster.

Figure 2 (a)-(d) report typical examples of the evolution of the opinions versus the timestamp. The Page Rank of a node is used to provide the weight of the respective node's opinion and initially, the size of the network is 500. Figure 2 (a) shows that no convergence found with the 0 threshold value and each node keeps its opinion. Figure 2 (b) shows that opinion converges in 19 timestamp to 310 clusters with a threshold value of 0.004. If the threshold value is very low then only few nodes change their respective opinions. Figure 2 (c) with a threshold value of 0.08 shows that the opinion converges in 12 timestamp to 5 clusters. In this case the threshold value is enough high so that groups of nodes share their opinions among them. Opinion values cover all the range of opinions uniformly at convergence. Indeed, observed values are 0.09,

0.24, 0.42, 0.70 and 0.91. Note that the cluster size are quite variable. Indeed, the biggest cluster is more than the three times bigger than the smallest one (cluster size= 51, 71, 83, 119 and 176). Finally, Figure 2 (d) shows that the consensus is reached in 5 timestamps with a threshold value of 0.5. In this case, all the nodes are influenced by the large number of its neighbors, hence, consensus is easily reached. Note that the polarization (2 clusters) is reached for $\alpha = 0.2$.

Figure 1 in blue and red color reports the results of the same set of experiments comparing page rank centrality with In-Degree centrality to weight the opinions instead of Page Rank. The fraction of top ranked nodes is still fixed at 0%. The similar behavior for the change in number of clusters against threshold value is observed in case of In-Degree as observed in Page Rank centrality. One can also observe that the opinions converge to the same or a higher number of clusters with In-Degree centrality as compared to the Page Rank centrality. The number of clusters is slightly higher at convergence when In-Degree centrality is considered instead of Page Rank centrality in the $[0.05, 0.08]$ and $[0, 0.01]$ threshold value interval. Very limited differences are found with the Page Rank centrality. Hence, the small differences between the two centrality measures are not visible as mentioned in Figure 1 in blue and red color. At all the other threshold values, the number of clusters in which opinions converge is quite comparable for the two centralities.

Figure 3 (a)-(d) reports typical examples of the evolution of opinions versus the timestamp using the In-Degree centrality. Figure 3 (a) shows that no convergence is reached with a threshold value of 0. Figure 3 (b) shows that the opinion converges in 19 timestamp to 310 clusters with a threshold value of 0.004. Figure 3 (c) shows that the opinion converges in 13 timestamp to 6 clusters with a threshold value of 0.08. It is higher than the 5 clusters which is obtained with the Page Rank centrality. Opinion values are 0.17, 0.32, 0.49, 0.59, 0.74, 0.89 in which they converge with cluster size as 165, 11, 126, 11, 94, 93. Finally, Figure 3 (d) with a threshold value of 0.5 shows that the consensus is reached. Note that polarization (2 clusters) is reached for alpha 0.2. These results suggest that the clusters in which opinions converge is quite insensitive to the centrality used to weight the opinion of the nodes.

Figure 1 reports the results of the same set of experiments comparing Page Rank centrality with In-Degree and Closeness centrality to weight the opinions of the nodes. The fraction of top ranked nodes is still fixed at 0%. Similar behavior for the variation of the number of clusters against threshold value is observed in the case of Closeness centrality as well as Page Rank and In-Degree centrality. One can also observe that the number of clusters at convergence is different with Closeness and In-Degree centrality as compared to the Page Rank centrality. The number of clusters is slightly higher when Closeness centrality is considered instead of Page Rank centrality in the $[0, 0.01]$ threshold value interval. The number of clusters is slightly lower at when Closeness centrality is considered instead of In-Degree

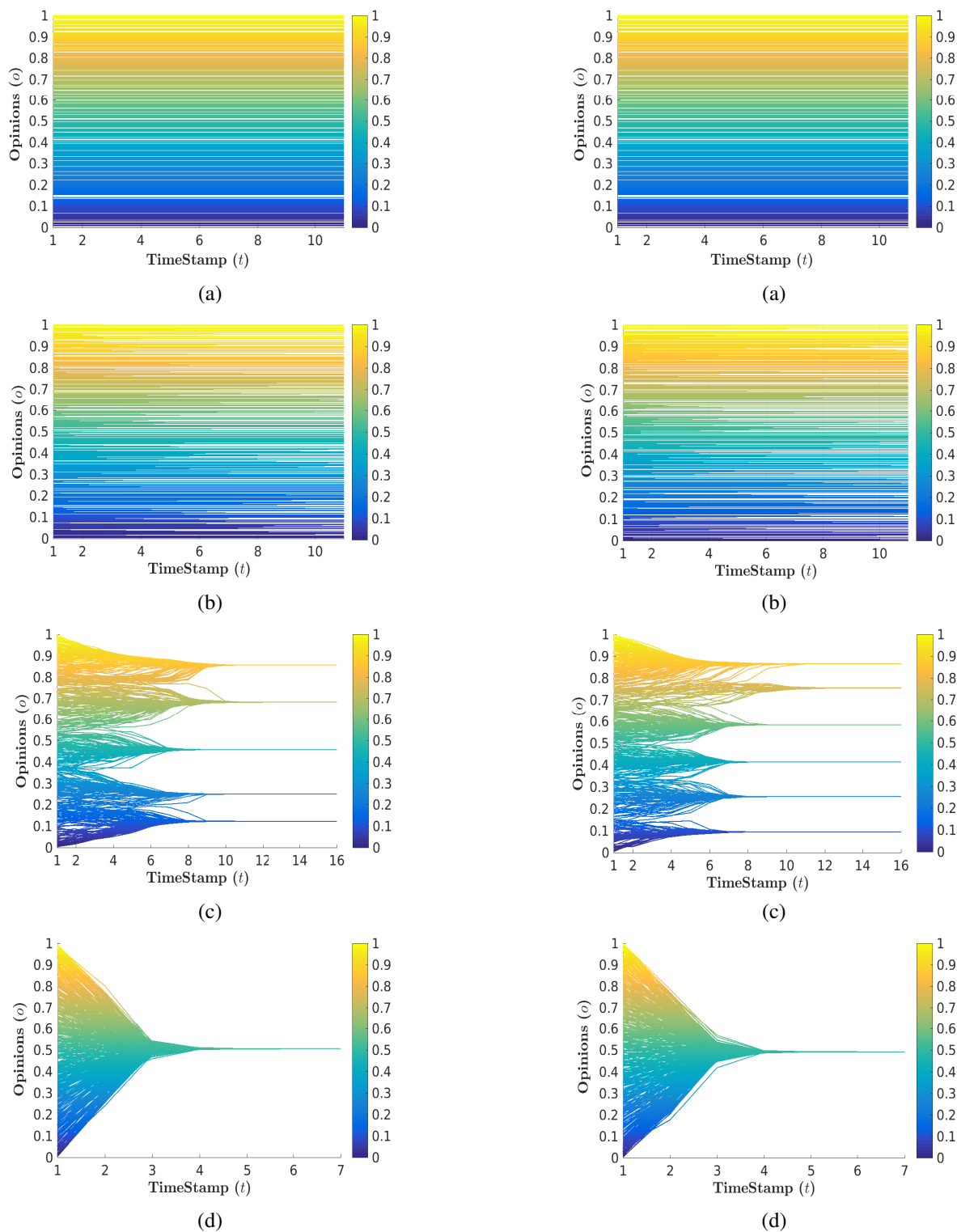


FIGURE 2. Evolution of opinion of the respective nodes versus timestamp until get convergence. The fraction of the top ranked nodes (β) fixed at 0%. The centrality used to weight the opinions is Page Rank on network of size 500. Different values of the threshold (α) are considered as (a) No Convergence with $\alpha = 0$ (b) Convergence with $\alpha = 0.004$ (c) Convergence with $\alpha = 0.08$ (d) Consensus with $\alpha = 0.5$. Different colors represent different opinion values in the range $[0, 1]$.

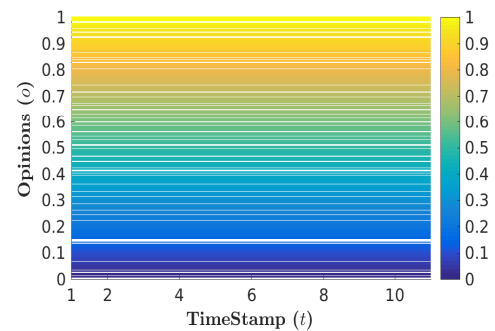
FIGURE 3. Evolution of the opinions versus the timestamp until convergence. The fraction of top ranked nodes (β) is fixed at 0%. The centrality used to weight the opinions is In-Degree centrality on a network of size 500. Different values of the threshold (α) are considered as (a) No Convergence with $\alpha = 0$ (b) Convergence with $\alpha = 0.004$ (c) Convergence with $\alpha = 0.08$ (d) Consensus with $\alpha = 0.5$. Different colors represent different opinions.

centrality in the $[0.05, 0.08]$ threshold value interval. Overall, very limited differences are found when comparing all the three centralities. Hence, the small differences between the three centrality measures are not visible as illustrated in Figure 1. At all the other threshold values, the number of clusters in which opinions converge is quite comparable for the three centralities.

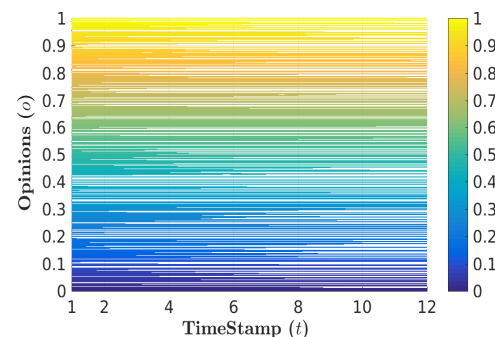
Figure 4 (a)-(d) reports typical examples of the evolution of opinions versus the timestamp using the Closeness centrality. Figure 4 (a) shows that no convergence is reached with a threshold value of 0. Figure 4 (b) shows that the opinion converges in 20 timestamps to 305 clusters with a threshold value of 0.004. Figure 4 (c) shows that the opinion converges in 13 timestamps to 5 clusters with a threshold value of 0.08. It is higher than the 12 timestamps to converge considering Page Rank centrality. The number of clusters in which opinions converge using the Closeness Centrality is similar to the Page Rank centrality but higher than In-Degree centrality. Opinion values at convergence are well distributed $(0.15, 0.39, 0.52, 0.69, 0.90)$ with cluster size varying from 36 to 128. Finally, Figure 4 (d) with a threshold value of 0.5 shows that the consensus is reached. Note that polarization (2 clusters) is reached for α 0.2. These results suggest that the clusters in which opinions converge are quite insensitive to the centrality used to weight the opinion of the nodes.

Figure 5 (a), (b) and (c) reports the distribution of the cluster size at convergence (S_c) for small values of the threshold using Page Rank, In-Degree and Closeness centrality. S_c is a set of size of all the clusters at convergence. Fifteen simulations are performed at threshold value, $\alpha = 0.08$. Bins of size 10 are used to compute the frequency of the opinion cluster size. Figure 5 (a) and (c) shows that there is a higher number of clusters in the high range of cluster size in comparison to the low range in the case of Page Rank and Closeness centrality. Figure 5 (a) shows that there is more clusters in the low range of cluster size in comparison to the high range in case of the In-Degree centrality. Opinions converge in more clusters with In-Degree centrality in comparison with the Page Rank centrality and Closeness Centrality. Hence, considering In-Degree centrality, there is more small size clusters as compared to Page Rank and Closeness centrality. One more thing is shown in Figure 5. A large number of medium size clusters emerge during simulations. In case of Page Rank, large number of clusters occurs in the range $[80-160]$, in the range $[80-120]$ in case of In-Degree, in the range $[70-140]$ in case of Closeness. This means there are few large and small size clusters in comparison to medium size clusters. Furthermore, there is comparatively a higher number of clusters in case of the In-Degree centrality.

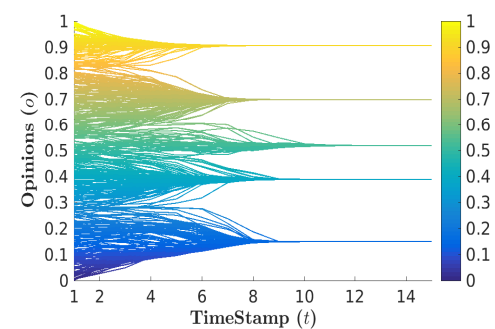
Figure 6 reports the estimated distribution of the opinion cluster values at convergence (O_c). O_c is a set of opinion values of all the clusters at convergence. Fifteen simulations are performed at threshold value $\alpha = 0.08$. Bins of size 0.2 are used. Page Rank, In-Degree and Closeness centrality is considered for the simulations in Figure 6 (a), (b) and (c). It is observed that the distribution of the opinion values is well



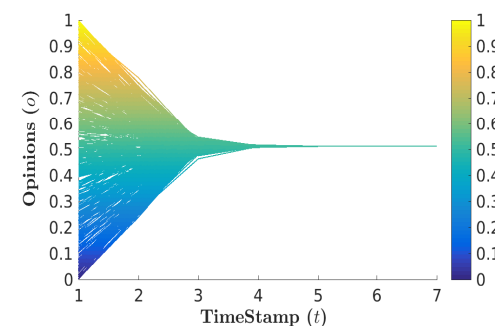
(a)



(b)

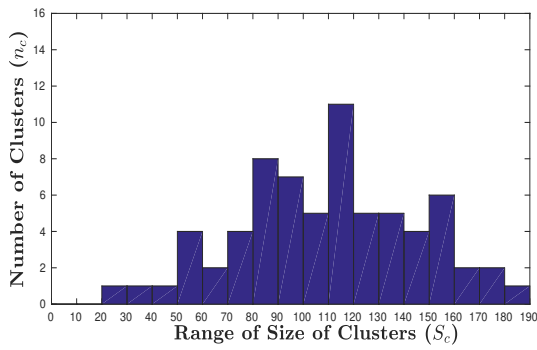


(c)

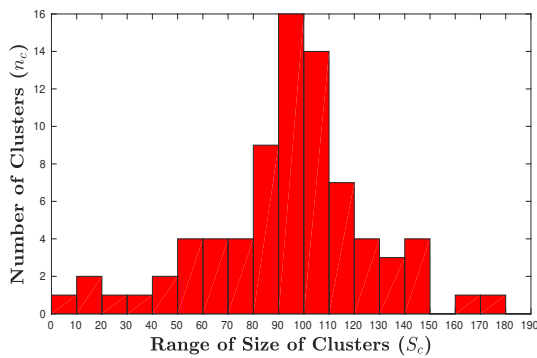


(d)

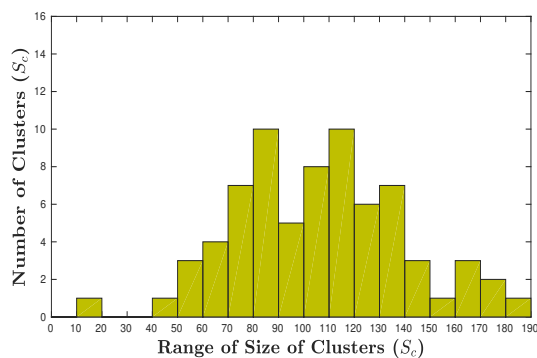
FIGURE 4. Evolution of the opinions versus the timestamp until convergence. The fraction of top ranked nodes (β) is fixed at 0%. The centrality used to weight the opinions is Closeness centrality on a network of size 500. Different values of the threshold (α) are considered as (a) No Convergence with $\alpha = 0$ (b) Convergence with $\alpha = 0.004$ (c) Convergence with $\alpha = 0.08$ (d) Consensus with $\alpha = 0.5$. Different colors represent different opinions.



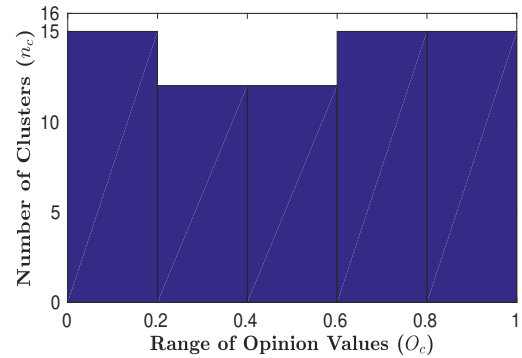
(a)



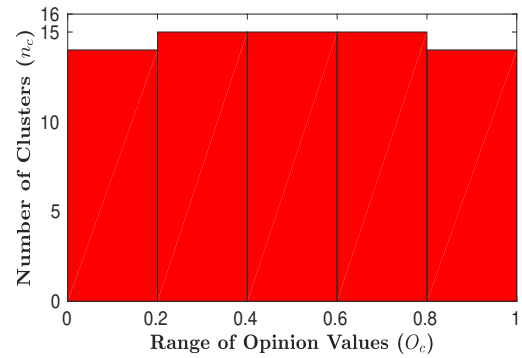
(b)



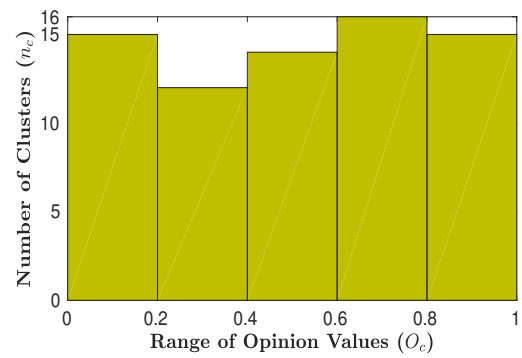
(c)



(a)



(b)



(c)

FIGURE 5. Histogram of the Cluster size at convergence (S_c) for small values of the threshold. The threshold value (α) is fixed at 0.08 and the fraction of top ranked nodes (β) is fixed at 0%. The centrality used to weight the opinions is (a) Page Rank, (b) In-Degree and (c) Closeness centrality shown in blue, red and green color. 15 simulations are performed on a network of 500 nodes.

FIGURE 6. Histogram of the cluster opinion values at convergence (O_c). The threshold value (α) is fixed at 0.08 and the fraction of top ranked nodes (β) is fixed at 0%. The centrality used to weight the opinions is (a) Page Rank, (b) In-Degree and (c) Closeness centrality shown in blue, red and green color. 15 simulations are performed on a network of 500 nodes.

approximated by a uniform distribution in all the cases.

2) Relating the threshold to the convergence time of the opinions

In this section, we study the number of iterations (Timestamps) needed to reach the convergence and how it relates to the threshold value. Let's first consider the Page Rank-based opinion model. Globally, it is observed that if the threshold value increases then the number of timestamp needed to reach convergence decreases as reported in Figure 7 in blue

color. Indeed, an increase in the threshold allows more and more nodes to share their opinion, the convergence is reached more quickly. Figure 7 reports the number of timestamps needed to reach convergence versus the threshold value. At $\alpha = 0$, none of the nodes change their opinions, hence convergence is considered at a number of timestamps equal to zero. As the threshold value increases to 0.001, the number of timestamps to reach consensus reached to 20. Then, the number of timestamps needed for the opinion convergence decreases till $\alpha = 0.5$. From $\alpha = 0.5$ to 1, a constant value of

5 timestamps is needed to reach the convergence (consensus). Indeed, a larger number of nodes try to share their opinions and this makes the convergence faster.

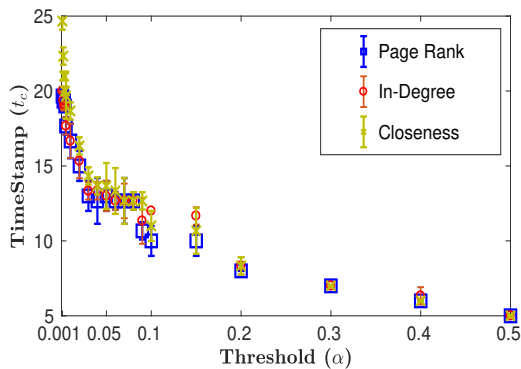


FIGURE 7. Plot representing the number of timestamps needed to reach convergence versus the threshold value (α). Page Rank, In-Degree and Closeness based opinion formation model are plotted in blue, red and green color respectively. The fraction of top ranked nodes (β) is fixed at 0%. Five simulations are performed on networks of size 500. Mean values and Standard deviation are reported.

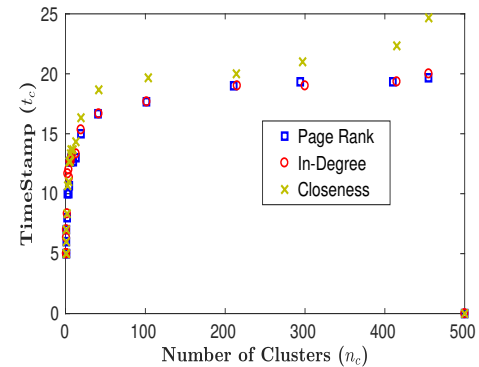
Figure 7 in blue and red color reports the results of the same set of experiments comparing page rank centrality with In-Degree centrality to weight the opinions instead of Page Rank. The fraction of top ranked nodes is still fixed at 0%. The similar behavior for the change in number of timestamps against threshold value is observed in case of In-Degree centrality as observed in Page Rank centrality. One can also observe that the opinions converge in an equal or higher number of timestamps when the In-Degree centrality is used instead of Page Rank. The number of timestamps at convergence is higher when the In-Degree centrality is considered in the threshold value interval $[0.2, 0.09]$ and $[0.04, 0.02]$. In the other intervals, the number of timestamps in which opinions converge is almost the same for In-Degree and Page Rank centrality.

Figure 7 reports the results of the same set of experiments comparing page rank centrality with In-Degree and Closeness centrality to weight the opinions instead of Page Rank. The fraction of top ranked nodes is still fixed at 0%. Results are quite similar for the three centrality measures. One can also observe that the opinions converge in an equal or higher number of timestamps when the Closeness centrality is used instead of Page Rank and the In-Degree centrality in the threshold value interval $[0, 0.2]$. In the other intervals, the number of timestamps in which opinions converge are almost the same for Closeness, In-Degree and Page Rank centrality.

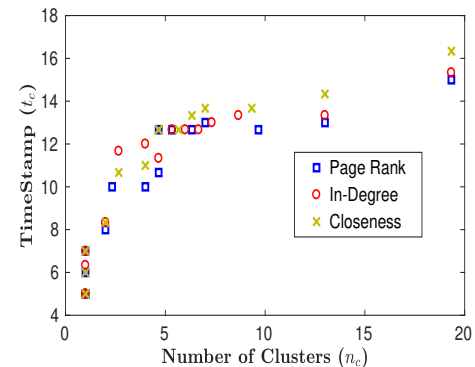
3) Relating the convergence time to the number of clusters at convergence

Figure 8 (a) in blue color shows the relationship between the number of timestamps at convergence versus the number of clusters using page rank centrality. The magnified view of Figure 8 (a) in blue color for the range of number of

clusters in $[0, 20]$ can be seen in Figure 8 (b) in blue color as it is difficult to have a clear view in Figure 8 (a) in blue color in this range of cluster values. For threshold values in the range $[0.3, 1]$, opinions converge to 1 cluster, but the convergence rate vary from 5 to 7 timestamps. As the threshold value decreases, the number of clusters increases and the number of timestamps in which opinion converge increases monotonically.



(a)



(b)

FIGURE 8. (a) Plot representing the number of timestamps needed to reach convergence versus the number of clusters with varying threshold value (α). The fraction of top ranked nodes (β) fixed at 0%. Page Rank, In-Degree and Closeness based opinion formation models are plotted in blue, red and green color respectively. Network size is 500. (b) Magnified view of (a) for the number of clusters in the range $[1,20]$

Figure 8 (a) in blue and red color reports the results of the same set of experiments comparing page rank centrality with In-Degree centrality to weight the opinions instead of Page Rank. The fraction of top ranked nodes is still fixed at 0%. The magnified view of Figure 8 (a) in blue and red color for the number of clusters in the range $[0,20]$ can be seen in Figure 8 (b) in blue and red color. The similar behavior for the change in number of timestamps against number of clusters is observed in case of In-Degree centrality as observed in Page Rank centrality. As the number of clusters increases, the number of timestamps in which opinion converge increases monotonically. Overall, the number of clusters or timestamps at convergence is higher when In-Degree centrality is considered rather than Page Rank but the difference is not very

large.

Figure 8 (a) reports the results of the same set of experiments comparing page rank centrality with In-Degree and Closeness centrality to weight the opinions. The fraction of top ranked nodes is still fixed at 0%. The magnified view of Figure 8 (a) for the number of clusters in the range [0,20] can be seen in Figure 8 (b). The similar behavior for the change in the number of timestamps against the number of clusters is observed in Closeness centrality as observed in Page Rank and In-Degree centrality. As the number of clusters increases, the number of timestamps in which opinion converge increases monotonically. Overall, the number of clusters or timestamps in which opinions converge is higher when In-Degree or Closeness centrality is considered rather than Page Rank but the difference is not very large.

B. INFLUENCE OF THE TOP RANKED NODES ON THE CONVERGENCE OF OPINIONS

In the previous experiments, we considered the centrality values of the nodes only to weight the opinions, and the top ranked nodes could not influence their neighbors directly. In order to take into account the influence of the fraction of top ranked nodes β in the opinion formation dynamics independently of the other parameters (case 4, 5, 6 in Table 2), a series of experiments are reported with the threshold value fixed at 0. In this case the threshold has no influence, and the confidence region depends only on the value of the top ranked nodes. Initially, there are as many opinions as nodes in the network. The opinions of the nodes are weighted using Page Rank, In-Degree and Closeness centrality.

1) Relating the top ranked nodes to the number of clusters in which opinions converge

In this section, we study how the number of clusters in which opinions converge evolves with the fraction of top ranked nodes that can influence their neighbors using Page Rank centrality on network of size $N=500$. When $\beta = 0\%$, nodes cannot take into account any of their neighbors opinion. Consequently, none of them changes its opinion, and there are as many clusters as initial opinions i.e. number of nodes. As β increases to 1%, opinions start reaching consensus (1 cluster). After this, there is no change in the number of clusters in which opinions converge as β increases. This is a quite different scenario as compared to the previous case. The reason for this observation is the consideration of top ranked nodes. In the case of the threshold value, different nodes are considered for the opinion sharing purpose for every node. Therefore, multiple opinions or clusters of opinions emerge. But in the case of top ranked nodes, it is the same limited set of nodes that is considered for opinion sharing purpose by the other nodes. The opinion of the top ranked nodes has a greater influence on the other nodes. Indeed, five top ranked nodes are enough in a network of size 500 to make all the population reach a consensus.

Let's now turn to the results obtained with the In-Degree and Closeness centrality. The same conditions lead to the

same behavior. Opinions converge to reach consensus (1 cluster) in any case when the In-Degree or the Closeness centrality is taken into consideration except in the case where $\beta = 0\%$.

2) Relating the fraction of the top ranked nodes to the convergence time of opinions

In this section, we study how the number of timestamps in which opinions converge evolves with the fraction of top ranked nodes using Page Rank centrality on network of size $N=500$. If $\beta = 0\%$, none of the nodes change its opinion, hence the convergence timestamp is considered as 0. As β increases, the number of timestamps in which opinions converge decreases. At $\beta = 0.4\%$, few top ranked node's opinion can propagate, so the number of timestamps needed to reach the consensus is large (33) because the influence of the Top Ranked nodes needs to propagate in the network. When β increases to the value 5%, the number of timestamps in which opinions converge decreases rapidly to the value 7. When β further increases to the value 50%, the number of timestamps in which opinions converge decreases up to 5 which is not a major decrease. A plot is reported in Figure 9 in blue color showing the number of timestamps versus the fraction of top ranked nodes in order to highlight this behavior observed when β ranges from 0% to 50%. It shows that for the low range values of β [0%, 5%] the number of timestamps needed to reach convergence exhibit more variation in comparison to the range [5%, 50%]. In the range [50%, 100%] opinions convergence is always reached in five timestamps. At this point adding new influential nodes does not increase the convergence rate.

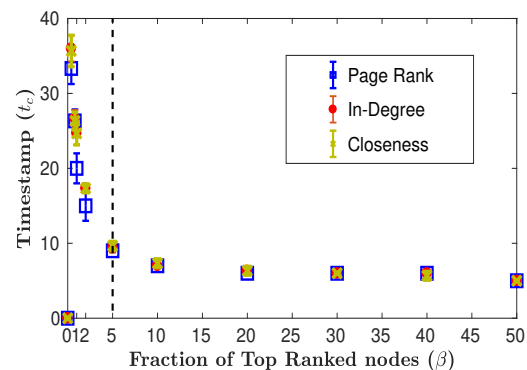


FIGURE 9. Plot representing the number of timestamps needed to reach convergence versus the fraction of top ranked nodes (β). Page Rank, In-Degree and Closeness based opinion formation model are plotted in blue, red and green color respectively. Vertical dashed line at point 5% shows the two observations: rapid and slow decrease in timestamp. The threshold value (α) is fixed at 0%. Five simulations are performed on networks of size 500. Mean values and Standard deviation are reported.

Figure 10 (a)-(c) reports a typical examples of the evolution of the opinions versus the number of timestamps. The centrality used to weight the opinions is Page Rank and the size of the network is 500. In all cases, consensus is reached. Figure 10 (a) with fraction of top ranked nodes fixed at 0.4%

shows that consensus is reached in 31 timestamps. Figure 10 (b) with a fraction of top ranked nodes as 1% shows that the consensus is reached in 20 timestamps. Finally, Figure 10 (c) with fraction of top ranked nodes as 10% shows that the consensus is reached in 7 timestamps. As the value of β decreases, the number of timestamps to reach consensus increases. It appears that when the number of top rank nodes considered is small, extreme opinion values evolve slowly making the convergence more difficult to reach.

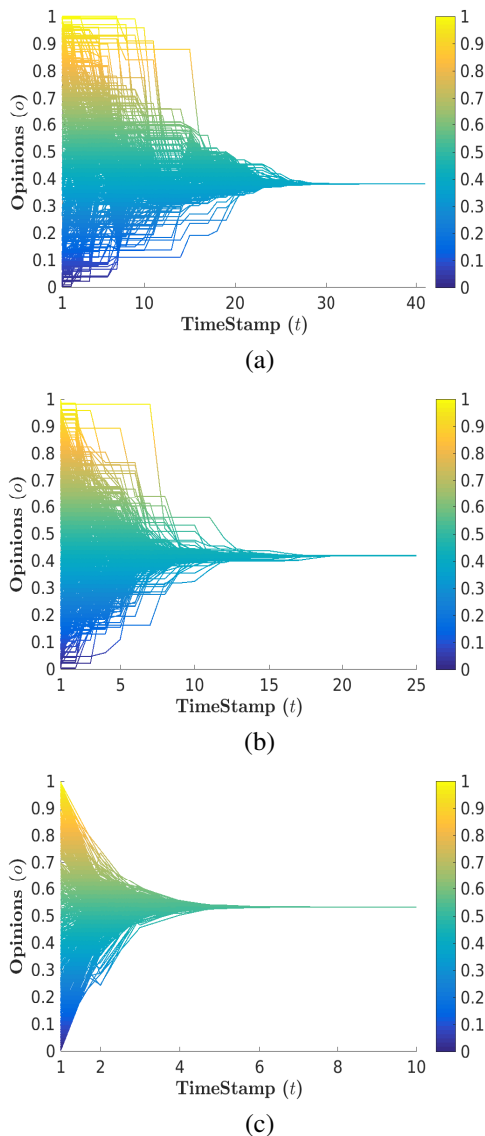


FIGURE 10. Evolution of the opinions reaching convergence versus the number of timestamp. The threshold (α) is fixed at 0%. The centrality used to weight the opinions is Page Rank on a network of size 500. Different values of the fraction of top ranked nodes β are considered. (a) Consensus with $\beta = 0.4\%$ (b) Consensus with $\beta = 1\%$ (c) Consensus with $\beta = 10\%$. Different colors represent different opinions.

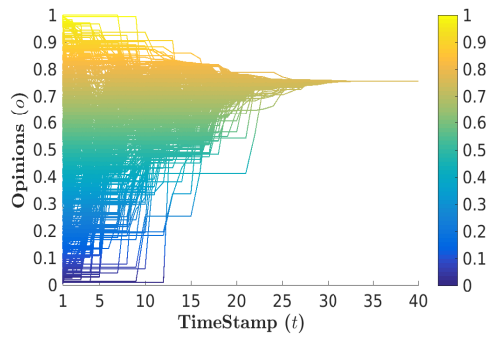
Figure 9 in blue and red color reports the results of the same set of experiments comparing page rank centrality with In-Degree centrality to weight the opinions instead of Page Rank. The threshold value is fixed at 0%. Similar behavior

for the number of timestamps against the fraction of top ranked nodes is observed in case of In-Degree centrality as compared to Page Rank centrality. One can also observe that the opinions converge in either the same or a higher number of timestamps when In-Degree centrality is taken into consideration instead of the Page Rank centrality. Results are quite similar for the higher values of top ranked nodes in comparison to its lower values. The number of timestamps at convergence is higher when the In-Degree centrality is considered compared to Page Rank centrality in the fraction of top ranked nodes interval [0%, 5%]. In the other intervals, the number of timestamps in which opinions converge is almost the same for In-Degree and Page Rank centrality.

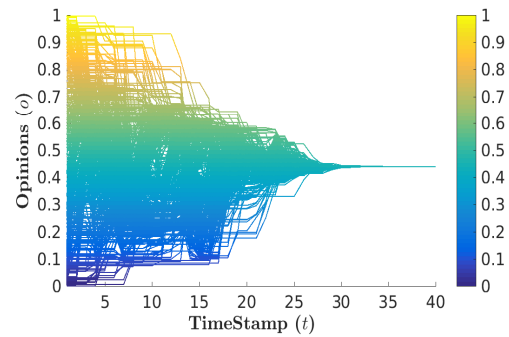
Figure 11 (a)-(c) reports a typical examples of the evolution of the opinions versus the timestamp. The centrality used to weight the opinions is In-Degree centrality and the size of the network is 500. Results are in the same vein that the ones obtained with Page Rank. Indeed, in all the cases, consensus is reached. Figure 11 (a) with fraction of top ranked nodes as 0.4% shows that the consensus is reached in 36 timestamps. Figure 11 (b) with a fraction of top ranked nodes as 1% shows that the consensus is reached in 25 timestamps. Finally, Figure 11 (c) with fraction of top ranked nodes as 10% shows that consensus is reached in 7 timestamps. As the value of β increases, the number of timestamps to reach consensus decreases. Note that the number of timestamps to reach consensus for In-Degree centrality is slightly higher in comparison to the Page Rank.

Figure 9 reports the results of the same set of experiments comparing Page Rank centrality and In-Degree centrality with Closeness Centrality to weight the opinions. The threshold value is fixed at 0%. A similar behavior for the number of timestamps versus the fraction of top ranked nodes is observed in case of the three centrality measures. One can also observe that the opinions converge in either the same or higher number of timestamps when Closeness centrality is taken into consideration instead of Page Rank. Results are quite similar for the higher values of top ranked nodes in comparison to its lower values. The number of timestamp at convergence is higher when the Closeness centrality is considered compared to Page Rank centrality in the fraction of top ranked nodes interval [0%, 5%]. In the other intervals, the number of timestamps at convergence is almost the same for both centralities. Results are quite similar when Closeness centrality is taken into consideration instead of the In-Degree centrality.

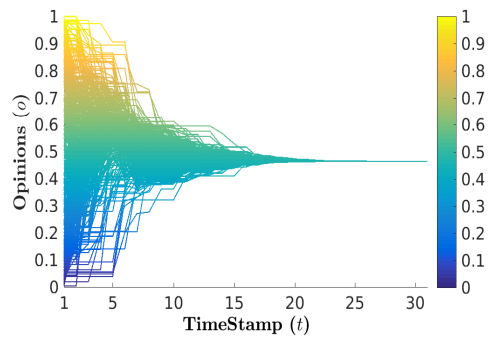
Figure 12 (a)-(c) reports typical examples of the evolution of the opinions versus the timestamp. The centrality used to weight the opinions is Closeness centrality and the size of the network is 500. Results are in the same vein that the ones obtained with Page Rank and the In-Degree centrality. Indeed, in all the cases, consensus is reached. Figure 12 (a) with the fraction of top ranked nodes as 0.4% shows that the consensus is reached in 36 timestamps. Figure 12 (b) with the fraction of top ranked nodes as 1% shows that the consensus is reached in 25 timestamps. Finally, Figure 12 (c) with the



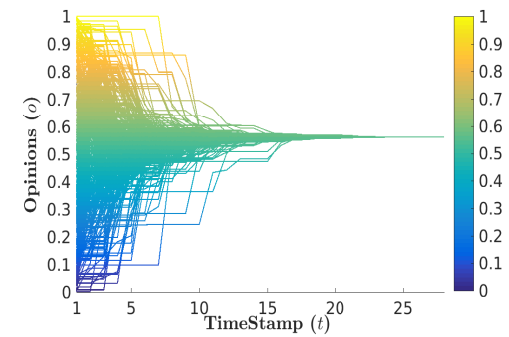
(a)



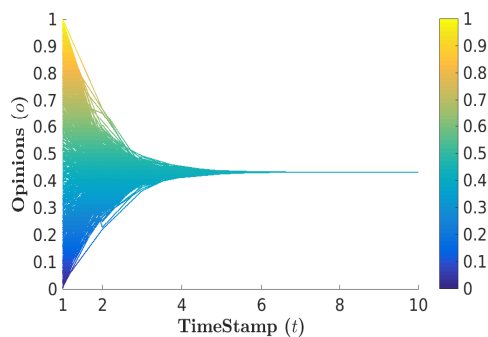
(a)



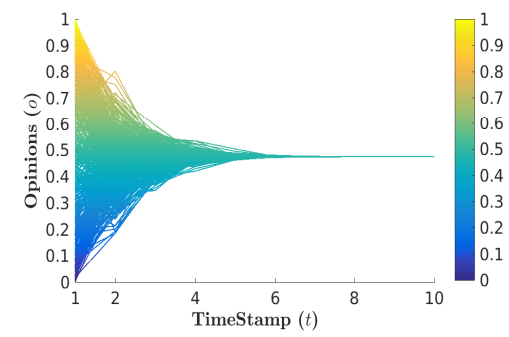
(b)



(b)



(c)



(c)

FIGURE 11. Evolution of the opinions reaching convergence versus the number of timestamp. The threshold (α) is fixed at 0%. The centrality used to weight the opinions is In-Degree centrality on a network of size 500. Different values of the fraction of top ranked nodes (β) are considered (a) Consensus with $\beta = 0.4\%$ (b) Consensus with $\beta = 1\%$ (c) Consensus with $\beta = 10\%$. Different colors represent different opinions.

FIGURE 12. Evolution of the opinions reaching convergence versus the number of timestamp. The threshold (α) is fixed at 0%. The centrality used to weight the opinions is Closeness centrality on a network of size 500. Different values of the fraction of top ranked nodes (β) are considered (a) Consensus with $\beta = 0.4\%$ (b) Consensus with $\beta = 1\%$ (c) Consensus with $\beta = 10\%$. Different colors represent different opinions.

fraction of top ranked nodes as 10% shows that consensus is reached in 7 timestamps. As the value of β increases, the number of timestamps to reach consensus decreases. Note that the number of timestamps to reach consensus for Closeness centrality is slightly higher than for the Page Rank but almost similar to the In-Degree centrality.

C. COMBINED INFLUENCE OF THE THRESHOLD AND THE FRACTION OF THE TOP RANKED NODES ON THE CONVERGENCE OF OPINIONS

Previous results show that we can consider three cases concerning the threshold values. Extreme small threshold values

in the range $[0, 0.005]$, medium threshold values in the range $[0.005, 0.3]$ and high threshold values in the range $[0.3, 1]$ can be considered. One can consider three situations concerning the fraction of top rank nodes: extremely small fraction of top rank nodes in the range $[0\%, 0.8\%]$, medium fraction of top rank nodes values in the range $[0.8\%, 5\%]$ and high fraction of top rank node values in the range $[5\%, 100\%]$. Therefore, in order to study the impact of the combination of both parameters we consider values in each range. A small threshold value (0.04), a medium threshold value (0.1) and a high threshold value (0.5), and a small fraction of top rank nodes (0.4%), a medium fraction of top rank nodes (1%) and

a high fraction of top rank nodes (50%) are considered.

1) Relating the combination of the threshold and the fraction of the top ranked nodes to the number of clusters at convergence

Whatever the values of threshold and fraction of top rank nodes, when both parameters values are different from zero, the opinions converge to a single cluster. The only case where opinion converge in more than one cluster is when the threshold values are in the range $[0, 0.2]$ keeping the fraction of top ranked nodes at 0%. For other values of threshold, keeping the fraction of top ranked nodes fixed at 0%, opinions converge in 1 cluster. Keeping threshold value fixed and considering any value of top ranked nodes, opinions converge in 1 cluster. This implies, the fraction of top rank nodes is the main parameter to converge the opinions towards consensus. This behavior is always observed whatever centrality measure is considered out of three.

2) Relating the threshold and the fraction of the top ranked nodes to the convergence time

The effect of both parameters on the number of timestamps at convergence is reported in Table 3. For a given a threshold value, we change the fraction of top rank nodes from lower to higher range. Until both parameters values are zero. There is no convergence and we denote the number of timestamp value as zero. For small fraction of top rank and small values of the threshold, convergence is very slow. For example, 180 timestamps are needed for a fraction of top rank nodes equal to 0.4% and a threshold value of 0.04. When the fraction of top rank nodes increases to a medium range value convergence accelerates, and this is also true when the threshold value increases from lower range to medium range values. Finally, when both parameters are in the higher range, convergence is always reached in five timestamps.

When only the threshold value (low range) is considered, few nodes change their opinions and opinions converge easily but in multiple clusters. But when the fraction of top ranked nodes is also considered in the low range, few nodes of high importance are very influential. In this case, opinions converge to a single cluster but this is done gradually in a higher number of timestamps. Indeed, different top ranked nodes go through different clusters of opinion with their neighbouring nodes before that all the nodes reach a global consensus. In other words, it takes time for all the nodes to share the same opinion. Furthermore, because of the threshold value, neighboring nodes other than the top ranked ones are also propagating their opinions. Hence, convergence of these opinions along with the opinions of top ranked nodes increase the time to converge. When the fraction of top ranked nodes increases further, the influence of small threshold values decreases and a smaller number of timestamps is required for convergence. This is because a higher number of important nodes share their opinions accelerating the convergence.

As the threshold value further increases to high range values, the effect of threshold value reduces the effect of

low fraction of top ranked nodes and a maximum of nodes are sharing their opinions and convergence is fast. If both parameters have low range values, opinion convergence is reached in a large number of timestamps because opinions do not propagate easily. If any one of the parameters has a high range value, opinion convergence accelerates because more nodes can share their opinions.

TABLE 3. Combined effect of the threshold value (α) and the fraction of top ranked nodes (β) on the number of timestamps at convergence. 1st row consists of β value. 1st column consists of α value. Every cell reports the number of timestamp at convergence for the corresponding α and β value considering the Page Rank centrality on a network of size 500.

α / β	0	0.4%	1%	50%
0	0	33	22	5
0.04	12	180	75	5
0.1	10	130	55	5
0.5	5	5	5	5

Table 4 reports the results of the same set of experiments using the In-Degree centrality. Globally, the similar behavior for the change in number of timestamps against threshold and fraction of top ranked nodes is observed as in Page Rank centrality. One can also observe that the convergence is always slightly slower in case of In-Degree centrality compared to Page Rank.

TABLE 4. Combined effect of the threshold value (α) and the fraction of top ranked nodes (β) on the number of timestamps at convergence. 1st row consists of β value. 1st column consists of α value. Every cell reports the number of timestamps at convergence for the corresponding α and β value considering the In-Degree centrality on a network of size 500.

α / β	0%	0.4%	1%	50%
0	0	36	25	5
0.04	13	185	75	6
0.1	12	135	60	6
0.5	5	5	5	5

Table 5 reports the results of the same set of experiments using the Closeness centrality centrality. Globally, the similar behavior for the change in number of timestamps against threshold and fraction of top ranked nodes is observed as in Page Rank centrality. One can also observe that the convergence is always slightly slower in case of Closeness centrality compared to Page Rank but almost similar to In-Degree centrality.

TABLE 5. Combined effect of the threshold value (α) and the fraction of top ranked nodes (β) on the number of timestamps at convergence. 1st row consists of β value. 1st column consists of α value. Every cell reports the number of timestamps at convergence for the corresponding α and β value considering the Closeness centrality on a network of size 500.

α / β	0%	0.4%	1%	50%
0	0	36	25	5
0.04	13	186	78	6
0.1	12	140	60	6
0.5	5	5	5	5

D. INFLUENCE OF THE NETWORK SIZE ON THE CONVERGENCE OF OPINIONS

In the initial experiments, we considered a network of size of 500. Now in order to analyze the influence of this parameter, we report the results of the experiments performed with a smaller network of size 100 and a bigger one of size 1000.

1) Relating the network size to the threshold at convergence

Figure 13 reports the evolution of the number of clusters versus the threshold value, keeping fraction of top ranked nodes fixed at 0% for the three size of networks under experiment. First of all, we can notice that we can consider three ranges of threshold value independently of the network size. For small threshold values, the number of clusters decreases slowly from the initial number of nodes. For medium threshold values, we observe a linear decreasing slope. As we use a log-log scale, it means that the number of clusters decreases exponentially with the threshold value. Finally, for high threshold values, the consensus is reached irrespective the network size.

Consider the case of small size network i.e. 100 nodes. Initially, when the threshold is 0, the number of clusters at convergence is equal to the total number of nodes i.e. 100. For small threshold values in range $[0, 0.01]$, convergence clusters for 100 nodes is less than 500 and 1000 nodes. For medium threshold values in range $[0.01, 0.3]$, convergence clusters for 100 nodes is equal to or little bit higher than 500 and 1000 nodes, but the difference is to small extent. For high threshold values in range $[0.3, 1]$, convergence clusters for 100 nodes is equal to 500 and 1000 nodes as consensus is reached. This observation can be explained as, at extremely small values of threshold, number of clusters in which opinions converge is near to 100 which is always smaller than values near to 500 clusters. As the value of threshold further increases, 100 nodes shared network split into multiple components and emerge in large number of clusters. As the threshold value further increases, irrespective of size of network, opinions reach to consensus state.

Figure 14 shows the number of timestamps at convergence versus the threshold value for networks of size 100, 500 and 1000. The fraction of top ranked nodes is fixed at 0%. For the case of small network size i.e. 100 nodes, and for small values of threshold i.e. in the range $[0, 0.002]$, opinions convergence is fast as the number of nodes that are changing their opinions is small (4 to 5) and this is done at early stage. When the threshold value increases the number of timestamps needed to reach convergence is large compared to bigger networks. Indeed, as there is a small number of nodes, many disconnected components arise while sharing the opinion. Sharing network (nodes and edges that exist according to the threshold value and fraction of top ranked nodes constitute the sharing network) remains disconnected for a long time even after addition and deletion of edges. As soon as it becomes connected, opinions converge. For the case of large size network i.e. 500, 1000, as the size of

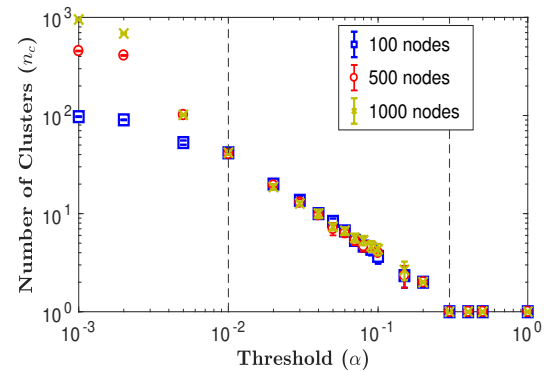


FIGURE 13. Plot representing the number of clusters of opinions versus the threshold value (α). Five simulations are performed on network of size 100, 500 and 1000 which are plotted in blue, red and green color respectively. Vertical dashed lines at point 0.001 and 0.3 shows the different threshold value ranges as low, medium and high. The fraction of top ranked nodes (β) is fixed at 0%. The centrality used to weight the opinions is Page Rank centrality. Mean values and Standard deviation are reported.

the network increases, the number of timestamps in which opinions converge increases but to a small extent.

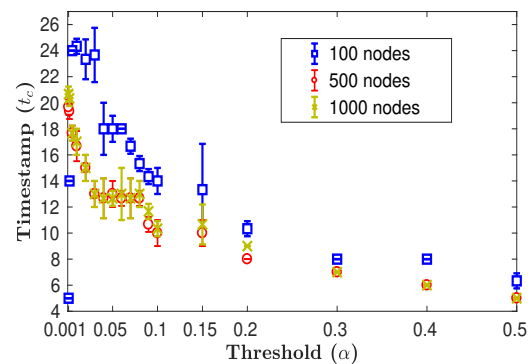


FIGURE 14. Plot representing the number of timestamps needed to reach convergence versus the threshold value (α). Five simulations are performed on network of size 100, 500 and 1000 which are plotted in blue, red and green color respectively. The fraction of top ranked nodes (β) is fixed at 0%. The centrality used to weight the opinions is Page Rank centrality. Mean values and Standard deviation are reported.

2) Relating the network size to the fraction of top rank nodes at convergence

Figure 15 shows the evolution of the number of timestamp needed to reach convergence versus the fraction of top rank nodes for networks of size 100, 500 and 1000. The threshold value is fixed at 0. Note that whatever the network size, the number of cluster at convergence remains the same, i.e. all the nodes reach a consensus (except for 0% top ranked nodes). As the size of network increases, the number of timestamps in which opinions converge decreases. As soon as sharing network becomes connected, opinions converge. In the case of top ranked nodes, sharing network of 100 nodes becomes connected comparatively late as compared

to the 500 nodes network. Hence, in 100 nodes network, opinions take more time to converge than 500 nodes network. Similarly, 500 nodes network takes more time to converge than 1000 nodes network.

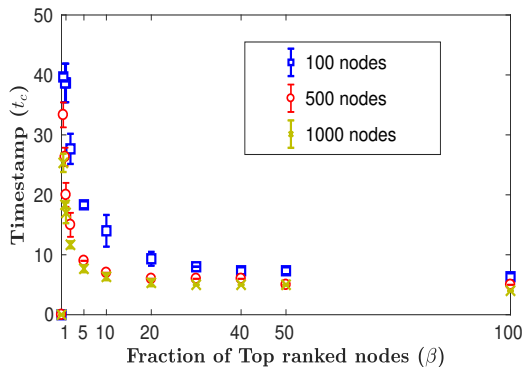


FIGURE 15. Plot representing the number of timestamps needed to reach convergence versus the fraction of top ranked nodes (β). Five simulations are performed on network size of 100, 500 and 1000 which are plotted in blue, red and green color respectively. The threshold value (α) is fixed at 0. The centrality used to weight the opinions is Page Rank centrality. Mean values and Standard deviation are reported.

3) Relating the network size to the combined effect of threshold and the fraction of top ranked nodes at convergence

In this section, we analyze the evolution of the number of clusters when both threshold and the fraction of the top ranked nodes are combined and the size of the network changes. First of all, one may notice that whatever the network size, the number of cluster at convergence remains the same, i.e. all the nodes reach a consensus. The only difference is how long it takes to reach convergence. Table 6 reports the number of timestamps needed to reach convergence for various values of the threshold and the fraction of the top ranked nodes for networks of size 100, 500 and 1000 respectively. When both parameters values are in the high range, convergence is fast. Indeed, 5 to 7 timestamps are needed for the all population to reach a consensus. In this case, we can conclude that convergence is almost independent of the network size. This is due to the fact that a large number of neighbors share their opinions. It takes a few more iterations for the smallest network because in some situations, isolated nodes may occur. The highest number of iterations needed to reach convergence is always observed when both parameters are in the smallest range, and it increases with the size of the network when the size ranges from 100 to 500.

Small number of iterations are required for networks of size 1000 as compared to networks of size 500. This is because network of size 500 converges quickly in comparison to 1000 nodes network in case of only threshold value and network of size 1000 converges quickly in comparison to 500 nodes in case of only fraction of top ranked nodes. But the effect of fraction of top ranked nodes is more in reducing the

timestamps for 1000 nodes network. Furthermore, increasing any of the parameter values improves the convergence rate. Indeed, more interactions allows to reach the consensus quicker.

VI. CONCLUSION AND FUTURE WORK

In this paper, we present and study a directed and weighted model for opinion dynamics on temporal networks. This model incorporates various parameters allowing the nodes to exchange their opinions with their neighbors. A node can be influenced by its neighbors if their opinions is not too far from its own opinion according to a threshold value or if these neighbors are globally influential in the network according to their centrality value. Three centrality measures are considered and compared in the proposed model: 1) Page Rank and 2) In-Degree centrality and 3) Closeness centrality. The first considers global information, the second considers local information and the third one is linked to the distance among the nodes. The underlying topology of the network is temporal and continuous opinions are considered. The threshold value, α and the fraction of top ranked nodes, β act as tuning parameters for the number of clusters in which opinions converge and for the number of timestamps needed to reach convergence. Various simulations and analysis are made to understand the effect of all the parameters. Opinions converge in more than 1 cluster only when low threshold values in the range $[0, 0.2]$ are considered without considering the fraction of top ranked nodes. As the threshold value increases in the given range, the number of clusters at convergence decreases. Otherwise, opinions converge in 1 cluster. As the threshold value increases, the number of timestamps to reach convergence increases and then it decreases. The number of timestamps in which opinions converge decreases with the increase of fraction of top ranked nodes. The number of timestamps to reach convergence remains same even if the fraction of top ranked nodes increases for the threshold value belonging to the high range. Similar scenario is observed while keeping the fraction of top ranked nodes fixed and changing the threshold value. Experimental results about the centrality comparisons (Page Rank, In-Degree and closeness centrality) show that the main differences are in the number of clusters at convergence and the number of timestamps needed to reach convergence. The local "In-Degree centrality" centrality measure takes more time to reach convergence with a larger number of clusters as compared to the global Page Rank centrality measure. Furthermore, it appears that opinions converge in an equal or larger number of clusters and the same or larger number of timestamps are needed to reach convergence when Closeness centrality is considered in comparison to Page Rank centrality. Results on networks of different size is analyzed. In small networks (100 nodes), convergence is slower compared to large size networks (500 and 1000). However, convergence is faster in network of size 500 as compared to networks with 1000 nodes when the fraction of top ranked nodes is fixed. In future work, we plan

TABLE 6. Combined effect of the threshold value α and the fraction of top ranked nodes β and the size of network on the number of timestamps at convergence. 1^{st} row consists of β value. 1^{st} column consists of α value. Every cell reports the number of timestamp at convergence for the corresponding α and β value considering the Page Rank centrality on networks of size 100, 500 and 1000 nodes.

α / β (Size of Network)	0.4% (100/ 500/ 1000)	1% (100/ 500/ 1000)	50% (100/ 500/ 1000)
0.04	63/ 190/ 170	50/ 75/ 70	7/ 6/ 5
0.1	49/ 135/ 135	48/ 68/ 56	7/ 6/ 5
0.5	7/ 5/ 5	7/ 5/ 5	7/ 5/ 5

to extend the model in various direction in order to increase its effectiveness. Instead of random opinion assignment to nodes, we plan to consider biased assignment. The impact of addition and deletion of nodes in the network need also to be investigated. Link weights may also be considered to contribute in the opinion sharing process [34]. Trust factor among the nodes may be used for defining the link weights. Instead of a fixed threshold value for every pair of node and at every timestamp, a variable threshold value can be considered for different pair of nodes which is also a function of time.

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