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# Stochastic Joint Replenishment Planning Optimization with an Iterated Local Search

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## 1. Context and Motivations

In this extended abstract, we discuss optimized operations of supply chains dealing with multiple items. Examples of such supply chains are omni-channel retailers, supermarkets and service supply chains, etc. Whenever more than one item is ordered from one supplier or they share the same transportation vehicle, *first order interactions* take place between the items. First order interaction can be due to a joint (common) fixed cost in addition to individual fixed costs. When the demands are known with certainty the problem is a deterministic Joint Replenishment Problem (JRP). On the other hand, if the demands are considered stochastic, it is called the stochastic joint replenishment problem. Other costs involved are the inventory holding costs and the shortage costs.

The periodic deterministic JRP is strongly NP-hard for infinite horizon and NP-hard for finite horizon (Cohen-Hillel and Yedidsion, 2018). No such proof is available yet for the stochastic JRP. However, it is commonly regarded as very hard due to its huge solution space. Structural results for the stochastic JRP show that the optimal solution cannot be expressed as a simple replenishment policy even for the two-item case (Ignall, 1969). Therefore, many authors have assumed some simple structures such as  $(s, c, S)$  (Johansen and Melchior, 2003),  $(s, c, d, S)$  (Feng et al., 2015) and Periodic Review  $(s, S)$  (Viswanathan, 1997), to facilitate practical implementation ( $S = \text{Order up to level}$ ;  $s = \text{Reorder level}$ ;  $c, d = \text{Can order levels}$ ). Even if the adopted method follows one of the above policies, any change in problem parameters requires re-evaluation of policy parameters. Such a process is also time consuming.

The real-world applications of stochastic JRP are numerous. Usually, for the stochastic JRP, replenishment planning solutions use heuristics that are independent for each item under the JRP or follow one of the joint policies mentioned in the previous paragraph. These methods are not optimal for the global problem. Since the stochastic JRP are very hard to solve optimally, we propose to use a stochastic local search (SLS) approach to find good quality solutions in reasonable time. The ordering decisions are then implemented under a rolling horizon planning environment. The quality of a solution obtained using our proposed methodology is always better than what can be obtained using item level heuristics because, it uses the item level heuristics for generating the initial solution and then conducts further improvements. We also conduct numerical studies to attest its performance against the joint policies.

## 2. Methodology

In contrast to the general assumption in the literature, non-stationary demand is encountered widely during application. Hence, our methodology follows a dynamic ordering strategy, i.e. orders are calculated during each ordering epoch. We first generate the initial solutions using different heuristics that are independent for each item then, use an iterated local search for further improvement. The details are explained in the following sections.

### 2.1. Representation of the Solution

The stochastic JRP is a multi-period inventory optimization problem involving multiple items. At the beginning of each review period (ordering epoch), the inventory levels of all items are recorded and order quantities for all items are decided. Under dynamic ordering strategy, the optimal order quantities during an ordering epoch can be expressed as a vector  $\mathbf{Q}$ , where each element of the vector corresponds

to the order quantity of the respective item. The order quantities can only take zero or positive integer values. Therefore, a solution of the stochastic JRP is a set of dynamically decided multi-item orders. It is optimal if it *minimizes the total expected cost over a period* for non-stationary demand or it *minimizes the expected cost per period* for stationary demand. Our proposed methodology is applicable in the cases of both stationary and non-stationary demands. The proposed SLS approach is presented next.

## 2.2. Proposed SLS Approach

### Steps of an iterated local search (ILS) for the stochastic JRP

1. *Initialization* – The proposed iterated local search uses either of the two following initializations: Random initial solution and initial solution obtained with a combination of single item heuristics.
2. *Neighborhood* – The second step involves a local search that uses three neighborhood operators: Positive/negative increment to one element,  $k$ -exchange and synchronized neighborhood operator (discussed later).
3. *Perturbation* – Multiple applications of the neighborhood operator to one or more randomly selected element(s).

The application of a neighborhood operator to a solution affects the order quantities for different items. The fitness of the new solution can be computed incrementally, and therefore is not time consuming. The performance of the above ILS approach depends significantly on the underlying neighborhood relation, and specifically, on the size of the neighborhood. For the stochastic JRP with  $N$  items, the size of the neighborhood for a  $k$ -exchange operator is in  $\mathcal{O}(N^k)$ , and for the simple increment/decrement (for example  $+1/-1$ ) the number of neighbors is  $2N$ .

### Synchronized Neighborhood Operator (SNO)

The idea behind SNO is derived from the analysis of structural properties of near optimal solutions for the stochastic JRP. It is observed that the near optimal solutions tend to generate orders for a group of items during each ordering epoch. Under stochastic environment it is impossible to determine the exact time until which the inventories would last. However, the property of a near-optimal solution is to always ensure that the inventories for a maximum number of items last until equivalent times. In other words items are ordered in groups to reduced the share of joint ordering cost, and in such a way that they are also ordered in group in the future.

The SNO works on the above principle. For  $N$  items there are at least  $q_{min}^N$  possible solutions, where  $q_{min}$  is the minimum of all possible number of orders for different items. The SNO provides a set of neighbors where a group of items has non-zero orders and the order quantities are synchronous to ensure that the next order is also grouped. Therefore, it eliminates the unnecessary searches in sub-optimal solution space.

## 2.3. Results

Preliminary results suggest further improvement of 2-7% over the best available joint policies in the literature for the two-item problem. More elaborate results will be presented during the workshop.

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