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Diagnosis of basic mathematical competencies in years 8 and 9

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In this paper we present a newly developed test with easy tasks which assesses basic mathematical competencies that are essential for vocational training. This test is used in years 8 and 9 in Germany. From both a content-oriented and a process-oriented perspective, preliminary test results reveal a substantial lack of these competencies in all academic tracks, and differential effects can be found with respect to year and class that need to be examined beyond curricular analyses.

Keywords: mathematics tasks, diagnostic test, assessment, basic competencies, vocational training.

Many professions require at least basic mathematical competencies to understand or to implement work processes, and such competencies are a prerequisite to successfully begin vocational training. Consequently, in mathematics these competencies should have been acquired beforehand by means of working on tasks. So far and especially in international comparative tests, the relation between tasks, basic mathematical needs and vocational training has hardly been examined as the few initial approaches to adult education primarily focus on social participation and necessary mathematical understanding. In this paper we present our task-based approach to testing secondary students’ basic mathematical competencies and raise ideas how to support these students adequately and in time.

The role of tasks in Mathematics

Especially in Mathematics tasks are essential and thus they are being used from various perspectives in both research and teaching. In any case they require engagement with a certain mathematical content (e.g. Christiansen & Walther, 1986), and they are understood here as orally or written set assignments to students to carry out a subject-oriented activity (Drüke-Noe et al., 2017).

In research, tasks can serve as documents of mathematics instruction. This applies e.g. to the project COACTIV in which tasks are used not only to assess teachers’ professional knowledge but also to get insight into the level of cognitive activation potentially realized in teaching or assessing mathematics (Kunter et al., 2013). Tasks also form the linking element between curricular standards and the professional knowledge of teachers on the one hand and the aimed-at knowledge, skills and competencies of students on the other hand (Neubrand, Jordan, Krauss, Blum & Löwen, 2013).

Furthermore, tasks are essential in tests to assess the achievement of learning goals. For this purpose, oral and especially written tests are regularly set in Germany. These tests, which are either set internally or externally, comprise a range of tasks of a certain cognitive demand to examine competencies either formatively or summatively. Due to their backwash effect on teaching as well as their impact on the students’ further educational career, centrally set external examinations at the end of various stages of education are quite prominent examples of summative written tests. Interestingly though, in several European countries as well as in different federal states of Germany such centrally set exams are often of a relatively low level of cognitive demand and cover only a limited range of competencies (Kühn & Drüke-Noe, 2013; Drüke-Noe & Kühn, 2017). This also
applies to internally written mathematics tests in Germany, so called class tests (Drüke-Noe, 2014). From a normative perspective, however, adequate compilations of tasks in tests are crucial as they “summarize the core components of an instructional unit and ultimately specify the level of mathematical achievement that teachers require” (Neubrand et al. 2013, p. 128) and thus make social expectations transparent for both teachers and students.

It can be concluded that tasks are “actively configurable content-related and didactic elements that serve to structure mathematics instruction” (Neubrand et al., 2013, p. 126; see also Zaslavsky, 2007, p. 434) that “provide the basis for students’ cognitive activities” (Neubrand et al., 2013, p. 127) and therefore play an essential role in teaching, learning and assessing mathematics.

**Tests and Basic Competencies**

In Germany, as a consequence from only mediocre results in international comparative studies like e.g. TIMSS and PISA, educational standards have been implemented in almost all subjects in the last fifteen years. In the course of this, the focus of attention shifted towards regular formative examinations of learning goals and competencies, and several centrally set formative tests were introduced nationwide in Germany that serve as diagnostic instruments. These tests are designed on the basis of the German Educational Standards and are in accordance with the standards’ conception. Therefore, their tasks cover all five content strands (quantity, measuring, space and shape, change and relationships, uncertainty and statistics), all six competencies (argumentation, problem solving, modeling, use of representations, working technically, communication) and all three levels of cognitive demand (low, medium, high). Two different kinds of tests are set: A first kind is compulsory comparative tests which are set in the years 3 and 8 (9-year- and 14-year-olds) which are administered as well as corrected within schools. The test results are to provide diagnostic information in which areas both teaching and learning should be improved to prepare students for a successful completion of compulsory education one or two years later at ISCED-level 2. A second kind of tests is centrally set every six years in the years 4 and 9. Its conception is standard-based, too, but this nationwide test is only taken by a representative sample of students and its results are only used by the education administration to monitor the school system with respect to the successful achievement of the educational standards. For both years 4 and 9 and especially for the latter one results reveal regularly that on average one quarter of all German students miss minimal educational standards (Pant et al., 2013). Very similar findings come up in a German longitudinal study on Bavarian secondary students of different school tracks (vom Hofe & Hafner, 2009) as well as in regular international studies like PISA (e.g. Neubrand & Neubrand, 2004).

To summarize, there is much empirical evidence for the mathematical achievement of year 9-students that firstly implies that 16-year-olds, who leave school at the end of lower secondary education and are about to enter a phase of vocational training, lack substantial basic skills such as a reflective use of knowledge and abilities. Secondly, these students presumably lack not only the basis for an understanding of routines beyond their mechanical application but also knowledge of problem solving activities. Consequently, it seems not only necessary to diagnose such basic competencies well beforehand but also – which is even more important – to support these low-achievers in time both on a class-level and individually, too. More efforts need to be taken so that in
the future less students fail minimal standards since a successful achievement of at least minimal standards is a precondition to successfully begin (and complete) a vocational training after the end of year 9 (or year 10).

These necessities form the basis of our task-based project „Diagnostic scaffolding of basic mathematical competencies for vocational training“. In this project we assess basic mathematical competencies well before students leave school. We test entire classes and can provide teachers with quantitative and qualitative feedback on the test results. Teachers can use this feedback to support their students to better achieve basic competencies by the end of compulsory education.

**Basic Competencies and the Start of Vocational Training**

Based on the German Educational Standards we designed a test that only assesses basic mathematical competencies relevant for vocational training. As an adequate selection of tasks for such a test substantially relies on both an understanding of basic competencies as well as of what is necessary to successfully start a vocational training, we briefly outline essential elements of both here.

In Germany, there are different empirical and normative approaches to defining basic competencies for the end of compulsory education, which is usually the end of lower secondary education irrespective of successfully obtaining a formal school leaving certificate. However, there is no consensus as to whether basic competencies characterize what is really necessary to be prepared for a life as a citizen, for a successful start and/or completion of a vocational training. It is equally diverse who defines basic competencies; is it teachers or researchers or those who continue working with these students after a certain stage of education? Some other task-based approaches to defining basic competencies which are relevant for our one and only focus on the end of lower secondary education are briefly outlined in the following: One primarily normative concept by Sill and Sikora (2007) considers empirical results from national comparative studies and defines basic mathematical competencies needed for further education, everyday life and society. It comprises three so-called competency levels that distinguish whether certain knowledge and abilities should be immediately and automatically applicable or be easily re-activated or be only exemplary and episodal. A second concept is essential for our approach as it is explicitly based on the German Standards. It combines a normative with an empirical approach and defines basic mathematical competencies as those

which all students of all educational levels must at least and permanently have at the end of compulsory education. These competencies are a prerequisite for sovereignly mastering everyday life and actively participating as politically mature citizens in social and cultural life. These competencies are equally a prerequisite for a successful and promising start into a vocational training and a pursuance of one’s profession. Those who do not have these basic competencies will presumably not sufficiently be able to get along in these situations without help. These students must timely and particularly intensively be supported (Drüke-Noe et al., 2011, p. 8).
A third German and predominantly empirical approach is the “Model of Competency Levels for the Educational Standards for a qualified school leaving certificate at the end of year 9 and at the end of year 10” which was passed by the Standing Conference of the Ministers of Education and Cultural Affairs. Based on results from several large scale studies and applying probabilistic test theories which relate task difficulties to students’ abilities, this model is based on a competency scale which distinguishes six competency levels. These levels describe students’ abilities on the basis of task properties. The lowest two levels, which we only focus on here, define the minimal standard as a set of competencies that all students must have achieved by the end of a certain educational stage. Students at or below this minimum standard can only comprehend or apply the most simple standard argumentations or problem solving activities, use simple models or well-known visualizations, apply one-step routines or extract single pieces of information from simple texts (Kultusministerkonferenz, 2011, p. 29ff).

Several other international approaches to basic mathematical competencies also relate structural errors to procedural processes: Brown and Burton (1978) already do so in their “Diagnostic Models for Procedural Bugs in Basic Mathematical Skills” and stress the close relation between basic skills, procedural and conceptual knowledge. In his analyses of structural errors with respect to procedural knowledge Wu (1999, p. 1) stresses that “in mathematics, skills and understanding are completely intertwined”. Ritter-Johnson, Siegler and Alibali (2001, p. 346) equally claim that “conceptual and procedural knowledge influence one another […] and develop iteratively, with increases in one type of knowledge leading to increases in the other type of knowledge”.

Consequently, a traditionally rather content-focused perspective on teaching (and testing) ought to be widened to a more comprehensive one which considers both content and processes not only when putting curricular regulations into practice but also when teaching and testing basic competencies which – amongst others – are necessary to work successfully on mathematics tasks. With respect to vocational training, Weißeno and colleagues (2016, p. 3) additionally point out that “mathematical skills play an important role in developing job specific competencies in a number of occupations that require intermediate qualifications.” Thus, learners do not have to be experts in the field of mathematics, but they must be able to implement the necessary basic skills, as well as be able to critically reflect upon and to apply processes in the course of solving tasks to correspond with the concept of “Higher General Education” as developed by Fischer (2001).

In our project, we designed a task-based test to assess students’ basic mathematical competencies relevant for vocational training. Due to our test conception we can derive information on these competencies from both a content- and a process-oriented perspective. Furthermore, this information and deeper test analyses allow us to deduce a specific feedback to support teachers and students.

**Test Conception and Task Features**

To both evaluate basic competencies and consider a prospective ability to take up vocational training, our test addresses students who aim at getting a qualified school leaving certificate at the end of year 9 or at the end of year 10 (ISCED-level 2). To allow for sufficient time for
compensation before leaving school, both versions of this test (see below) can be taken well before, i.e. in year 8 (14-year-olds) and/or in year 9 (15-year-olds) and can be set in all school tracks.

Since the test’s conception is based on the Educational Standards, it covers all five content strands, all six competencies and all three levels of cognitive demand (see Table 1). All tasks were selected correspondingly and thus allow analyses from a content- and a process-oriented perspective. Based on cognitive analyses we can describe the content addressed and the competencies needed when solving a task, which we briefly outline below by means of three examples. As only basic competencies are to be assessed, only relevant content areas of the curriculum of the preceding years 7 and 8 are tested. This curricular validity has been approved by experts. Almost all tasks are presumed to be empirically easy (i.e. at least 80% correct answers are expected) as they were developed in accordance with characteristics of the two lowest levels of the competency level-model (KMK, 2011). However, from our perspective relevant basic competencies not only comprise the ability to apply certain routines and procedures to show how something works but also to use them critically and/or to explain why something works: Therefore, we intentionally included some presumably empirically more difficult tasks in the test and only for these we anticipated less than 80% correct answers.

### Figure 1: The tasks “Triangle” (left), “Parallelogram” (middle) and “Savings book” (right)

Three tasks (see Figure 1) are to illustrate our approach: The task „Triangle“ is cognitively easy and it tests how something works. The representation immediately leads to a procedure to calculate the area by means of the given lengths of the triangle’s base and height. This task ought to be empirically easy as calculating the area of this elementary figure requires just one step with small numbers. The task “Parallelogram” represents those that test why something works. The rule to determine the area of this quadrilateral is explicitly named. Using the representation students need to develop a strategy to explain why the area can be calculated by means of this rule. According to the competency level-model (KMK, 2011) it must be assumed, though, that this is no empirically easy task as it requires an argumentation. In the task “Savings book” students must initially extract relevant information from a simple text (communication) to then design a strategy and apply a basic mathematical model to calculate the amount of interest (problem solving, modeling, working technically).

<table>
<thead>
<tr>
<th>Test version</th>
<th>Competence</th>
<th>Level of cognitive demand</th>
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<tbody>
<tr>
<td>Test version</td>
<td>Argumen-</td>
<td>Problem solving</td>
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<td></td>
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<td></td>
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<tr>
<td>B</td>
<td>4</td>
<td>2</td>
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</table>
Table 1: Numbers of tasks that require one or more of the six competencies and numbers of tasks of the given level of cognitive demand

The two test versions (B: basic version, E: extended version) comprise 15 tasks each with three tasks per content strand that all require open answers. Eight tasks are identical in both versions and almost all the remaining seven ones are parallel versions of each other which differ either only as regards their presumed task difficulty or as regards the track-specific curriculum. The students have 45 minutes to work on the test, and they may only use pens, a ruler and a device to measure angles. Before the test was finally used, it was tried and tested beforehand and improved further.

By means of a coding scheme and applying a dichotomous coding with double digits, different types of correct, partly correct or wrong answers are identified. Further analyses of the test results allow us to provide teachers with quantitative feedback (charts on types of answers with respect to content strands) and with qualitative feedback on missing basic competencies. Thus, this diagnostic test with its combined content- and process-oriented design and the subsequent detailed feedback may not only close a gap PISA leaves but also provides teachers with class-specific information on (missing) basic competencies of prospective school leavers. Teachers can then use this detailed information to develop suitable interventions to better support their students to acquire basic competencies.

Sample and preliminary results

The test was set to a non-representative sample of students (N=367) in 16 academic and nonacademic classes (see Table 2) from two German states (Baden-Wurttemberg, Rhineland-Palatinate). The test took place at the beginning of the second semester in the school year 2016/17.

<table>
<thead>
<tr>
<th>Test version</th>
<th>Academic track</th>
<th>Nonacademic track</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>---</td>
<td>Baden-Wurttemberg: 2 classes in year 8 (N=52), 2 classes in year 9 (N=45)</td>
</tr>
<tr>
<td>E</td>
<td>Rhineland-Palatinate: 2 classes in year 8 (N=52), 8 classes in year 9 (N=189)</td>
<td>Baden-Wurttemberg: 2 classes in year 9 (N=29)</td>
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</table>

Table 2: Sample of classes and students that took the diagnostic test

In accordance with the standard-based test conception we present preliminary findings firstly from a content-oriented perspective and then from a process-based one. Irrespective of track and year, hardly one quarter of all students (23 %) correctly solved both tasks on the area of elementary figures (see Figure 1), and almost equally many students (26 %) solved none of these two correctly. It is interesting to note that the task „Triangle“, which rather focuses on procedures (knowing how), was solved considerably better by students from the nonacademic track (nonacademic track: 76 %, academic track: 64 %). However, considering both this task’s features and the predominance of routines in Germany, a much higher percentage of correct answers could have been expected. Less surprising, far more students from the academic track were able to solve the task „Parallelogram“ (nonacademic track: 6 %, academic track: 42 %) which rather focuses on understanding (knowing why). As expected, in both tracks students of year 9 solved both tasks better than those in year 8 but
from a content-based perspective the overall results of both tasks are rather astounding as the area of such basic figures is an integral part of every curriculum. Error analyses reveal that students typically fail when distinguishing formulae (here: area of a rectangle and that of a parallelogram) or when explaining a rule (see “Parallelogram”) although it is explicitly given here.

For analyses from a process-oriented perspective we focus on modeling here. Detailed analyses of the students’ work rely on steps of the modeling cycle by Blum and Leiß (2005). As expected and irrespective of track and year, in all six modeling tasks students primarily find it difficult to understand the real situation and then to constitute a situation model. Furthermore, validating a result proves to be another difficult step even for students of the academic track. Surprisingly, in year 9 only 39.2 % of the students in the academic track solved “Savings book” correctly (non-academic track: 20.7 %) and many typically failed when validating an even obviously wrong result like e. g. an annual amount of interest of 9000 €. Based on detailed analyses of this kind we provide teachers with both quantitative and qualitative feedback on their students’ basic competencies. The qualitative feedback is based on cognitive analyses of all tasks which serve as a basis for class-specific descriptions of the most frequent mistakes per task. All these descriptions help to characterize existing and lacking basic mathematical competencies within each class and can support teachers in their successive work.

**Summary and Implications**

The preliminary results of our test correspond with results from representative national and international studies. Beyond this, our test which is curricular valid and addresses basic mathematical competencies relevant for vocational training once more highlights a discrepancy: Though these competencies are broadly considered to be essential, considerably high proportions of students can neither use nor apply them and thus lack crucial preconditions to successfully start a vocational training and succeed in their later professional careers. This leaves many things to do: From both a scientific and a diagnostic perspective, further analyses beyond curricular ones are necessary to explain unexpected differential effects as regards tracks, years and classes. Secondly, far greater efforts are needed to turn all students into citizens of age (cf. Fischer, 2001) and to provide them with basic mathematical competencies. We are currently developing adaptive lesson material to be implemented (and evaluated) in teaching. This material specifically addresses students’ typical difficulties as regards acquiring and keeping basic mathematical competencies. The material aims at supporting students to better solve tasks on how something works as well as those on why something works, which means that all students should be able to write down argumentations or to validate results to name just a few examples. Last but not least it will be necessary to improve not only the tasks but also the kinds of feedback we provide the teachers with.

**References**


