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# Classroom assessment tasks and learning trajectories

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*Abstract. One of the purposes of assessment is to inform the classroom teacher about students' current understanding in order to improve the teaching and learning processes. Learning trajectories present a developmental progression towards increasing understanding of mathematical ideas and are commonly found in curriculum materials to assist teachers in planning instruction. Assessing students' learning along the trajectory could serve as a mediator for adjusting the pacing and the selection of opportunities during the classroom enactment. We propose a framework for designing different types of assessment tasks to elicit evidence about how students respond to the mathematical ideas presented in the trajectory. We illustrate the application of the framework by drawing on a learning trajectory of fraction division for sixth grade students and discuss how the enactment of the trajectory in classroom could be adjusted to students' current understanding.*

*Keywords: Classroom assessment, learning trajectories, assessment tasks.*

## Introduction

Research evidence indicates that assessment could enhance learning, double the speed of learning and reduce the achievement gap between low and high achievers (e.g. Black & Wiliam, 1998; Kluger & DeNisi, 1996). The aforementioned is commonly referred to as formative assessment or assessment for learning. Even though there are differences in the use of these terms (Swaffield, 2011), their focus is on providing feedback to the teacher and student to improve the teaching and learning. In this paper, we use the term “classroom assessment for learning” to highlight that the classroom is the context in which assessment takes place and that the purpose is to elicit evidence about students' current understanding for making instructional decisions to enhance the learning processes.

The learning trajectory can affect and enhance the classroom assessment for learning (Ebby & Petit, 2018; Sztajn, Confrey, Wilson & Edgington, 2012). According to Clements and Sarama (2004), a learning trajectory includes the learning goal, the developmental progressions of thinking and learning, and the sequence of instructional tasks. A learning trajectory takes into account theoretical and empirical data to form a learning route that involves processes to develop increasing sophistication in understanding, which is linked with an instructional sequence (Heritage, 2008; Clements & Sarama, 2004). Classroom assessment could then be closely linked with these cognitive pre-designed learning trajectories as they inform the teacher about the desired learning goal and a progression towards developing understanding. Thus, they can help teachers interpret how students respond to these goals and how they move along the progression path by making informed decisions (Ebby & Petit, 2018).

## Literature review

Assessment tasks could provide information about where students stand regarding the learning trajectory and students' levels of understanding and misunderstanding (Bennett, 2011). The design of assessment tasks has been mainly an issue of discussion for high-stake assessments and

standardized tests but it is time to include clear descriptions of the form and key features of classroom assessment in order to enrich the current research base (Kingston & Nash, 2011).

Different types of tasks elicit evidence about various facets of students' understanding. Existing frameworks of assessment tasks rely on categorizations based on the format of tasks or the level of processes students are expected to engage with. Phelan, Choi, Vendlinski, Baker and Herman (2011) categorized tasks into basic computational tasks, partially worked examples, word problems, graphic problems and explanation tasks. Webb, Alt, Ely and Vesperman (2005) and deLange (1999) identified levels of assessment tasks. Webb et al. (2005) suggested that tasks could engage students in recall of information, skills and concepts, strategic thinking, and extended thinking. deLange (1999) categorized tasks into those of reproduction, procedures, concepts and definitions; connections and problem solving; mathematization, mathematical thinking, generalization and insight.

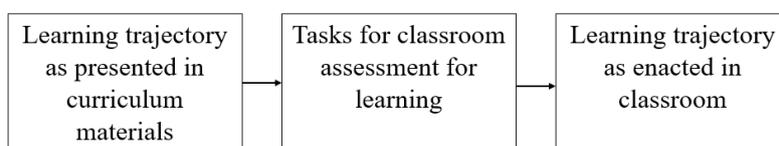
It seems that there is consensus that assessment tasks should elicit evidence from students' engagement with recall procedures to complex reasoning. However, the field lacks a systematic way for making connections between assessment tasks, increasing complexity and learning trajectories. In this paper, a framework is presented towards this aim in order to contribute to the discussion of learning trajectories and assessment for learning. The framework describes types of tasks for classroom assessment for learning which are linked with the learning trajectories aiming (1) to elicit evidence about how students respond to the mathematical ideas presented in the learning trajectory and (2) to inform as well as adjust accordingly the enactment of the learning trajectory as it unfolds in classroom.

Sztajn et al. (2012) mentioned that teachers make sense of students' ideas and respond accordingly by relying on students' reasoning when they follow a learning trajectory since they help teachers identify common misconceptions beforehand. Then, the levels of learning "could serve as reference points for assessments designed to report where students are along the way to meeting the goals of instruction and perhaps something about the problems they might be having in moving ahead" (Daro, Mosher & Corcoran, 2011, p. 29). In addition, it is critical to interpret appropriately the evidence for planning instructional adjustments. The majority of studies, so far, do not elaborate on the possible instructional adjustments based on the evidence from various assessments approaches (McMillan, Venable & Varier, 2013). Instructional adjustments include differentiation for remediation, reteaching using different strategies and changing the pacing of instruction (Hoover & Abrams, 2013).

## **Development of framework**

The framework was formulated by relying on existing categorizations of tasks (i.e., deLange, 1999; Webb et al., 2005), international studies (e.g. TIMSS, PISA) and online resources (e.g. [map.mathshell.org](http://map.mathshell.org)). We identified key points of learning, critical features of tasks, ways to capture increasing complexity and then designed tasks for grades 4 to 6. In total, 161 tasks were piloted in real classroom settings during a school year. Based on the feedback from teachers' comments and the coding of students' responses, the types of tasks were refined through several cycles of revisions. In this paper, we present only assessment tasks for a selected learning trajectory for sixth grade.

The framework describes different types of tasks for the purpose of classroom assessment for learning and presents the role of learning trajectories in designing assessment tasks and in planning instructional adjustments. Learning trajectories are foundational since assessment elicits evidence of how students' learning develops towards the expected goal and supports teachers in deciding for their next teaching actions (Heritage, 2008). A learning trajectory as presented in the curriculum could inform the development of assessment tasks and the evidence from the assessment tasks then shapes the enactment of the learning trajectory in classroom (Figure 1). The fact that teachers in the Cypriot educational context follow closely the learning trajectories as presented in the curriculum materials and particularly in textbooks facilitated the process of designing assessment tasks that were aligned with the learning trajectories by identifying key points in learning. However, the classroom enactment is not expected to be identical to the intended curriculum. Indeed, the evidence from students' responses in the assessment tasks could contribute in adjusting the enactment in classroom to students' ongoing needs.



**Figure 1: Classroom assessment tasks and learning trajectories**

We define a task for classroom assessment to consist of a single request of what students are expected to do and the necessary information to respond to that request. Three different types of tasks were formed and two task features were selected. Table 1 presents the three types of tasks with their respective description. The different tasks aim to elicit evidence regarding students' progression along the learning trajectory and be aligned to the increasing expertise and sophistication as expected by the trajectory. These tasks are not intended to be seen as consecutive and linear steps of learning (e.g., it is not assumed that necessarily “apply and connect” tasks will be used after “recall and reproduce”). These tasks could be used in classroom according to how the learning trajectory unfolds and inform of the ways students develop increasing expertise.

<b>Type of Task</b>	<b>Description of Tasks</b>
Recall and Reproduce	Students are able to respond to these tasks because they can use taught procedures and technical skills.
Apply and Connect	Students are able to respond to these tasks because they can apply the mathematical ideas in various contexts and make connections between representations and mathematical ideas.
Reflect and Generate	Students are able to respond to these tasks because they can investigate mathematical ideas, structures and contexts, and develop their own mathematical products.

**Table 1: Types of classroom assessment for learning tasks**

Familiarity and complexity are two features that characterize the tasks. The familiarity of the task is determined by students' previous experiences with the requests of the task and the information provided. The complexity of the task is determined by (a) the number of steps to complete the

assessment task, (b) the amount and nature of information that students need to handle, and (c) the possible deviations that alter the complexity according to the mathematical topic (e.g. the place of the unknown in an equation, the choice of numbers).

### Application of the framework

In this paper, we discuss the application of the framework to design tasks for a learning progression on fraction division, which serves as the context to present examples of the three different tasks for assessment for learning. Fraction division was selected since on one hand it is not a difficult topic when students work only procedurally to invert and multiply. On the other hand, it is conceptually demanding since fraction division could be represented using various ways and connected with other mathematical ideas (Li, Chen & An, 2009).

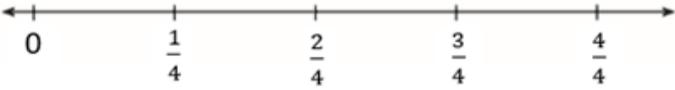
There are different approaches in the learning trajectories in textbooks around the world (e.g., Li et al., 2009). Only the trajectory found in the Cypriot curriculum materials is presented since it shaped the design of assessment tasks. This trajectory is not presented as an exemplary one but rather as a medium to exemplify the types of assessment tasks, to illustrate how the increasing sophistication in understanding could be captured and to discuss possible instructional adjustments. The selected trajectory is designed to engage students first with whole numbers divided by fractions by making links with quotative division of whole numbers. Then, students are engaged with fractions divided by whole numbers by making links with partitive division of whole numbers. Afterwards, they are introduced to fractions divided by fractions, where again the quotative division appears and they engage also with division of mixed numbers. Students are expected to have the opportunity to learn different procedures (i.e., invert and multiply, convert to fractions of common denominator and divide the numerators), represent fraction division using number line and area models, interpret representations, and solve word problems.

The first type of task aims to assess whether students can reproduce the procedure of fraction division. The procedure is expected to have been taught and practiced. Hence, students would have extensive familiarity. Figure 2 presents an example of “recall and reproduce” tasks. These are different mathematical expressions in order to identify which of these students can complete fluently and flexibly. The last two expressions have increased complexity due to the position of the unknown. This example consists of expressions with different number characteristics and it is not intended to present a step of the instructional sequence but a medium for capturing increasing understanding.

Find the unknown $v$ .	
(a) $6 \div \frac{3}{4} = v$	(b) $\frac{3}{10} \div 5 = v$
(c) $\frac{1}{3} \div \frac{1}{6} = v$	(d) $2\frac{5}{6} \div 1\frac{1}{2} = v$
(e) $1\frac{1}{3} \div v = \frac{2}{3}$	(f) $v \div \frac{1}{4} = \frac{4}{5}$

Figure 2: Example of “Recall and Reproduce” tasks

The second type of assessment tasks intend to assess whether students understand the underlying mathematical concept, in this case, the way in which fraction division could be interpreted and represented. It is anticipated that this type of task will bring to the surface students' reasoning and misconceptions. Figure 3 shows examples of “apply and connect” tasks. Students are asked to represent the fraction division using an area model and a number line. In the first case, students would need to interpret the fraction division as “ $\frac{3}{4}$  divided by 2” and represent it (e.g., divide the rectangle in 4 equal stripes, shade 3 out of the 4 and then divide the shaded area into two equal regions). In the second case, students would need to interpret the fraction division as “how many  $\frac{1}{4}$ 's are in  $\frac{3}{4}$ ,” and represent it (i.e., draw three jumps of  $\frac{1}{4}$ ). Deviation to the complexity of the tasks could be achieved without providing the area model and the number line or asking students to provide more than one representation of the mathematical expression (e.g., verbal and pictorial). The last example is a word problem in which students would not only need to identify which procedure to pursue but also to interpret and communicate their thought process.

<p>Represent the mathematical expression using a rectangular model and find the quotient.</p> $\frac{3}{4} \div 2 =$ <div style="text-align: center; margin: 10px 0;">  </div>
<p>Represent the mathematical expression on the number line and find the quotient.</p> $\frac{3}{4} \div \frac{1}{4} =$ <div style="text-align: center; margin: 10px 0;">  </div>
<p>Mr. Apostolos has a piece of wood of <math>\frac{5}{6}</math> m length. He wants to cut it into equal pieces of <math>\frac{1}{8}</math> m. Explain the way in which Mr. Apostolos can work to find beforehand whether it is possible to make 7 such pieces.</p>

**Figure 3: Examples of “Apply and Connect” tasks**

Students are expected to have worked on representing fraction division using a variety of representations and solving word problems. Hence, the requests of the tasks would be familiar to the students. However, depending on the given context of representation and word problem, students would need to decide how to apply the procedure of fraction division and how to interpret it.

The third type of assessment tasks intend to assess whether students can investigate independently mathematical ideas, structures and contexts, and develop their own mathematical products. For example, students could engage in evaluating arguments, generating justifications, forming examples and non-examples. Students are expected to have limited familiarity with the request of

the tasks in order to engage in generating an idea instead of recalling it. Figure 4 presents examples of “reflect and generate” tasks. The first task asks students to generate a word problem based on the given mathematical expression. Students are anticipated to have had opportunities in solving word problems but they are now asked to formulate their own word problem by taking into account the given numbers and an appropriate real life situation. The next task, which was adapted from Askew et al. (2015), presents a statement and students are asked to explore whether it is always true, sometimes or never. Students would need to compare the quotient and the dividend in different cases of numbers (e.g., division of two integers, division of two fractions).

Write a word problem that corresponds to the mathematical expression $\frac{6}{9} \div \frac{1}{3}$ .
Marios says: “When I divide two numbers, the quotient is always smaller than the dividend” Do you think that this is always, sometimes or never true? Explain.

**Figure 4: Examples of “Reflect and Generate” tasks**

A less complex task could be designed by asking students to complete the description of a word problem based on a given mathematical expression while a more complex task could ask students to write a word problem for a mathematical expression consisting of two operations (e.g.  $5 + (\frac{6}{9} \div \frac{1}{3})$ ). Another option is to modify the nature of the information by reducing the complexity such as “Find the cases for which the division of two numbers gives a quotient larger than the dividend”. In this way, students would need to identify cases of numbers that confirm the statement instead of exploring all possible cases of numbers to decide whether Marios’ argument holds true in all, some or none of these cases.

## Discussion

The framework aims to contribute to the discussion about how classroom assessment could be linked with learning trajectories (e.g., Ebby & Petit, 2018; Heritage, 2008). Different types of tasks were formed based on the pre-designed trajectory in curriculum materials that aim to elicit evidence about students’ understanding along the increasing expertise of the learning trajectory in order to make informed decisions of how the enactment of the learning trajectory in classroom could be adjusted. Hence, we rely on students’ understanding to discuss about teachers’ instructional adjustments.

The adjustments elaborate on differentiation, reteaching and changing the pace of instruction (Hoover & Abrams, 2013). Students’ responses to “recall and reproduce” tasks could inform the classroom teacher whether students reproduce the procedure of dividing fractions correctly. If students use the procedures incorrectly (e.g. invert the dividend instead of the divisor and multiply), the teacher might need to provide more opportunities in reteaching the procedure and in using other procedures (e.g. converting into like fractions) to develop understanding. If students complete incorrectly the last two mathematical expressions (in which the place of the unknown differs), then the teacher might make explicit links between division of whole numbers and division of fractions. If students seem to make minor arithmetical mistakes (e.g. in converting mixed numbers to

improper fractions), then either they face difficulties with previously taught mathematical concepts or they need further practice. Hence, along the enactment of the learning trajectory in classroom, more time and relevant opportunities could be planned. The evidence elicited from “apply and connect” tasks could indicate whether students are facing difficulty in making sense of fraction division, in using different representations, in identifying fraction division in various contexts and even making connections with fraction multiplication. If students find difficulties with the area model and the number line, then either more opportunities are needed to make sense of what fraction division means or more opportunities are needed with the particular representational formats (e.g. by corresponding various fraction division expressions with their respective representations). If students solve the problem incorrectly (e.g. they mention that it is possible to make 7 such pieces), then either more opportunities are needed to address previously taught concepts (e.g., converting into like fractions) or more opportunities to develop understanding of how fraction division could be interpreted. The “reflect and generate” tasks could inform teachers whether students could extend their understanding of mathematical ideas and exhibit higher-order thinking. If students find difficulties in writing a word problem that corresponds to the given mathematical expression, then students might need further opportunities to reflect on the structure of relevant word problems and opportunities to write word problems using for example integer numbers. Students who are not able to respond fully or complete at all the last task might need further opportunities that engage them in reflecting about the structure of numbers and to develop arguments that justify or refute a statement.

Decisions about instructional adjustments would also be regulated by the number of students facing constraints and time limitations. The number of students will determine whether adjustments will involve the whole class, groups of students or individual students. The teachers’ role is quite demanding in making decisions for the pacing of instruction and for the selection and amount of learning opportunities. Particularly, teachers find most difficult the process of using assessment information to plan instructional actions (Heritage, Kim, Vendlinski & Herman, 2009) while some teachers may not even use the information to adapt their instruction (Zhao, Van den Heuvel-Panhuizen & Veldhuis, 2017).

The presentation of the framework, in this paper, is limited to its application in the design of assessment tasks based on a learning trajectory for fraction division. The framework could be validated and further refined by developing tasks for other grade levels. Further empirical evidence could provide insight into the potential and limitations of the framework from its application in real classroom settings. Further research could also explore what kind of evidence is purposeful for teachers to be elicited from assessment tasks and how decisions about instructional adjustments could be planned to move students towards increasing expertise along the learning trajectory.

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