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# Conceptualising tasks for teacher education: from a research methodology to teachers' knowledge development

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*In this paper, we discuss the process of conceptualising tasks for teacher education as a research methodology that may improve the understanding of the features of teachers' knowledge, as well as refine the specificities of the Mathematics Teachers' Specialized Knowledge (MTSK) conceptualisation. We focus our discussion on a task for teacher education conceptualised to develop teachers' knowledge of connections between measurement and fractions. The discussion of the process of designing a task emphasises the role of using a teacher's knowledge conceptualisation (MTSK) as a tool for such design, in a dialectic relation with the aim to develop a methodological tool to be employed in shaping teacher education and to develop teachers' knowledge.*

*Keywords: Teacher education, task design, teacher knowledge, mathematical connections.*

## Introduction

Teacher education involves a broad set of dimensions and may thus be investigated from different perspectives. One of the core aspects of mathematics teacher education is related to identifying the most problematic areas, not only in terms of the topics that present difficulties to both students and teachers—when learning and teaching, respectively—but also in terms of how the teachers' knowledge should be considered and developed.

It is well known that teachers' knowledge has a great impact on pupils' knowledge/learning (Grossman, 2010). In that sense, it is essential to focus teacher education on the core aspects of such knowledge, which demands a particular attention to both the process of conceptualising tasks and to how they are implemented. Moreover, it is necessary for the tasks conceptualised for teacher education to be in a direct relationship with practice, thus assuming a practice-based approach. One of the ways of achieving this aim is to include pupils' productions (containing errors or not but, in particular, those considered as non-standard), whenever aiming to contribute to the development of what has been termed as Interpretative Knowledge (Mellone, Tortora, Jakobsen, & Ribeiro, 2017).

Considering that the principles related to the school mathematics (NCTM, 2000) provide a set of five standards of mathematical process—connections, representations, problem solving, reasoning, and communication—teacher education must be concentrated on developing teachers' knowledge of those principles. Amongst them, connections and representations are particularly relevant for the work we are developing, as it focuses on teachers' knowledge. Since the representations and their use are directly linked to the learning process in mathematics, it is required to broaden and deepen the understandings about what kind of relationships can be established between various forms of representations, such as verbal, pictorial, numerical, symbolic, algebraic, and graphical.

In terms of mathematical topics in which representations play a central role, rational numbers are considered the most problematic, both in terms of learning and teaching, mainly because of the different meanings they assume across different contexts. When we consider the variety of interpretations of rational numbers—part-whole, measure continuous quantities, quotient, operator and ratio—it is not surprising that pupils experience difficulties in understanding and dealing with this concept. Besides that, because of the intrinsic relationship between the construct of rational numbers and the phenomenological process of measurement—since it is one of the meanings of the fraction representation for rational numbers (Charalambous & Pitta-Pantazi, 2005)—it is essential to focus on the connections between these two topics (rational numbers, fractions in particular, and measurement) in order to propose effective interventions in the context of teaching and learning, and specifically in the context of teacher education.

Considering, therefore, the central role that teachers' knowledge plays in pupils' learning, and the importance of improving such knowledge, allowing teachers to help students in developing the understanding about connections and (between) representations, this work focuses on the question: *which are the features required in conceptualising a task for developing the specificities of teachers' knowledge?*

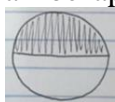
## **Theoretical Framework**


Several conceptualisations of teachers' mathematical knowledge have been developed (e.g., Ball, Thames, & Phelps, 2008; Carrillo et al., 2018), most of which include, explicitly or implicitly, connections as a part of the dimensions of teachers' knowledge. This inclusion can be perceived as an awareness of the central role that connections assume in teachers' practice, for example, to establish a coherence on the work plan, that is, to the sequence of the tasks they prepare and implement. Another aspect related to the teachers' practice in which connections are essential is the need to give sense to students' productions (Mellone et al., 2017), treating them as a starting point to develop and broaden their mathematical knowledge.

We treat connections as a dimension of teachers' knowledge, which is related to the relationships teachers, consciously and deliberately, establish among different constructs within the same topic (intra-conceptual connections) and/or among different topics (inter-conceptual connections), in order to develop students' mathematical knowledge. In that sense, we assume a teachers' knowledge conceptualisation (Mathematical Teachers' Specialised Knowledge – MTSK – Carrillo et al., 2018) that considers such differentiation, both in terms of the type of connections, and the intentionality behind establishing such connections. The MTSK conceptualisation is conceived as a theoretical and analytical tool to better understand teacher's knowledge specificities, from two main dimensions, named Pedagogical Content Knowledge (PCK) and the Mathematical Knowledge (MK) domains. Each domain includes three subdomains related to: the content itself; the connections between topics and how one proceeds in mathematics (MK); the features of teaching and learning each topic and the awareness of the curriculum (PCK).

In this work, we focus on three of the six subdomains—Knowledge of Topics (KoT) and Knowledge of Structures of Mathematics (KSM)—related to, respectively, intra-conceptual and inter-conceptual connections and Knowledge of Features of Learning Mathematics (KFLM).

The KoT subdomain includes the teachers' knowledge on definitions, procedures, characteristics of results, foundations, properties, distinct types of representations, and phenomenology and applications. In the scope of rational numbers, it includes knowledge of the different meanings associated with the rational number (e.g., part-whole, ratio, operator, quotient, and measure – Charalambous & Pitta-Pantazi, 2005). Another aspect included in the KoT refers to the knowledge of the models and representations, for example, within contexts like percentage, ratio, and proportion, in which rational numbers can be applied. When a teacher establishes relationships



amongst the representations 50%,  $\frac{1}{2}$ , and , connecting them to the notion expressed by the term “a half,” and intentionally contextualises these representations in different situations, we would say that he/she reveals knowledge of intra-conceptual connections (KoT). In this context, the specialisation of the knowledge is related to connecting, intentionally and deliberately, the concept image (in the sense of Tall & Vinner, 1981) associated with a construct to its concept definition (as specific equivalence class), in each particular case, which is thus associated with intra-conceptual connections (KoT).

The KSM subdomain concerns the knowledge related specifically to (inter-conceptual) connections, and it comprises four categories. Because of the space limitation, we will discuss only two of such categories: the auxiliary and transverse connections. The auxiliary connection category is related to teachers' knowledge of the need to consider a notion (procedure or construct) as a support to the process of developing students' understanding of a certain concept (procedure or notion). In this case, we can take an example of a kindergarten teacher implementing a task involving a measurement activity, whereby he/she evokes the procedures of measuring (iteration of the unit and assigning a quantity to the number of times the unit fits within the whole).

The transverse connections pertain to the knowledge grounding the establishment of relationships between several topics with common features, even if those common features are not necessarily evident at first sight. An example of this concerns connecting the experience of measuring a magnitude, using different units of measurement, with the inverse proportional relationships between the number of units of measurement used and the correspondent values obtained for such measurement. In this case, considering the same whole, the smaller the unit of measurement, the bigger the number expressing the result of the measurement.

In relation to the work we are developing focusing on teachers' knowledge on connections, one other core aspect concerns the knowledge on students' mathematical thinking, learning and knowledge development when engaging with mathematical tasks, which is part of the KFLM subdomain from the MTSK conceptualisation. It also includes the knowledge of the types of tasks and examples, and common errors or areas of difficulty, as well as misunderstandings and misconceptions when proceeding in mathematics. For instance, a teacher should know the students' misunderstanding of a rational number as a measurement (quantity) and not only as a part-whole relationship.

Such specialised knowledge grounds teachers' practice, and in order to develop such practice having as a starting point what their students know and how they know it, it is required that the

teachers interpret and give meaning to the students' productions and comments. When considering teachers' knowledge required to interpret students' productions, the notion of Interpretative Knowledge (IK) has been developed (Mellone et al., 2017). It is defined as "the knowledge that allows teachers to give sense to pupils' answers, in particular to 'non-standard' ones, i.e., adequate answers that differ from those teachers would give or expect, or answers that contain errors", (Mellone et al., 2017, p. 2949). The aim of the educational process should be to develop teachers' IK, so that they transition from an evaluative perspective (traditional—associated with a teacher's establishment of a relation between the students' answer and the elements of his/her own (im)possible and mathematically (in)adequate set of solutions for a problem, named *space of solutions*) to a real interpretation for the educational design, that is, when a teacher "revises his/her mathematical formalization in order to ensure that it is coherent with students' productions" (Mellone et al., 2017, p. 2950).

Considering, therefore, the specificities and particularities associated with teachers' knowledge, in the context of teacher education, the tasks conceptualised to develop such knowledge must also have a specialised nature (Ribeiro, 2016). This means that the MTSK and the IK perspectives, as tools for supporting the process of designing tasks conceptually, on one hand, aim at accessing and developing some aspects of the specialised knowledge related to the mathematical topic(s) to be taught. On the other hand, including students' productions from which the mathematical reasons that sustain eventual errors or non-standard responses can be explored, the task conceptualisation relies on the notion of broadening teachers' *space of solutions* and, ultimately, developing teachers' ability to support the development of pupils' mathematical knowledge, starting from their own reasoning.

## **Context**

The current work is a part of broader research project which focus on the dimensions of teacher's knowledge related to connections. The goals of such research project include identifying and describing some categorisations of connections that teachers elaborate (or should be developed) when discussing tasks specifically conceptualised for teacher education.

All the tasks conceptualised are conceived to be implemented in different contexts of teachers' training programs and they comprise of two parts (described in detail in the next section). The first part of the task discussed in this paper (denoted as "A half") was designed to last about four hours, including both teachers' reflections and the whole group discussions. The second part of the task was designed to last about two hours, also including teachers' reflection and the whole group discussions.

## **Analysing the conceptualisation of a task for teacher education**

The MTSK conceptualisation considers, as mentioned before, six subdomains of teacher's knowledge, with several associated descriptors. When conceptualising a task for teacher education using the MTSK as a tool for approaching the complexity of teachers' knowledge, those descriptors will serve as guiding principles for the work of accessing and developing (this last process occurring specifically in the context of the task implementation) teacher's specialised knowledge. Thus, after identifying the mathematical subject matter to be explored in the task, the task designer

must choose the main aspects of teacher's knowledge he/she wants to develop, and then must associate it to the descriptors included within the subdomains.

For instance, one of the questions of first part of the task (named "A half") aimed at accessing (and not assessing) teachers' knowledge of the meaning of the concept/construct of "a half": a rational number that can be understood as a quantity resulting from the phenomenological process of measurement; the meanings of a fraction; and/or result of a division (operation – which is a mathematical process). *"Imagine you are on the street and someone stops you and poses a question: 'What is a half?' What would your answer be? Respond to this question for yourself, using your own mathematical knowledge."* – KoT. Another question included in this part of the task aimed at accessing teachers' knowledge about types of representations associated with a concept (*"Present two distinct representations for 'a half.' Justify why you consider these representations as distinct from each other."*) – KoT. Moreover, they were considered in the first part of the task the type of problems/different contexts the concept/construct of fraction can be applied to, and procedures associated with, the characteristics of the result (number sense – number as a quantity) contexts in (*"Pose two distinct situations (or word problems) in which the term (and the concept) 'a half' is explicit."*) – KoT and KFLM.

As the background of the aspects approached of the questions aforementioned, they were considered as contents of the teacher's knowledge to be accessed the role of the whole when measuring and the foundations of the measuring activity (What is "to measure" something? What can be measured?) – KSM.

The conceptualisation involved in the tasks we have been assuming for teacher education includes the perspective that any situation students would encounter must be experienced by the teachers. In the "A half" task, two situations for students were included, denoted as "The secret envelopes." Specifically, three different envelopes containing a stick and a piece of string are distributed to students and each pupil is asked to measure the stick using a piece of string. In the first situation, the three envelopes contain sticks of the same length, but the length of the strings corresponds to 4, 2 and  $1/2$  of the stick length, respectively. In the second situation, the envelopes contain both sticks and strings of different lengths, but the relation between the stick and string lengths in each envelope is always  $\frac{1}{2}$ .


Teachers are stimulated to solve the students' task by assuming that they are not involved in a teaching context. Then, they are invited to reflect on the teaching process involved in the students' task from questions explicitly linked to PCK subdomains. For doing so, questions such as *"What kind of knowledge do students need to solve the task?"* and *"How do you consider a 2nd grade student would solve the task? Try to present some examples of possible difficulties, mistakes, or misunderstandings a child could experience."* are posed – KFLM.

In order to access teachers' knowledge on: i) connections between the measurement process and the concept of a fraction as a number (quantity) including the different types of representations associated to it (KoT); and ii) connections between the inverse proportional concept and the characteristics of results obtained by a measurement process (KSM) a question linked to the students' task was included (*"What are the mathematical topics/contents (that can be) explored*

within these tasks? Do you consider those topics/contents as related to each other? Give examples.”).

The second part of the task is conceptualised according to the IK perspective (Mellone et al., 2017). For doing so, a transcription (Fig. 1) of a discussion between a teacher and a 2nd grade for “the secret envelopes” is presented<sup>1</sup>. Teachers are invited to interpret the last student’s statement, considering the mathematical adequacy and correction. They are subsequently instructed to propose a continuity of the dialogue to develop the student’s mathematical knowledge related to the topic of the task.

When interpreting the last student’s statement, teachers must evoke their MTSK related to the meaning of the construct “a half”, that is, as a rational number that can be represented by a quantity (resulting from the phenomenological process of measurement). This teacher’s specialised knowledge is associated to the KoT and KSM subdomains explored on the first part of the task. For instance, if in the first part of the task a teacher provides a representation of “a half” that is somehow associated with a magnitude (length, for example), he/she would be able to understand that the student is not comprehending the construct “a half” as a number because he/she is not considering a number as a magnitude (a quantity yielded by a measurement process).

One of the principles of the measurement process is to attribute a value to the measure, that is, a number. Therefore, in order to develop student knowledge about the number sense related to the concept “a half,” teachers should be aware of the need to redesign the educational process (Mellone et al., 2017), establishing connections among the numeric representation  $\frac{1}{2}$ , the pictorial representation  and the notion of a phenomenological process of measuring. Moreover, a teacher should be reflecting on how the special role played in the society by the rational number  $\frac{1}{2}$  is reflected in the natural language. Indeed, in many (all?) languages, we can find at least two ways to express it, like "a half" and "one on two", and while this second way suitably arranged works for all the fractions, the first is only for  $\frac{1}{2}$ , stressing its special role.

Obviously, this (re)design of the educational process is associated with teachers’ specialised knowledge about the connections, both between different types of representation of a construct (in this case “a half”) and the meaning(s) of such construct (fraction as a measure). In that sense, it is

#### Part II

During “The secret envelopes” task implementation with a 2<sup>nd</sup> grade children, teacher Alice has a dialogue with one of her students, as it is followed:

*Alice: Matheus, how many [pieces of] string you needed to measure the stick?*

*Matheus: I need a half...*

*Alice: Cool. So, you go back to your seat and do not tell anything to the girls, right? You must keep this secret! You only must keep this number in your mind, so that we can discuss after, ok?*

*Matheus: Number? Which number? A half is not a number.*

**Figure 1: The second part of the “A half” task**

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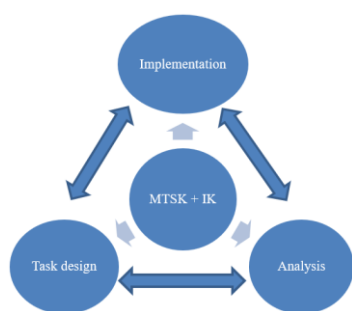
<sup>1</sup> In the context of a task conceptualisation we have been developing, students’ responses as written or oral commentaries are considered as “student productions.”

important to highlight that, in order to develop the IK in teachers, it is necessary to simultaneously develop their MTSK.

## Final Comments and Remarks for the Future

In this paper, we presented a process of conceptualising a task based on a methodological and analytical tool focusing on the teachers' specialised knowledge (MTSK) and its relation to the process of intentionally developing their Interpretative Knowledge. Considering the features of the teacher's knowledge—which differs from students' knowledge and from that well-educated individuals possess—tasks aimed at teacher education must explore and develop the diverse dimensions and features of such knowledge (specialised and interpretative). The first part of the task involves a combination of the knowledges the students are expected to possess that is rooted in a deeper, formal, and rigorous understanding. The second part of the task is conceptualised by the IK perspective and is essential the inclusion of student's productions, preferably those in which the mathematical aspects can be explored in a broaden and deepen way.

When implementing a task conceptualised from these two perspectives about teacher's knowledge in a teachers' training context (initial or continuous), it is essential to emphasise the role of the teacher educator, who must be aware of the need for improving the specialised and interpretative teachers' knowledge. This should be done by not only following the questions posed in the task, but also by scrutinising the diverse types of (possible) interactions that teachers should present with the task. This awareness that teachers' educator must provide during the implementation is related to both his/her own mathematics specialised knowledge about teachers' knowledge and his/her own Interpretative Knowledge. In the context we have been working, the teacher educator (who is also a researcher) uses the particularities of both teacher's knowledge perspectives (MTSK and IK) as conceptual bases in the process of designing the task, implementing it in the teacher's training programs and analysing the data gathered from such implementation.



**Figure 2: Relationships to be considered in teacher education**

The process of conceptualising a task for teacher education described here is one of the several ways to perceive the methodology of designing tasks aiming at accessing and developing the specificities of teacher's knowledge. Both MTSK and IK perspectives are in process of development in different contexts (Spain, Norway, Italy, Brazil, etc.). In that sense, it is understandable (and desirable) that different teacher's knowledge conceptualisations are being used as tools for designing tasks for teacher education, from different perspectives (e.g., Montes, Climent, Carrillo & Contreras, 2019). In the context of the broad research we have been developing, we have been considering it extremely relevant to focus on the relationships between what we consider to be three pillars which sustain the process of

conceptualising a task for teacher education: the task design process and its implementation; the task implementation and the analysis; and the results of the analysis and the task design process. All these aspects are certainly influencing and are being influenced by the MTSK and IK perspectives,



since both perspectives are considered in a dialectic relation with the aim to develop a methodological tool to be employed in shaping teacher education.

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