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► **To cite this version:**

Juan Luis Piñeiro, Elena Castro-Rodríguez, Enrique Castro. Prospective primary teachers' knowledge of problem solving process. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02430483

HAL Id: hal-02430483

<https://hal.science/hal-02430483>

Submitted on 7 Jan 2020

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Prospective primary teachers' knowledge of problem solving process

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Studies on teachers' understanding of processes such as problem solving have focused primarily on their beliefs and conceptions to the detriment of features indicative of their professional knowledge. This study analyses the knowledge about problem solving process revealed by prospective primary teachers. A questionnaire specifically designed for the study was administered to 61 undergraduates beginning and 53 ending their training. Although characterising solving as a dynamic process, both groups, in their description of stage characteristics and strategy identification, revealed that their knowledge was essentially theoretical.

Keywords: Problem solving, Teacher knowledge, Pre-service teachers

Researchers have engaged in the study of teachers' knowledge in an attempt to identify the type of expertise required to teach mathematics effectively. Problem solving (PS) forms part of such expertise, as it is a fundamental process in classroom mathematics, which should cover both problem-solving skills and elements that help students become better problem solvers.

To teach PS, teachers need to know what to do, when to do it and the implications of their actions (Lester, 2013). Unfortunately, teachers have been found to exhibit limitations in that regard (Andrews & Xenofontos, 2015). Research in the area has targeted their ideas and their own problem-solving skills (Lester, 2013), eclipsing factors indicative of their professional knowledge. These considerations led us to pose the following research question: what knowledge do prospective primary teachers have about PS process? Specifically, this study analyses and describes the differences in the theoretical knowledge about the PS process exhibited by prospective primary teachers at the beginning and end of their training with a view to helping fill that gap.

Teachers' knowledge of mathematical PS.

Mathematical knowledge for the teaching is understood as a wider knowledge than the mathematical subject. Therefore, "the knowledge needed to effectively teach PS should be more than general problem-solving ability" (Chapman, 2015, p. 19). Lester (2013) pointing out that the teachers' ability to solve complex, cognitively demanding problems does not suffice to guarantee appropriate PS instruction. In addition this author points out that it is necessary to clarify what aspects, other than the teacher's competence as a problem solver should be part of the knowledge of the mathematics teacher. However, shortcomings for determining teachers' professional proficiency with previous theoretical frameworks have been identified, regarding to overlap or the need to broaden the theoretical approach to supplement analyses. More specifically, research on teachers' ability to teach PS is not organised in the usual manner (Chapman, 2015), which leads among other limitations to omissions around the nature of PS (Foster, Wake, & Swan, 2014).

Teacher knowledge models have a tendency to be more content-focused, provoking the omission on some aspects of the nature of the PS. PS could be part of mathematics practice (in MTSK' terms,

Carrillo et al. 2018), in the sense that allows us to do mathematics (Codes, Climent, & Oliveros, 2019). Carrillo et al., (2018) establish that this sub-domain "encompasses teachers' knowledge of heuristic aids to PS and of theory-building practices" (p. 245). However, for a math task to become a problem, it necessarily needs the solver' perspective. PS is a process with personal nature, in which a problem is defined according to the solver. Under this perspective, tension emerges in the description of a mathematical knowledge disconnected from the students.

The knowledge for teaching PS in primary education needs to embrace more than the algorithms involved, how to apply them efficiently or possible mental calculation or representational strategies. To use the problem to its full potential teachers must also understand it as such, as well as the process for solving it. More specifically: a) the *problem*, i.e., the underlying mathematics, but also its type and format and the extent to which it may be a problem for students; b) *solving the problem*. The phases needed to solve: understanding what each data item means, how they are inter-related... But also is necessary the applicable strategies may vary, from algebraic; guess and check, or even tables or diagrams to visualise the relationships among quantities; c) the *disposition* it may generate in students; d) the *mistakes* they may make; e) the *potential* for developing cognitive features such as the various strategies, and non-cognitive features such as the belief that problems can be solved in different manners or that discussion of a problem is part of the learning process; f) the *changes* that can be made in the amounts of the variables or their inter-relationships to render the problem harder or easier; and g) classroom *organisation*, i.e., the approach to use or emphasise. Given that the characteristics of the problem accommodate the consideration of different strategies, prioritising teaching *about* PS would appear to be the most suitable approach.

Identifying professional knowledge about PS teaching calls, firstly, for addressing teachers' knowledge of processes rather than their mathematical content knowledge, the perspective adopted in traditional teachers' knowledge models. Based on mathematical and PS proficiency theories, PS is understood here to mean the action taken by a subject who identifies a problem, proceeds to solve it by deploying a strategy involving a series of not necessarily linear steps and confronts the challenge with a favourable disposition. Based on this idea, teaching PS proficiency draws on a complex network of interdependent knowledge. Chapman (2015) proposes a model consisting of a) teacher PS proficiency; b) knowledge of problem content, solving and posing; c) pedagogical knowledge of students as problem solvers and of teaching practice; and d) a dimension comprising affective factors and beliefs that impact teaching and learning this skill.

We have applied the Chapman' framework to six curricular guidelines in order to identify the knowledge required to teach PS (Piñeiro, Castro-Rodríguez, & Castro, 2016). This analysis has led to modifications (Piñeiro, Castro-Rodríguez, & Castro, 2019), in which we distinguish three elements on teacher's knowledge, its own proficiency to solve problems, and two related to teaching (one referred to PS theoretical knowledge, and another to aspects of learning and teaching). Figure 1 shows the components of our framework related to PS knowledge and PS pedagogical knowledge. Here we focus on teachers' PS process knowledge, distinguishes four key components: PS stages and their characterization, strategies, metacognition, and non-cognitive factors (Piñeiro et al, 2019). This study is confined with static knowledge related to PS, that is, the theoretical aspects of PS process (Blanco, 2004).

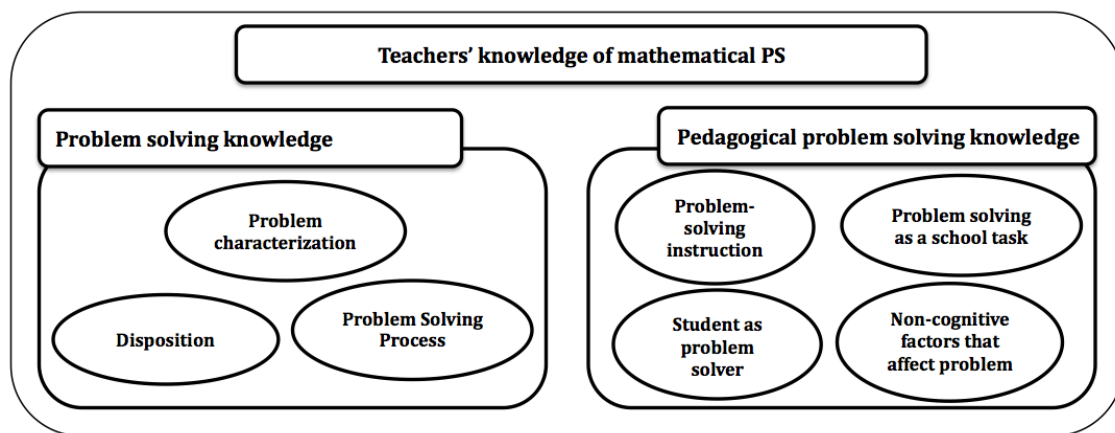


Figure 1: Teacher knowledge components for teaching problem solving

Theoretical knowledge about PS process.

In our approach to Teachers' PS knowledge, PS process can be broken down into four areas: solving stages and their characterisation, solving strategies, metacognition, and affective factors (Piñeiro et al, 2019).

The first area, solving stages, adapts naturally to Pólya's (1945) postulates on how solvers proceed: comprehension, planning, action and evaluation. Awareness of those stages helps teachers adapt the assistance needed to the circumstances. One factor common to all four is their configuration as personal cognitive processes, not observable directly but only through what the solver says or does in each stage (Mayer & Wittrock, 2006). The process is non-linear, for as Wilson, Fernandez, and Hadaway (1993) explained, it is flexible, accommodating both forward and backward movements. Teachers aware of these elements can stand by their students and mediate in their development of PS proficiency.

In connection with the second area, solving strategies, Schoenfeld (1985) distinguished two types of decision-making. Strategic decisions include the definition of objectives and the decision to adopt a course of action. Tactical decisions are geared to implementing strategic decisions. Whilst together they constitute what is understood as strategy, singly they are of no use for a number of reasons, including the role of metacognition (Schoenfeld, 1985). Strategies must be taught carefully, for that endeavour covers all the overlapping components addressed in this section. More specifically, decision-making on what to do and how to do it depends on an understanding and mental representation of the problem. It is also affected by metacognition, for the success of the strategy is partly determined by its conscious use. Backtracking further reinforces this process and helps determine the aptness of the initial decision. The entire process is mediated by the emotions that may arise, the attitudes prompted and the beliefs held during PS.

The third area is metacognition. Schoenfeld (1985) expanded research perspectives by showing the importance of metacognition and affect. Metacognition is described as the manner in which solvers self-regulate, monitor, and control; their heuristics and mathematical knowledge to solve a problem, enabling them to apply appropriate decisions to the task at hand.

The fourth and last area covers non-cognitive factors and their essential role, for they determine how the solver confronts problems. It is generally agreed that depending on the suitability of the challenge posed to students, they bring their emotions into play, which in turn mobilises their intellect (Mason, 2016).

Method.

To characterize the knowledge about PS of prospective primary teachers, we have used a questionnaire for the purpose that this type of instrument to collect information to, among others, describe the knowledge of the people (Fink, 2003). The questionnaire was designed and administered to university undergraduates beginning and ending their pre-service teacher training to analyse and describe the differences between them. A dual analysis was subsequently performed.

Context and participants.

The participants in this study were 114 undergraduates working toward a degree in primary teacher education at the University of Granada. They were grouped by the training received, 61 first year (G1) and 53 fourth year (G2) undergraduates. G1 had received no university mathematics training. G2 had taken three requisite subjects on mathematics teaching: classroom mathematics content; teaching and learning core classroom mathematics topics; and the primary education mathematics curriculum. In these three subjects, PS is treated as a transversal goal. Specifically, when discussing meanings and modes of use of mathematical concepts. The activities in which they have involved are mainly of two types: master classes in which are presented, guided and synthesized some of the courses' topics. The second type, called practical activities can have two orientations, laboratory and TIC. In laboratory practices, work with manipulative materials, and TIC practices will focus on the management of educational software and Internet resources. They had also taken an elective addressing (among others) PS content, in which they were introduced to strategies and heuristics, problem posing and strategies for teaching PS. Specifically, in activities such as: a) characterization and exemplification of the role of PS in the learning of mathematics and its link with mathematical competence, b) development and application of strategies and heuristics for PS, c) application of criteria for posing problems, and d) analyze appropriate teaching strategies for teaching PS.

Instrument.

The questionnaire was developed following phases: a) theoretical analysis on the notion of PS proficiency; b) study on the primary education curricular syllabus related to PR; c) research review about PS with primary teachers; d) construction of the pilot version of the instrument; e) review by expert judgment and pilot application; and f) construction of the final version of the questionnaire. These phases originate a specific questionnaire with two sections and 66 items. A closed binary design was adopted to elicit ideas that would denote the presence or absence of certain types of knowledge (Fink, 2003). The first section of the questionnaire (Figure 2) was sub-divided into PS stages and their characterisation, metacognition, and non-cognitive factors. The second sought to determine trainees' ability to recognise specific strategies in students' hypothetical answers to problems, in which, eight items were formulated as multiple choice questions. The options were: 1a) building a table; 1b) work backwards; 2a) draw a diagram; 2b) guess and check; 2c) look for a pattern; and 2d) operating.

1. A student solved the problem set out below as shown:

A dragonfly, the fastest-flying insect, can fly 50 metres in 2 seconds. How long would it take it to fly 375 metres?

Dragonfly => 375 metres => time? → 1

Dragonfly flies 25 metres per second → 2

It needs 375/25 seconds to fly that distance → 3

$375:25 = 15$ → 4

It takes the dragonfly 15 seconds to fly 375 metres → 5

$25+25+25+25+25+25+25+25+25+25+25+25+25+25+25$
 $\underbrace{\hspace{10em}}_{375}$ → 6

a. The problem was identified when the student described what the dragonfly does and the distance it needs to fly and posed the question about time. (Step 1)

b. The student drew up the solving plan when mentally found the distance per second and proposed the operation: $375/25$ (Steps 2 and 3)

c. The student implemented the plan when divided 375 by 25. (Step 4)

d. The student verified the answer when counted or summed up the metres per second. (Step 6)

2. When solving a classroom mathematics problem, students advance toward the solution without backtracking.

3. Problem solving is a dynamic process in which the solver backtracks if necessary to find the solution.

4. Students solve problems step-by-step.

5. Which of the following do you believe might be present when solving a problem?

a. Representing the problem
 b. Reading the problem
 c. Doing the operations
 d. Understanding the problem
 e. Exploring different ways to solve the problem
 f. Solving similar problems
 g. Verifying the solution
 h. Writing out the answer

6. A classroom mathematics problem must be understood to be solved.

7. A solving plan can be suitably implemented without understanding the problem.

8. Representing a problem in different ways may help understand it.

9. Understanding a problem entails determining what information is available and how it is structured.

10. Drawing a diagram or graph is a useful tool for understanding a problem.

11. A student solved this problem. Answer YES or NO to the assertions, as appropriate.

A dragonfly, the fastest-flying insect, can fly 50 metres in 2 seconds. How long would it take it to fly 375 metres?

As follows:

- The dragonfly can fly 50 meters in 2 seconds
- 4 seconds would be 100 meters
- Then 6 seconds would be 200 meters
- 8 seconds would be 300 meters
- 9 seconds would be 350 meters
- As there are 25 meters left, we put half
- Then the dragonfly takes 9 seconds and a half to fly 375 meters

a. No error, the answer is right.

b. The student understood the problem.

c. There are operational errors.

d. The student verified the answer.

12. When someone solves a problem they perform mental exercises that reveal when they've made a mistake.

13. An awareness of one's own knowledge helps choose the most suitable way to solve a problem.

14. When tackling a problem it's helpful to know several ways to solve it.

15. A strategy is a procedure that guides the choice of the knowledge that should be used in each stage of a problem.

16. Once a problem is solved, it's advisable for the solver to know what would happen if some of the data were changed.

17. If a student finds the answer to a problem, no more action is needed.

18. Re-reading the question after finding the solution helps determine the suitability of the procedures and the result.

19. Once the answer to a problem is found, alternative ways to solve it should be sought.

20. Identifying the characteristics of a problem enables solvers to create families of problems that can be solved in similar ways.

21. It's important to know how to solve problems, but it's more important to want to.

22. A problem can be successfully solved even when there's no motivation to do so.

23. When students are confronted with a problem-solving situation, it's important for them to be motivated.

24. Students don't need to want to solve a problem: what they need to do is listen to and obey the teacher.

25. Students don't need to be interested in solving a problem. What they need is to understand the mathematical content.

Figure 2: Items on section 1 of the questionnaire

In the validation process, we contemplate two aspects. The first one is related to ensuring that the content is relevant and was made from the selection of knowledge related to PS on Primary Education school curricula. We have discussed this process somewhere else (Piñeiro et al., 2016). The second aspect corresponds to a test of the validity of the items, for which it was submitted to expert judgment. The experts' judgment makes possible a qualitative evaluation of the statements. Five Spanish mathematics education experts conducted the process. Also, a pilot application was made with the main goal was to increase reliability, validity and feasibility (Cohen, Manion, & Morrison, 2011). Our piloting was focused on assessing aspects such as the adequacy of the total time, clarity and comprehension of the statements.

Analysis and results.

This study aims to characterize the theoretical knowledge about the PS process manifested by prospective primary teachers. This has motivated the use one of the forms of interpretation of multidimensional scaling, allowing identifying the groupings that emerge from their answers, describing the common feature of these and labelling the attribute present in them (Bisquerra, 1989). Therefore, respondents' replies were processed first with dimensional scaling multivariate analysis using ALSCAL (SPSS) software, in which the dimensions defined were *agreement* or *disagreement* with the item (from here on, these terms will be used only to refer to the dimensions found). A second descriptive analysis was subsequently performed, in which responses were reviewed in terms of inter-group differences and the ideas defended in the literature.

PS stages and their characterisation.

This answers elicited high levels of agreement, although the percentage of agreement was higher in G2. Both groups acknowledged that the process could comprise several stages. For example, on the

stages' identification on a hypothetical solution, agreement percentages increase in the G2. Specifically, in the G1 a 97% identifies the understanding phase, 82% planning, 90% on carrying out the plan, and 80% on revision. In G2, 94% is the lowest percentage of recognition in any phases. At the same time, both groups characterize the process as flexible (98% in both groups), in which progress is made towards the solution (G1: 97%; G2: 100%), and they also admit the possibility of moving backwards if necessary (G1: 79%; G2: 83%). Both groups accept several moments on the PS process. Particularly, G1 states that, for example, representations (93%), reading (97%), calculations (95%) or verifications (95%) may appear. Likewise, in the same questions, the G2 presents positive answers in approximately 98% of the students. However, in this section one of the notorious differences can be found. In G1 only 46% respond positively to the possibility that similar problems can be solved, and 77% to the exploration of other resolution' paths. Conversely, G2 states that they can appear in 94% and 80%, respectively. On the questions that delve into the phases' features, there is majority agreement in both groups. For example, they recognize the usefulness of the representations to understand a problem (98%; 100%), and also that problems should not be solved without understanding them (97%; 98%). Nonetheless, in the questions that inquire about the value of the review phase, a difference was observed: only 49% of G1 state that it is advisable to pose similar problems, while 92% of the G2 states that it is necessary. Likewise, 67% on G1 indicates that it is appropriate to look for alternative forms of solution, as opposed to 91% on G2.

Knowledge of strategies.

The strategies that best recognize G1 were: operating (95%), draw a diagram (92%), building a table (90%), and look for a pattern (74%). A lower identification' degree was found on strategies works backwards (64%) and guess and check (46%). The strategies that best identify on G2 were: operating (93.6%), draw a diagram (87.2%), works backwards (83%), and look a pattern (80.9%). The strategies that have a lower percentage of recognition were building a table (72.3%), and guess and check (40.4%). Interestingly, in G1, work backwards was mistaken for operating by 31% of respondents, whilst 34% confounded guess and check with building a table and 15% with look for a pattern. Similarly, 30% of G2 confounded guess and check with look for a pattern and 21% with building a table.

Knowledge of metacognition.

Although most of the responses to the metacognitive items were agreement, a number of disagreements were recorded. Specifically, both indicate 92% state that the existence of monitoring that allows awareness of the errors committed. However, there are also some interesting answers. For instance, in one of the questions (11) on the reasons for operational errors in answers to problems, both groups identified the error. Their replies nonetheless differed about whether it should be attributed to a misunderstanding of the problem or to the calculation itself. In addition, both groups agree that the adequacy of the response was not verified.

Knowledge around non-cognitive factors.

Most of the items on non-cognitive factors elicited agreement. The percentage of agreement was slightly higher among the G2, however. However, the agreement percentages are slightly higher in

the G2. For example, in a question that inquires into the disposition to solve a problem, 77% of the G1 agrees, while the percentage rises to 91% on G2. Also, there are some doubts that need to be highlighted. Disagreement was expressed in a number of items. In particular, although the future teachers in both groups deemed motivation to be important to PS, a smaller percentage believed it to be instrumental to a successful outcome. For example, both groups state that it is important to be motivated to face an PS process (G1: 90%; G2: 96%), however, only 39% on G1 and 40% of the G2 believe that this can determine success in finding a solution.

Discussion and conclusions.

The findings showed that participants' replies were in line with the knowledge reported in the literature. They nonetheless co-existed with ideas that may prevent the translation of such knowledge into teaching practice.

Both groups' replies were consistent with a dynamic, cyclical interpretation of the stages described by Pólya (Wilson et al., 1993). The differences between the two groups studied in connection with knowledge of solving stages and their characterisation had to do with the variety of elements that may be involved in the solving process, specifically in the review stage. However, they do not consider posing problem as part of the PS process, one of the key moments to understand the process as cyclical (Wilson et al., 1993). That characterisation may fail to have practical implications, however, for the stages of PS about which respondents showed least agreement were the exploration of different solutions and problem posing. Given that those two approaches are determinants in characterising the process as dynamic and cyclical (Wilson et al., 1993), respondents may possibly be presumed to hold contradictory belief. Simultaneously, the present findings also seem to imply that the university training received has a scant impact on a command of PS theoretical knowledge, for the replies to the items on strategies between the two groups were essentially similar. The strategies with which both were most familiar were operating and drawing, whilst neither was well acquainted with guess and check. The inference is that although their knowledge was compatible with an understanding of solving as a dynamic and cyclical process, their approach continued to be linear. That conclusion is supported by the fact that the strategy least recognised, guess and check, is related to authentic PS.

In general, findings on teacher' PS process knowledge show little differences between groups. This seems to indicate that the knowledge they have, does not change during their training, despite having specific courses. This result could be explained because it is one of the areas stressed in curricula and textbooks (Wilson et al., 1993). Hence, we believe it is necessary for training programs to be concerned about this aspect, fostering skills that make it possible a deeper understanding of the PS process. On the other hand, the contrast between that finding and the actual belief exhibited by future teachers when confronting PS activities (Andrews & Xenofontos, 2015) raises the question of whether the relationship between mathematical knowledge and pedagogical knowledge is the same when a process such as PS is involved.

Acknowledgement

Spanish National R&D Project EDU2015-70565-P; and Chilean PhD. scholarship folio 72170314.

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