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Constructing tasks for primary teacher education from the perspective of Mathematics Teachers' Specialised Knowledge

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Developing tasks to help prospective teachers construct professional knowledge is an under-researched area of teacher education. In this paper we focus on describing the process of generating tasks in a research project at the University of Huelva. Using the model Mathematics Teachers' Specialized Knowledge we designed tasks to prompt the construction of specific elements of knowledge necessary for teaching mathematics, with the involvement of educators and prospective teachers selected ad hoc. As an example, we describe elements from a task focused on the notion of a polygon.

Keywords: Mathematics Teacher Education, Tasks, MTSK, Polygons.

Introduction

Mathematics teachers' knowledge has been the subject of discussion for over thirty years during which time various forms of modelling this knowledge have emerged, representing a wide variety of perspectives, foundations and proposals (see Montes, Ribeiro, & Carrillo, 2016). In this study we take on the challenge of applying insights from this body of research into professional knowledge to our teacher education programmes (Neubrand, 2018).

We are currently working on a research project at the University of Huelva which aims to develop a series of tasks for working with students following a degree in Primary Education. In the medium term, our objective is to structure the course content (with respect to mathematics) around research findings. Here we offer a progress report on the development of tasks for the prospective primary teachers which focus on the construction of professional knowledge, and which are underpinned by the research model, Mathematics Teachers' Specialised Knowledge (Carrillo et al., 2018). The first stage of development involved an experimental one-to-one teaching session (Cobb & Gravemeijer, 2008) with a specially selected group of students working alongside a team of experienced and less experienced teacher educators.

First we describe the theoretical foundations of the model of professional knowledge underpinning the study; then we give a detailed description of the first stage of the process for creating the tasks; finally, we offer some reflections on the development process, and the implications, limitations and potentials in taking it forward. This will contribute to give some light to our research question: how can a teacher's knowledge framework contribute to the design of tasks in teacher education? In particular, here we show the design process of a task concerning mathematical knowledge.

Theoretical Foundations

There are three main aims in studying mathematics teachers' professional knowledge: enriching our understanding of its nature, evaluating its quality, and studying how it relates to performance through the study of competencies.

In terms of the first of these, the majority of models currently available attempt to identify the different components of knowledge which a mathematics teacher needs to carry out his or her work. Shulman's (1986) proposal, dividing teachers' knowledge into three broad domains (Subject Matter Knowledge, Pedagogical Content Knowledge, and Pedagogical Knowledge), has served as the basis for a multitude of subsequent conceptualizations offering more detailed subdomains, components and facets. Among these, the work of Ball, Thames and Phelps (2008) deserves mention for its identification of the kind of mathematical knowledge which distinguishes practitioners of mathematics education from all other professionals within the field of mathematics.

Among studies describing teachers' professional knowledge, particularly those focused on the extent to which teacher training enables teachers to construct the knowledge required for teaching the subject, we can highlight the TEDS-M comparative studies (Tattoo et al., 2008), which evaluate initial teacher training programmes, alongside the development of evaluative measures and instruments in studies at national scale (e.g. LMT – Learning Mathematics for Teaching – Hill, Ball, & Schilling, 2008, which aims to measure the knowledge of teachers practising in the US).

Finally, several studies have been carried out focussing on the connection between knowledge and action, in particular through the use of the notion of competence, understood as knowledge with the potential for use in different professional practices (Godino, Batanero, & Font, 2011). The aim of the research described here was to develop properly grounded tasks for training primary teachers consistent with design research centred on professional development (Cobb, Jackson, & Dunlap, 2016). Studies such as these draw on theoretical frameworks for the interpretation of how participants engage in the dynamic of professional development; in our case, we use the Mathematics Teachers' Specialized Knowledge (MTSK) model, developed by the team of educator-researchers who designed the task.

Mathematics Teachers' Specialized Knowledge (MTSK)

The MTSK model derives from the aim to understand and analyse the teacher's knowledge specific to the teaching and learning of mathematics, and seeks to construct a detailed analytical tool. Since its inception (Carrillo, Climent, Contreras, & Muñoz-Catalán, 2013) the model has been discussed in various forums and we have given considerable attention to its theoretical foundations (e.g. Montes, Ribeiro, & Carrillo, 2016); at the same time, the model has been put to use with teachers in different educational contexts to analyse their work across a wide variety of mathematical content, and as a result, we have been able to develop a system of analytical categories within the subdomains constituting the model (Carrillo et al., 2018). The model itself contemplates three domains – mathematical knowledge, pedagogical content knowledge and beliefs and conceptions about mathematics and its teaching and learning, the first two of which acknowledge the legacy of Shulman (1986) and Ball et al., (2008), among others. Mathematical knowledge (MK) encompasses a thoroughgoing knowledge of mathematical content (Knowledge of Topics, KoT), its structure

(Knowledge of the Structure of Mathematics, KSM) and its syntax (Knowledge of Practices in Mathematics, KPM). Pedagogical content knowledge can be broken down into knowledge of mathematics teaching (KMT), knowledge of the features of learning mathematics (KFLM) and knowledge of mathematics learning standards (KMLS). As mentioned above, one of the features of the model that we would highlight is the detailed classification of categories within the subdomains, which enables the model to make a fine-grained analysis of the knowledge deployed by the teacher. Other features of note are the redefinition of mathematical knowledge and pedagogical content knowledge as intrinsic to mathematics; the integration of the teacher's beliefs into the model; and the intention to recognise that knowledge involves an intricate pattern of connections between subdomains.

Task design from an MTSK perspective

In our view, the process of designing a task should involve representatives of all those participating, with different degrees of responsibility. In the case we present here, those involved were: three experienced primary educators with more than twenty years' experience in training and research into primary education; two primary educators with from five to ten years' experience in training primary and/or secondary teachers, both also researchers into mathematics education; two new entrants to teacher education with less than three years' experience, and in the process of completing their doctoral studies in the group; one student on a master's course who was studying the process; and three primary trainee teachers (PTTs), one in their final and two in their penultimate year of study. The educators constituted virtually the sum total of educators specialising in mathematics education at the University of Huelva. The PTTs were selected *ad hoc*, with an emphasis on those students with high grades in subjects related to mathematics, with a good capacity for reflection and analysis, a willingness and availability to meet with (part of) the team of educators, and a disposition to both provide and receive positive criticism regarding the training programme in which they were participating.

Tasks have a form, a function, and a specific focus (Grevholm, Millman, & Clarke, 2009). The task we describe here pertains to a set of teacher training tasks designed to stimulate the trainees' construction of professional knowledge of geometry, specifically the concept of a polygon. To achieve this goal, three tasks were designed around exemplification, the notion of a polygon, definition and classification. In acknowledgement of the potential of video as a tool for initial training (Schoenfeld, 2017), we decided that each set of tasks would be prefaced by a recording of pupils responding to some relevant mathematical content as a way of leading into the task. This obviously meant that any recorded material that might be used need to be carefully analysed first for its potential. In this instance, we used a recording of a teacher introducing her pupils to the notion of polygon by presenting a variety of cardboard shapes for them to group together and subsequently arrive at an acceptable definition of a polygon. The teacher included a number of non-standard shapes, and by presenting them as cardboard cut-outs, and hence non-static, she avoided the risk of standard positioning influencing the pupils. The video illustrated how the teacher acted as guide, largely leaving the pupils free to make groupings according to their own criteria, and encouraging them to reflect on these.

The process of planning the associated task followed several stages:

- 1) *Watching the video*: the three representative PTTs watched the recording and recorded their impressions against a checklist of the following items: teaching strategies used by the teacher; pupils' thinking strategies, intuitive ideas and difficulties; mathematical content in play and specific aspects of the content brought to the fore; task type; resources used – potential, limitations and use; appropriateness for the syllabus; examples given, representations of the content and resultant problems; teacher's knowledge of the content; teachers knowledge of the teaching and learning of the content. The checklist, drawing on the MTSK model, was designed to focus trainees' attention on relevant aspects of the excerpt while they watched, always with a mind to the mathematics involved. The PTTs were asked to follow up the viewing with an individual analysis in the form of a written report.
- 2) *Informal seminar*: The three PTTs met with one of the educators to discuss the video and identify the key items which had sparked their reflection. The educator took the role of interviewer, drawing out from the collaborating students their impressions of what they had watched. The aim of this session was to explore what the educators might reasonably expect trainee teachers to make of the recording, and in fact the PTTs' perception of the potential of the recording for use with trainees was also discussed, making them participants in the design process. The complete seminar was video recorded.
- 3) *Designing small group tasks*: The team of educators agree a focus for each of the three tasks and establish a sub-group for each one, making sure that each contains an experienced as well as a novice educator. The foci derive from areas of interest arising from the students' analysis (issues deemed by them to be relevant to their professional training). Keeping in mind the previously agreed objectives, each group selected the subdomains and categories of the MTSK model which they felt were most compatible with each objective, and created sub-tasks dealing with each objective, along with an 'ideal' solution to the problem specifying which elements of knowledge they expected to be brought into play (making reference to the MTSK indicators).
- 4) *Educators discussion group*: Each sub-group presented its task to the whole group, and this was discussed and refined, returning to step 3 if necessary. The discussion aimed to ensure that (i) the task matched the agreed objectives, (ii) the MTSK indicators facilitated the objectives, and (iii) the sub-tasks contributed to the construction of the elements of knowledge reflected in the indicators. The main objective of these discussions was to improve the task, but at another level there was a secondary aim to aid the development of the novice educators. When the task was generally considered to be ready to use, it was given to the PTTs to be carried out individually.
- 5) *Small group implementation*: Once the PTTs had done the problem individually, they met with an experienced and novice educator to talk through their solutions. The PTTs took turns to present their solutions, which the group then discussed, also taking the opportunity to consider any digressions from the task which the educator deemed interested. After the discussion, the PTTs were asked to reflect on (i) the potential and limitations of the task for use on their training programme, (ii) the likely response of their fellow trainees to the task

and how they would likely tackle it, and (iii) the extent to which they felt the task was appropriate to their training.

- 6) *Closing discussion*: All the educators met to evaluate whether the task had enabled the students to reach the agreed objectives. To do this, the knowledge deployed by the three students was measured against the MTSK model, on the basis of which the task was further fine-tuned into its final version ready to be used with the whole group, and was added to the collection of completed tasks. The process then looped back to stages 3 and 4 until the remaining tasks were completed in the same way.

One of the tasks, dealing with the definition of a polygon and focusing only on mathematical knowledge, is described below as an example.

In their analysis of the recording of the primary lesson on polygons, the PTTs provided contrasting definitions and images of a polygon. The key features they agreed on were that a polygon was a closed, flat figure with straight sides, and could be considered in terms of both the interior and boundary. To this extent they largely agreed with the definition which was constructed in the course of the lesson (a polygon “is a flat shape, it has angles, it has vertices, it has straight sides and no curves; all the sides are joined at their extreme points”), barring the tautological “straight lines and no curves”. They consider this definition to “be close to the correct one” (which, it seems they believe, is unique). They also dismiss size as integral to the concept of a polygon, differentiating between mathematical qualities (which play a role in the definition) and those qualities which they denominate “physical” (which play no role in the mathematical concept). They consider that a polygon has to have the same number of sides, vertices and angles. A doubt emerges as to whether a segment of curved line can be called a side (for example, can we say that a circular sector has three sides?). In this respect, one of the PTTs considers that “a curved side is infinite sides” since, as they go on to explain, “the circumference is the limit of a succession of polygons in terms of the number of sides.” For the educators, this contribution to the discussion represented an opportunity within the task for studying degeneracy. For their part, the PTTs recognise as polygons rectangles, squares, triangles, regular and irregular polygons, and concave and convex polygons; and they reject as polygons circumferences, circular sectors and polyhedra.

We saw the opportunity from the above to take the students further into the concept of polygon, its key features and possible definitions. With this in mind, we planned an activity which started by asking them to discuss different definitions of a polygon (giving examples of figures which complied with the definition and those which didn't). They were also asked to compare each definition with the one they had previously deemed correct. Below is a selection of the definitions they were given:

- a) A polygon is a flat figure with an edge.
- b) A polygon is a continuous line with an angle between each segment.
- c) A polygon is the flat region delimited by a jagged line where I can differentiate the inside from the outside.
- d) A polygon is the flat region delimited by a closed polygonal line, and which has the same number of vertices, sides and angles.

- e) A polygon is a region delimited by a closed polygonal line, such that given any two points on it, the segment joining the two is always within said region.

The aim of this activity was to encourage the PTTs to explore the properties of polygons, the interconnections between these, and what can be considered essential properties. In particular, it sought to highlight the notions of concavity and convexity, connected components, examples and counterexamples of polygons (KoT – definitions, properties and foundations), the uniqueness of a concept's definition and the equivalence of definitions (KPM), geometric vocabulary (KoT – registers of representation) and comparison of definitions (KoT – procedures). In the course of the activity, the PTTs become immersed in a process of coming up with examples and counterexamples, attempting to find what they themselves term 'unusual polygons'. Some of the definitions, such as (a) and (b), struck them as very natural; in others, such as (e), they had to think back to the case of concave polygons. The discussion of the remaining definitions led them to consider whether two shapes (Figure 1) were polygons.

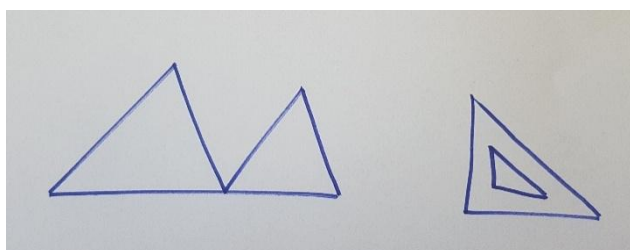


Figure 1: Shapes suggested by the primary trainee teachers

Student 1: For me, the shape that looks like a 'set square' [referring to the technical drawing aid] is very striking. I think it is a polygon, but it's got that hole, which is 'unusual'.

Educator: And what would the sides be?

[Student 1 indicates the 'exterior' boundary]

Student 2: Yeah, it's the same for me, those should be the sides, but the ones inside... What could they be? It's the same with the two triangles joined at the base, is that one side or two? If you turn one of the triangles slightly, leaving them joined at the vertex, then they'd be two, but the number of vertices, sides and angles don't add up.

This debate led them to consider the relationship between sides, vertices and angles that a polygon must fulfil (KoT), or, more formally, the number of connected components that the interior of a figure should have in order to be a polygon (KoT). At the same time, they also recognised that in mathematics it is possible to look for relationships between different definitions of the same object so as to tell if they really define the same object (KPM). For the educator, this presented an opportunity to consider the limits of the degree of complexity that can be demanded from the students' understanding.

The discussion that ensued revealed it had been interesting for the PTTs to consider which of the figures – among those that struck them as common, or even particular to the interaction of the

pupils with the educational material (as in the case of the set square) – prompted a richer discussion of mathematics, and they recommended exploring examples such as these as a means of extending trainees’ knowledge beyond the standard examples of school mathematics.

In summary, the confluence of carefully developing the task, using the MTSK indicators, and drawing on the experience of the educators and the impressions of the specially chosen PTTs enabled professionally-oriented training tasks to be devised, which tap into the specific elements of specialised knowledge required for teaching. Including the contributions of prospective primary teachers in the development process will undoubtedly help to ensure that future students engage in the tasks with a high degree of motivation.

Final reflections

One of the chief aims of primary teacher education should be the construction of the knowledge base which will enable newly qualified teachers to meet the challenge of their first classroom experiences with success. To this end, the work described here uses a model of professional knowledge to define suitable aims and points of focus. The model, MTSK, guides the choice of elements to include in the process of construction, such as the properties of polygons in the task described above (KoT), and the coordination of different definitions (KPM). A remaining challenge is how to evaluate the knowledge aimed to be developed with this task and the relationship between the elements of the task and the knowledge built, which will be addressed further. Likewise, we assume a second challenge, consisting in the necessity of scaling up from task designing for/some PPT to design for ‘many’ of them (Prediger, Schnell, & Rösike, 2016).

A good starting point to face this necessity is the design of tasks developed from one-on-one teaching experiments (Cobb & Gravemeijer, 2008), which can then be presented to the whole group. The involvement of high profile PTTs for appraising the task in terms of its training potential provides an opportunity to collate suggestions on how it might be improved and implemented. Likewise, the dynamic of the discussion group, involving both experienced and novice educators represents a context for constructing trainers’ knowledge, MTSK also playing a structuring role in this training, as suggested by Kilpatrick and Spangler (2016).

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