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► **To cite this version:**

Marlova Estela Caldato, Miguel Ribeiro. An analysis of the nature of the knowledge disseminated by a mathematics teacher training policy: The PROFMAT case. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02430445

HAL Id: hal-02430445

<https://hal.science/hal-02430445>

Submitted on 7 Jan 2020

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An analysis of the nature of the knowledge disseminated by a mathematics teacher training policy: The PROFMAT case

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This paper discusses the nature of the knowledge disseminated by the Brazilian public policy PROFMAT for training teachers on natural numbers, based on the analysis of one of the textbooks that outline the formative activities developed in this program. This discussion was structured and developed from a conceptual and analytical model of the mathematical knowledge of teachers- Mathematics Teacher's Specialized Knowledge (MTSK). The analyses and discussions developed suggest that this public policy favors only part of the knowledge taken by the literature as essential for the practice of the mathematics teacher in the school.

Keywords: Mathematics teacher's specialized knowledge, PROFMAT, public policy.

Introduction

Educational policies are configured as sources pertinent to the investigative processes that seek to identify the specificities of the teacher's knowledge (Stylianides & Ball, 2004), since they demonstrate the response produced by the government bodies to an issue that permeates the educational discussions related with what teachers need to know in order to teach mathematics adequately (Ball, Lubienski, & Mewborn, 2001).

Thus, the object of discussion of this article is a Master's Program in Mathematics (PROFMAT), aimed at the continuous training of mathematics teachers (lower and upper secondary), developed in all Brazilian states and financed by the country's government – assuming the character of Public Policy. According to the official documents, such a Program aims to provide in-depth and relevant mathematical training for teaching in Basic Education, seeking to give teachers a certified qualification for the mathematics teacher profession (Bsm, 2017).

The role played by the mathematical knowledge of teachers to improve the quality of learning opportunities justifies, at least in part, the growing focus of research on teachers' mathematical knowledge (Stylianides & Stylianides, 2014). This knowledge has been widely studied and, from different points of view, diverse conceptualizations of teacher knowledge have emerged (e.g., Knowledge Quartet - Rowland & Turner, 2007, MKT - Ball, Thames & Phelps, 2008; COACTIV – Kleickmann et al., 2015; MfT-Stylianides & Stylianides, 2014; MTSK-Carrillo et al., 2018). In this text, we focus on the Mathematics Teacher's Specialized Knowledge - MTSK conceptualization (Carrillo et al., 2018) as the analytical model of this type of knowledge. Thus, we will support the MTSK to discuss teacher training developed within the scope of PROFMAT, seeking to answer the following question: what knowledge about Natural Numbers and their teaching are presented (and not presented) in the textbook that guides a part of the training developed in PROFMAT?

Theoretical framework

The MTSK is an analytical model that assumes that the professional practice of the math teacher requires a set of specific knowledge strictly related to the teaching of mathematics. This includes the meanings, properties and definitions of particular topics, means that favor the construction of the understanding of the subject, connections between mathematical items, knowledge of mathematics teaching and characteristics associated with learning mathematics, among others (Carrillo et al., 2018). In this sense, the MTSK model (Figure 1) considers three subdomains in Mathematical Knowledge (MK) and three in Pedagogical Content Knowledge (PCK). In this way, MK is subdivided into: Knowledge of Topics (KoT), Knowledge of the Structure of Mathematics (KSM) and Knowledge of the Practice of Mathematics (KPM), while PCK is subdivided into: Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM) and Knowledge of Mathematics Learning Standards (KMLS).

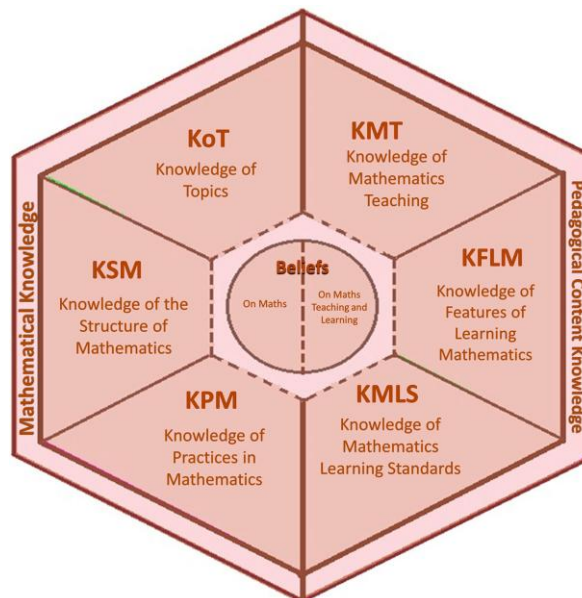


Figure 1: Sub-domains of the MTSK
(Carrillo et al., 2018)

The KoT includes phenomenological aspects, meanings of specific concepts and examples that characterize specific aspects of the topic addressed, aside from contemplating the disciplinary content of the mathematics addressed by textbooks and other materials of a pedagogical nature. KSM refers to the understanding that the knowledge of teachers, besides including concepts as isolated elements, must integrate them into a system of connections, which allows the teacher to understand certain advanced concepts from an elementary perspective and develop certain concepts from an advanced perspective.

KPM covers aspects related to mathematical thinking, such as knowledge of the different ways of defining, arguing or proving in mathematics, as well as knowledge of mathematical syntax. The teaching action involves the knowledge of how this teaching can and should be developed, so the KMT contemplates knowledge as knowing different teaching strategies that allow the teacher to foster the development of procedural and conceptual mathematical capacities. In the same way, this

subdomain predicts that the teacher needs to know examples that awaken the intuition in the student about some concepts, as well as resources that allow the teacher to induce his students to learn, through manipulation, certain mathematical concepts. Knowing how students learn mathematical content is knowledge that every teacher should possess. Thus, KFLM encompasses the knowledge of the characteristics of the process of understanding the different contents by the students, the errors, difficulties and obstacles associated with each concept and the language used by students in relation to the concept worked in the classroom. The KMLS refers especially to the teacher's knowledge of the curriculum adopted by the institution in all stages / levels of education. This knowledge can be complemented with information present in the productions originating from research in the area of mathematics education, with information provided by experienced teachers about the expected learning in each stage (For further deepening and detailing the subdomains see Carrillo *et al.* 2013, Flores-Medrano *et al.*, 2016 , Carrillo *et al.* 2017).

The context of the study: The PROFMAT

PROFMAT is a teacher training program promoted by the Brazilian Federal Government and developed in partnership with 96 higher education institutions (universities and colleges) that train teachers of mathematics (lower and upper secondary), organized in the format of professional **masters**. This program has a length of 2 years and is basically composed of a list of 9 subjects, each with a 120-hour workload. Of these, 7 of them are considered as mandatory and titled "Real Numbers and Functions", "Discrete Mathematics", "Arithmetic", "Geometry", "Problem Solving", "Calculus Fundamentals" and "Analytical Geometry". The "elective" disciplines can be chosen by students from the following list: Topics of Mathematics History; Topics of Number Theory; Introduction to Linear Algebra; Differential and Integral Calculus Topics; Mathematics and Actuality I; Mathematical Modeling; Algebraic Polynomials and Equations; Spatial Geometry; Topics of Mathematics; Probability and Statistics; Educational Evaluation; Numerical Calculation; Mathematics and Actuality II. The final course work refers to the production, by the academic under the guidance of a university professor, of a product/work that can be presented in different formats (dissertation, literature review, article, patent, applications, teaching and instructional materials , media programs) and that addresses topics relevant to the Mathematics curriculum of Basic Education and promotes the impact on didactic practices in the classroom.

These disciplines are based on the collection of textbooks entitled "PROFMAT Collection", whose elaboration is the responsibility of mathematicians, and each discipline is based on a textbook from the collection, which is adopted by all 96 institutions where PROFMAT is developed.

Here we focus on analyzing and discussing one of the elements that make up the teacher training process developed in the scope of PROFMAT from the analytical model MTSK. However, the analyzes presented here are part of a project that has analyzed a multiplicity of elements of the PROFMAT: policy texts, textbooks that guide the training activities; class episodes; and PROFMAT academic productions (answers to questionnaires, works, etc.).

In this analysis, we classify all the contents that compose the textbooks (definitions, axioms, theorems and demonstrations, properties, activities, exercises, explanations, etc.), from the subdomains of the MTSK framework, with the objective of identifying the presence and absence of

knowledge related to the teacher's practice in mathematics teaching, as proposed by Carrillo and contributors (Carrillo *et al.*, 2013, Flores-Medrano *et al.*, 2016, Carrillo *et al.*, 2017, Carrillo *et al.*, 2018).

It is highlighted in this scenario that, through the following discussion presented, we will seek to note indications of the nature of the knowledge disseminated by PROFMAT, since the knowledge produced from a textbook depends on the manipulation that is made of this didactic material, particularly, the manipulation developed by the teacher who uses it.

Due to limitations of space, we focus here on two chapters of one of the textbooks used in PROFMAT called "real numbers and functions" (which describes the work developed in the mandatory subject that has the same name). We chose to analyze this textbook because it guides the teaching activities of one of the four compulsory subjects developed during the course of the PROFMAT, besides addressing mathematical themes that permeate themes also common to the school curriculum, such as numerical sets and functions (NCTM, 2000). In this analysis, we will focus on "Natural Numbers" and for the analysis, we use MTSK as a theoretical lens as a way of discussing the nature and focus of such a course. With this, in the sequence, we will present some representative examples of the analyzes that we developed.

Analysis and discussion

The textbook "Real Numbers and Functions" (Lima, 2013) is structured from the following sequence of themes (chapters): Sets; Natural Numbers; Cardinal Numbers; Real Numbers; Related Functions; Quadratic Functions; Polynomial Functions; Exponential and Logarithmic Functions; Trigonometric Functions. This organization is justified by the author, who states "Mathematics deals primarily with numbers and space. Therefore, the sets most frequently found in Mathematics are numerical sets, geometric figures (which are sets of points) and the sets that derive from them, such as the function sets, those of matrices" (Lima, 2013, pp. 02). This structure presents opportunities for the development of MK, in particular KSM, by teachers in training, since it favors the (potential) development of connections between the numerical sets and functions that establish relations between subsets of \mathbb{R} .

The chapter "Natural Numbers" is initiated by the author with a description of the meaning of the term "numbers" as being "abstract entities, developed by man as models that allow to count and measure, therefore to evaluate the different quantities of a magnitude" (Lima, 2013, pp. 22). "Mathematical definition" is described as "a convention consisting of using a name, or a brief sentence, to designate an object or property, the description of which would normally require the use of a longer sentence" (Lima, 2013, pp. 23), axiom and primitive concept. These descriptions can be associated with MTSK by means of KPM, as it returns to the fundamentals associated with mathematical proceedings.

The author also presents a description of the axiomatic method and in relating it to the school argues that "[...] it is not appropriate to present Mathematics in an axiomatic form. But it is necessary for the teacher to know it can be organized in the manner outlined above" (Lima, 2013, pp. 23). As this fragment shows, the author argues that it is necessary for the teacher to know that mathematics can be organized in the axiomatic form (KPM), but it does not elucidate, for example,

why it is necessary for the teacher to know this form of mathematical exposition (KoT), nor "how" (KMT) and "when" (KMLS) this method would aid in the teaching of mathematics.

Lima (2013, pp. 26) links natural numbers to historical aspects (potential indications of KPM) by saying that "Slowly, to the measure that it has become civilized, mankind has taken over this abstract model of counting (one, two, three, four, ...) which are the natural numbers". The author also argues that, currently, the set of natural numbers can be described as "concise" and "precise" by Peano's theory, which states that "N is a set, whose elements are called natural numbers. The essence of the characterization of N lies in the word "successor" (if $n, n' \in N$, then n' is the successor of n means that n' comes soon after n , there being no other natural numbers between n and n'), which is considered a primitive concept because it is not explicitly defined. The author then mentions that "everything" about natural numbers "can be demonstrated" from Peano Axioms¹.

According to Lima (2013, pp. 27), "an ingenious process, called a decimal number system, allows us to represent all natural numbers with the help of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. In addition, the first natural numbers have names: the successor of number one is called two, the successor of two is called three, etc." This presentation of decimal numbering systems can be linked to KoT (knowing the set) on the one hand, and on the other, to KPM (knowing the role of the successor and the associated mathematical structure – e.g., continuity, density). Meanwhile, the author avoided discussing, for example, the different aspects of the mathematical knowledge underlying the construction and use of the decimal system (KoT), such as the notion of grouping, the language involved in the reading of numbers and the idea of positional value (KoT; KPM). In this case, it is important to highlight the lost opportunity for elaborating some discussions on, for example, the resources and examples in and for teaching such topic (KMT), concerning the students' difficulties (KMLM) and the content of the official documents (KMLS).

Lima (2013, pp. 27) then argues that the set $N = \{1, 2, 3, \dots\}$ is a "sequence of abstract objects that are, in principle, empty of meaning" and that "each of these objects has only one place determined in this sequence". Since "Every number has a (single) successor and, with the exception of 1, it also has a single predecessor. Viewed in this way, we can say that natural numbers are ordinal numbers: 1 is the first, 2 is the second, etc.". In this fragment, Lima presents a powerful scenario for an exploratory work of KoT (when discussing the definition of natural numbers); however, although the author recognizes that the objects of the sequence of natural numbers are abstract and are in principle empty of meaning, he does not discuss, for example, what the teacher's work could be in addressing this content in the school environment (PCK), in addition to the symbol of "..." being placed in the representation of the set of natural numbers, but without making any comment about the meaning of this symbol (KoT and KPM). Lima also does not present indicatives of "how" (KMT) and "when" (KMLS) Peano axioms could be addressed in math classes at school.

¹ I) Every natural number has a single successor; II) Different natural numbers have different successors; III) There is a single natural number, called one and represented by the symbol 1, which is not a successor to any other; IV) X is a set of natural numbers (that is, $X \subset N$). If $1 \in X$ and if, in addition, the successor of every element of X still belongs to X, then $X = N$.

The author also defines the "fundamental" operations among natural numbers, and the addition of numbers $n, p \in \mathbb{N}$ corresponds with the sum " $n + p$ " and multiplication associates the product " np " (Lima, 2013). It also re-elaborates them in such a way as to present the sum " $n + p$ " as "the natural number that is obtained from applying n p times following the operation of taking the successor", so that " $n + 1$ is the successor of n , $n + 2$ is the successor of the successor of n , etc." (Lima, 2013, pp. 29) and presents, as an example, the addition of $2 + 2 = 4$, where " 4 is the successor of the successor of 2 ". For the product and defined as $n \cdot 1 = n$ and, when $p \neq 1$, " np is the sum of p plots equal to n ". In this context, it is argued that "until we know how to use natural numbers to make counts, there is no point in speaking in 'p times' and 'p plots'. Therefore, fundamental operations must be defined by induction" (Lima, 2013, pp. 29). In the following, Lima defines Addition and Multiplication by induction (KPM - from a level of knowing how to perform). However, the operations are not linked to the practices of mathematics teachers, which could have been done by discussing how these operations are approached in the school curriculum (KMLS), what kind of resources - and their intentionality - could be used to explore such operations with students (KMT) in order to understand what they do and why; and the students learning processes linked to understanding multiplication mainly as a solely repeated additions (KFLM).

In addition, the author constructs the set of natural numbers and only two of their operations (addition and multiplication), omitting the definition of the operations of subtraction and division between natural numbers (KoT), which are also used by teachers in school. The author also does not link the definitions of addition and multiplication (and consequently of subtraction and division) to the algorithms commonly used by school materials (KMLS) and used in calculating these operations (KoT). Another relevant observation refers to the non-linking of this work with one of the main practices of the Mathematics teacher, to give meaning to the operations among the natural ones through problem situations and problem history (KMT).

In addressing the Natural Numbers, Lima (2013) presents the order relationship between natural numbers in terms of addition ("Given $m, n \in \mathbb{N}$, it is said that m is less than n , and we write $m < n$ to mean that there is some $p \in \mathbb{N}$ in which $n = m + p$ ") and presents the properties of Transitivity, Trichotomy, Monotonicity of Addition and Multiplication, and Good Ordination. This fragment highlights the potential of the work in the development of KoT and KPM. However, the author does not present any indication of discussions regarding PCK subdomains.

In the thread of the work the author presents and demonstrates properties of \mathbb{N} : Associative property of Addition, Commutative property of Addition, Distributive property, Commutative property of Multiplication, Cancellation Law for Addition, Transitive Order Relationship, Trichotomy, Monotonicity, Cancellation Law for Inequalities and the Principle of Good Ordination. Lima argues that the adoption of this approach was because it expresses "[...] some basic facts about natural numbers, which are used very often, most of the time without us stopping to ask how to prove them." He argues further that the "[...] objective is to show how such facts result from Peano axioms. There is no creative reasoning or elaborate methods to prove them. In all demonstrations, the central role is played by the Induction Axiom" (Lima, 2013, pp. 31).

This approach of N properties again highlights the potential of the material in relation to the development of KoT and KPM. However, at no point in the work does the author present any discussion that may be associated with the development of PCK. For example the author does not discuss, whether these demonstrations are accessible to students of basic education, whether they need to be adapted (KMT) to become accessible to school students (KFLM) and in which years it is possible to use them (KMLS).

Final comments

The developed analyses of a textbook that guides the formative practice in a discipline of PROFMAT show indications of the nature of the knowledge disseminated through this public policy of continuous training of teachers that aims to impact the quality of the learning of Brazilian students. Thus, based on the MTSK, it was found that elements of MK are considerably favored, particularly those associated with properties and definitions (KoT) and mathematical practice (KPM), such as the forms of validation and demonstration and the use of formal language. On the other hand, this material presents very few indications of the presence of knowledge related to the PCK, such as those related to the characteristics of the teaching (KMT) and learning (KFLM) of Natural Numbers and how this content is included in the school curriculum (KMLS).

The results presented provide us with indications that this public policy favors only part of the knowledge taken by the literature (Carrillo *et al.*, 2013; Flores-Medrano *et al.*, 2016; Carrillo *et al.*, 2017; Scheiner *et al.*, 2017; Carrillo *et al.*, 2018) as essential for the practice of the mathematics teacher in school. With this, it is possible to affirm that PROFMAT will not be able to fully achieve the objectives it proposes, which is to change the mathematical education of the country qualitatively and quantitatively, especially if we consider that the work developed in this program follows rigid evaluation standards that are guided by the textbooks (see Caldato, 2015, Caldato, Pavanello & Fiorentini, 2016).

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