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# Emergent model for teachers' conceptions of argumentation for mathematics teaching

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*In recent years, there has been a growing appreciation of the importance of incorporating argumentation into the mathematics classroom. Whereas considerable research has been done on argumentation, little has specifically focused on teachers' conceptions. This paper presents an exploratory study as part of ongoing research into teachers' conceptions of argumentation for teaching mathematics. Drawing on the literature on argumentation and empirical data, we propose an emergent model, presented as a 5-by-2 construct featuring five types of conceptions across two dimensions: structural and dialogic. This paper illustrates the emergent model at this initial stage of the research using an individual case study of a teacher. The overall aim of the research is to theorize a model that can be used to analyze and characterize teachers' conceptions of argumentation for teaching mathematics.*

*Keywords: argumentation; teachers' conceptions; mathematics teaching*

## Introduction

In recent years, there has been a growing appreciation of the importance of incorporating argumentation into the mathematics classroom. Existing research suggests that participation in argumentation activities that require the student to explore, confront, and evaluate alternative positions, voice support or objections, and justify different ideas and hypotheses, promotes meaningful understanding and deep thinking (Asterhan & Schwarz, 2016). This view is reflected in recent educational reform documents all over the world, and in Israel in particular, that underscore argumentation as one of the important goals for students. However, argumentation in the mathematics classroom is not yet commonplace (e.g., Bieda, 2010). Recently, there has been considerable research dedicated to argumentation; however, little of that work has specifically focused on teachers' conceptions of argumentation (Ayalon & Hershkowitz, 2018; Staples, 2014). Considering the fact that teachers' conceptions impact the way in which this key practice is implemented in the classroom, there is a strong need to learn what characterizes them. This paper addresses this need. It presents an exploratory study as part of ongoing research into teachers' conceptions of argumentation for teaching mathematics. Drawing on the literature on argumentation and empirical data, we propose an emergent model, presented as a 5-by-2 framework comprising five categories of conceptions of argumentation for teaching mathematics across two dimensions: structural and dialogic. This paper illustrates the emergent model at this initial stage of the research.

## Theoretical Background

### Argumentation

There are diverse definitions of argumentation in the education literature. A commonly accepted definition is that of van Eemeren and Grootendorst (2004) who argue that argumentation is "a verbal, social, and rational activity aimed at convincing a reasonable critic of the acceptability of a

standpoint by putting forward a constellation of propositions justifying or refuting the proposition expressed in the standpoint” (p. 1). According to this definition, argumentation entails generating claims, providing evidence to support the claims, and evaluating the evidence to assess their validity. It posits argumentation in a social context and, if incorporated in classroom discourse, affords a venue for the articulation and critical evaluation of alternative ideas, eventually supporting collaborative knowledge construction (Asterhan & Schwarz, 2016). Argumentation that "balances between critical reasoning and collaborative knowledge construction” (Asterhan & Schwartz, 2016, p. 167) is considered to be productive for learning (Felton, Garcia-Mila, & Gilabert, 2009).

Following these descriptions, the present work considered argumentation as having two important meanings – structural and dialogic (McNeill et al., 2016). The structural meaning of argumentation focuses on the aspect of discourse in which a claim, presented as an idea, conclusion, hypothesis, solution etc., is supported by an appropriate justification which, in our case, represents the types of justifications that are valued within the mathematics community. The dialogic meaning regards argumentation as the interactions between individuals when they attempt to generate and critique each other’s ideas. This meaning aligns with the common view of mathematics as a social enterprise whereby mathematicians are part of a community with established norms of argumentation for advancing mathematical knowledge.

### **Teaching for argumentation**

Mathematics teaching that encourages argumentation provides students with opportunities to take an active part in both structural and dialogic meanings – to construct arguments, share, consider others' ideas and critically evaluate their validity (Ball & Bass, 2003). Such teaching requires the teacher to make sense of students' ideas and interactions, identify and evaluate the strengths and weaknesses, promote adherence to standard disciplinary criteria for determining the truth of a claim, encourage students to elaborate their thinking, and lead them to listen to each other, critique and question ideas (Nathan & Knuth, 2003). It entails the teacher's drawing on various resources, such as the mathematics involved, student thinking, socio-cultural background, affect, and curriculum-related aspects (Ayalon & Even, 2016; Ayalon & Hershkowitz, 2018; Staples, 2014). As noted above, argumentation in the mathematics classroom is rare (e.g., Bieda, 2010), which suggests that teaching for argumentation is perhaps challenging. Whereas research on argumentation is rapidly growing, little research specifically focuses on teachers’ conceptions in the context of argumentation.

This study draws on existing research to propose a preliminary working definition for mathematics teachers' conceptions of argumentation for mathematics teaching, with the intention of further developing this definition as a research goal. The use of ‘conceptions’ refers to both knowledge and beliefs, according to Thompson (1992), who described teachers’ conceptions of the nature of mathematics as their combined knowledge and beliefs pertaining to the discipline of mathematics. Following this definition, the present study focuses on teachers’ conceptions of argumentation in mathematics teaching, under the assumption that in order to develop teachers' instructional practices for argumentation, we need to better understand their conceptions. In particular, we focus on teachers' conceptions as they relate to both the structural and dialogic aspects of argumentation.

## **Methodology**

The data for the overall research were collected through observations of lessons and semi-structured interviews which preceded and followed the lesson observations. For this paper, we focused on data obtained from the first part of the interviews conducted prior to the lesson observations, in which the teachers were asked to express their views on argumentation for teaching mathematics and to provide examples of argumentation, as manifested in their own teaching.

### **Research participants**

Eight middle-school mathematics teachers, each having more than five years of teaching experience, participated in this study. The decision to focus on this particular school population stemmed from the emphasis placed on argumentation in the middle-school curriculum in Israel.

### **Data collection**

The data used for this paper consisted of individual, semi-structured interviews with the teachers. The interview lasted approximately one hour. During the interviews, the teachers were presented with a written quote from the national mathematics curriculum that reflects the importance ascribed to students' involvement in argumentation activity in the mathematics classroom. According to this quote, one main goal of the curriculum is that students will engage in justifying their claims, communicate them to others, and critique their own and their peers' arguments. The teachers were asked: (1) What do you think about this quote? Do you agree or disagree? Why? (2) What strategies, if any, do you use in your classroom to achieve this goal? (3) What have you found supports or hinders you in achieving your goals? The teachers were urged to provide detailed responses as well as instances from their own classroom. The interviews were audio-recorded and subsequently transcribed.

### **Data analysis**

We combined directed content analysis and inductive content analysis (Patton, 2002). Directed content analysis included a classification of teachers' statements about argumentation in mathematics teaching into one of the two aspects of argumentation: structural or dialogic. The structural aspect dimension included discourse pertaining to elements of arguments, such as claims and justifications and what counts as an appropriate justification in class. The dialogic aspect dimension included discourse associated with students' interactions when generating and critiquing arguments. We then used inductive content analysis for devising categories for the two dimensions. During this phase of analysis, we first identified initial categories based on some of the data collected for a particular teacher. We then refined and expanded the initial categories based on more data collected for the same teacher. Finally, the devised categories were refined by the analysis of data collected for all the participating teachers. This process resulted in five categories of teachers' conceptions of argumentation: (1) what is argumentation; (2) teaching strategies for argumentation; (3) mathematical task characteristics; (4) student characteristics; and (5) socio-cultural characteristics. We ultimately received a 5-by-2 framework featuring these five categories for each of the two dimensions: structural and dialogic.

In this paper, we provide examples that illustrate the way in which several of the framework's different components were devised based on the teachers' discourse about argumentation in

teaching. The examples of categories are taken from an interview with one teacher named Adam (pseudonym), as we found his interview rife with illustrations for various categories. Note that each category was identified in other teachers' interviews as well. Naturally, one citation may refer to more than one aspect of argumentation.

## Findings

Adam is a middle-school teacher (grades 7-9) with 25 years of experience in teaching mathematics. He serves as the mathematics teaching coordinator and is considered by his colleagues to be a leading teacher in his school. When asked for his opinion of the quote taken from the curriculum with regard to the importance of engaging students in argumentation, he replied:

The things said in the quote are very important in the learning of mathematics. This [argumentation] helps to promote students' understanding of the material; the teacher can understand, identify and emphasize the thought process of each student who raises claims in class and justifies his or her answer [1-1] ... Also, critiquing others' arguments helps students to develop their mathematical thinking [1-2].

This initial response from Adam implies *what argumentation means* to him. His response relates both to the structural aspects of argumentation and to its dialogic aspects. He mentions claims and justifications (structural, 1-1), alongside critical assessment of one another's arguments (dialogic, 1-2). According to Adam, this activity allows the teacher to learn about students' thought processes, and helps to develop students' mathematical understanding. When asked how he uses argumentation in his classes, he replied:

In my classes, I always use argumentation activities. The students raise ideas, explain, and bring justifications [2-1]. Different solutions for the same problem arise, and a fruitful dialogue develops in which the students explain their solutions, try to convince others, and find mistakes in others' solutions, and critique each other [2-2]. The students listen to each other and give criticism politely and respectfully [2-3]. This atmosphere encourages students to participate in math classes, so that the student feels like a central part of the lesson [2-4]. Also, a student who correctly justifies her/his solutions will feel greater self-confidence [2-5].

Adam's answer informs us more of *what argumentation means* to him, both from a structural perspective (2-1) and a dialogic perspective (2-2, 2-3). This answer expands on his previous response by relating, also, to persuading others of the correctness of one's views, and to assessment that is not only critical, but also respectful (2-3). There is mention here of *class norms* of respectful critical dialogue (2-2, 2-3). He also mentions *students' emotional characteristics* – student engagement and the student being in the “center” – that are connected to the student's participation in dialogue (dialogic, 2-4) and to self-confidence stemming from justification of a solution (structural, 2-5). When he was asked to give more details regarding the way in which he encourages argumentation activities in his class, Adam said:

In general, I try to give the students open, multiple-solution tasks [3-1], and give students space to express themselves and to present their solutions and examine them together with the rest of the class, and reach agreements together [3-2]. For instance, tasks involving word problems, or problems that involve finding generalizations – there are students who manage to come up with various hypotheses and solutions, some correct and some not, and there are students who

struggle [with the challenge] [3-3]. What's important is that when we work with these tasks in my class, it always turns into a mathematical discussion [3-4] which the students enjoy [3-5], and in which students listen to each other's solutions and explanations and critique them, try to challenge their arguments and also to defend arguments [3-6]. When I give them problems to solve individually, my sense is that the students feel like it's a competition, who will find a solution first and give a correct justification [3-7] - after all, I don't accept answers without a clear, written explanation and mathematical justification [3-8] - and who will present their solution to the class [3-9] ... I mean "competition" in a positive sense.

In Adam's answer, he notes the *characteristics of mathematical tasks* that encourage argumentation in class. Open, multiple-solution tasks serve Adam's purposes when it comes to the dialogic activity he mentioned earlier [3-1]. Here, Adam also discusses *students' skills* in coming up with hypotheses and various methods of solving (3-3). Two additional characteristics of the dialogic aspect of argumentation, which Adam did not mention earlier, come up in the context of working with these types of tasks: one relates to students working towards consensus (3-2); the other relates to defending one's arguments (3-6). This response also makes note of the *socio-cultural norms* that pertain to argumentation activity, in Adam's mention of mathematical dialogue: "when working with these tasks in my class, it *always* turns into a mathematical discussion" (3-4), and in the expectation in his class for a justification of the claims (structural, 3-9). He also mentions the norm of writing arguments clearly (3-8). Adam previously mentioned students' engagement in argumentation activities; here, he adds a competitive but friendly, good-spirited atmosphere between the students, challenging who will solve the problem first and present the solution, with its justification, to the whole class (3-9). He also mentions enjoyment (3-5).

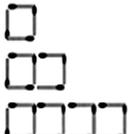
Adam was then asked to give an example of a hypothetical implementation of an argumentation activity in his class. He chose to talk about employing the known "match train" task, which includes an examination of concrete cases involving small numbers (1, 2, and 3 wagons) and then the formation of an algebraic expression. Figure 1 presents part of a script Adam wrote down for an imaginary dialogue in his class focused on finding the number of matches needed for any number of squares.

When discussing his script, Adam emphasized: "I took into account ways in which students' thinking about building generalizations might be incorrect, such as employing empirical methods or using invalid proportional reasoning" [4-1]. He also referred to his approach of "prompting as many arguments as possible" [4-2] as well as "bringing students' ideas to the class for judgment" [4-3], while "emphasizing important ideas given by students, such as the use of counterexample" [4-4]. In addition, Adam said: "I would use various strategies for providing students with scaffolding in generating arguments [4-5]. For example, by using real matches to help students develop a sense of the situation [4-6], suggesting counterexamples to use in refuting students' claims [4-7] ... and using a table of values to support students' efforts to reach a generalization and to identify invalid claims, such as the one suggested by student #3" [4-8].

Adam's concrete example of implementation of a task provides us with more ideas about the way he conceives argumentation: First, in terms of the structural aspect, with claims and justifications requested in class and distinguishing between accepted and unexpected justifications (4-1). Secondly, in terms of the dialogic aspect, his approach here is very similar to the one he raised with

regard to argumentation in his earlier responses (4-2, 4-3). In this response, we can also see Adam referring to *student characteristics* related to common ways of thinking (4-1), along with a reference to *teaching strategies* associated with promoting argumentation, such as prompting students' dialogue (4-2, 4-3), explicating important mathematical argumentation ideas in class (4-4), and (4-8

Let's make "trains" from matches.



S1: I will continue counting.  
 S2: It is not an efficient method.  
 Teacher: Do you agree? If we would like to build a train with 1,000 wagons, will you count the number of matches?  
 Students: no, it is too hard.  
 Teacher: So what will we do?  
 S3: We will multiply the number of matches by four.  
 Teacher: Students, what do you think about it?  
 S4: I think it is not okay. If we have for example two squares we will need 7 matches and not 8.  
 Teacher: Very good argument. We have here a counterexample for the claim that the number of matches is the number of squares multiplied by four.  
 S5: For each additional wagon we need more three matches.  
 Teacher: What do you think class?  
 Students: yes yes  
 Teacher: Let's draw on the board... Follow me and think about the generalization...

**Fig. 1** A part of the script Adam wrote for an imaginary dialogue in his class

Overall, the above statements from Adam's interview illustrate the way in which several of the different components of the framework were devised. Figure 2 presents the emergent framework for teachers' conceptions on argumentation for mathematics teaching. Several of the categories were identified in Adam's statements as presented in this paper, and therefore appear in the model, accompanied by their indexes in brackets. Other categories were not illustrated in this paper; however, they will be further elaborated upon in the conference presentation.

<b>Mathematics teachers' conceptions of argumentation</b>	
<b>Structural aspects</b>	<b>Dialogic aspects</b>

<b>What is argumentation?</b>	<ul style="list-style-type: none"> <li>• Elements of argumentation: claims and justifications (1-1, 2-1, 4-1)</li> <li>• Types of justifications valued (4-1)</li> </ul>	<ul style="list-style-type: none"> <li>• Students raising different point of view (2-2, 3-2, 4-2)</li> <li>• Students listening critically to each other's arguments (1-2, 2-2, 4-3)</li> <li>• Students defending their ideas (3-6)</li> <li>• Students attempting to convince others of their opinion (2-2)</li> <li>• Students listening to each other respectfully (2-3)</li> <li>• Students working towards consensus (3-2)</li> </ul>
<b>Teaching strategies</b>	<ul style="list-style-type: none"> <li>• Encouraging and scaffolding students' justifications (e.g., through questioning, using concrete examples and tools) (4-5, 4-6, 4-7, 4-8)</li> <li>• Encouraging self-evaluation of claims</li> <li>• Providing criteria for justification</li> <li>• Explicating important mathematical argumentation ideas (4-4)</li> <li>• Requiring written justifications (3-8)</li> </ul>	<ul style="list-style-type: none"> <li>• Encouraging students to present different points of view (4-2)</li> <li>• Encouraging students to respond critically to each other's arguments (4-3)</li> <li>• Giving value to students collaborating on, generating and critiquing arguments</li> </ul>
<b>Task characteristics</b>	<ul style="list-style-type: none"> <li>• Open tasks that afford various solutions (3-1)</li> <li>• The types of justifications that the task invites</li> </ul>	<ul style="list-style-type: none"> <li>• Open tasks that afford various solutions (3-1)</li> </ul>
<b>Student characteristics</b>	<ul style="list-style-type: none"> <li>• Students' ways of mathematical thinking (e.g., tendency to use empirical-based justification) (4-1)</li> <li>• Students' skills of generating arguments, including difficulties (3-3)</li> <li>• Students' self-confidence (2-5)</li> </ul>	<ul style="list-style-type: none"> <li>• Students' skills of presenting arguments to the class</li> <li>• Students' skills of responding to others' ideas</li> <li>• Students' skills of revising their arguments based on the class/group discussion</li> <li>• Students' engagement (2-4)</li> <li>• Students' enjoyment (3-5)</li> <li>• Students' competition: who will present her/his work to class (3-9)</li> </ul>
<b>Socio-cultural characteristics</b>	<ul style="list-style-type: none"> <li>• Norm of providing justifications for claims (3-8)</li> <li>• Norm of writing arguments clearly (3-9)</li> </ul>	<ul style="list-style-type: none"> <li>• Norm of collaborating on generating and critiquing arguments (2-2, 3-4)</li> <li>• Norm of respectful dialogue (2-3)</li> </ul>

**Fig. 2** Emergent model for teachers' conceptions of argumentation in teaching mathematics

## Conclusion

This paper presented an exploratory study as part of ongoing research into teachers' conceptions of argumentation in the teaching of mathematics. It proposed an emergent 5-by-2 framework comprising five categories of conceptions of argumentation for teaching mathematics across two dimensions: structural and dialogic. The study opened with a presentation of the theoretical distinctions related to argumentation, focusing on structural and dialogic aspects as the main dimensions of argumentation (McNeill et al., 2016). The empirical investigation of the teachers' discourse on argumentation in their teaching resulted in five categories of teachers' conceptions of

argumentation: (1) what is argumentation, (2) teaching strategies for argumentation, (3) mathematical task characteristics, (4) student characteristics, and (5) socio-cultural characteristics. These categories reflect the complex process of establishing argumentation in the mathematics classroom and the roles that teachers need to fill in order to facilitate argumentation (Ayalon & Even, 2016; Ayalon & Hershkowitz, 2018; Mueller, Yankelewitz, & Maher, 2014; Yackel, 2002), and adhere to notions of productive argumentation that promotes learning (Asterhan & Schwarz, 2016).

This is the beginning of learning about mathematics teachers' conceptions of argumentation in the mathematics classroom. We intend to continue our research, basing the work on the categories developed in this paper. First, the model will be applied to characterize each participating teacher's conceptions. Second, since the research literature shows that teachers' declarative conceptions can be different from those realized in their actual teaching (e.g., Lev-Zamir & Leikin, 2013), we intend to use the model for analyzing the teachers' lessons, and for ascertaining the relationships between teachers' declarative conceptions and their conceptions 'in-action' with special attention devoted to argumentation. The analysis will also search for new categories, in order to refine the model to be used to analyze and characterize teachers' conceptions of argumentation in the teaching of mathematics.

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