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Teachers' actions in classroom and the development of quantitative reasoning

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The main goal of this paper is to understand how teachers' actions in classroom influence the way students solve a task involving quantitative additive reasoning. The study used a qualitative methodology and a teaching experiment was carried out. One task was proposed to two different classes (2nd and 3rd graders), of the same public school, with two different teachers. The results show that 2nd grade students solved the task with ease, but 3rd graders had difficulties. Teachers' actions of guiding and challenging and how teachers developed the process of communication in classroom do influence students' performance.

Keywords: Quantitative reasoning, teacher's actions, students and teacher interaction.

Introduction

This paper reports on part of the research carried out by a project focused on flexible calculation and quantitative reasoning developed by teachers of the Higher Education Schools of Lisboa, Setúbal and Portalegre in Portugal. The research question for this paper is: How do the teachers' actions in whole class discussion influence the way students solve a task involving quantitative additive reasoning? We present the whole class discussion of a task involving quantitative difference in a 2nd and a 3rd grade classes and analyse how the teachers' actions of guiding and challenging influence students' quantitative additive reasoning (Thompson, 1993).

Theoretical framework

The communication processes that take place in the classroom, as well as the tasks proposed to students, are essential aspects of teachers' practices (Ponte & Quaresma, 2016; Boaler, 2003). Franke, Kazemi and Battey (2007) stress the importance of processes that support students' language development, like resaying - saying the same idea in a different way, usually closer to formal mathematical language, questioning the meaning and supporting the development of students' mathematical thinking. The questions posed by the teacher are a fundamental aspect of communication, with inquiry questions being particularly important, admitting a range of legitimate responses. Related to discourse, Ruthven, Hofmann, and Mercer (2011) propose a dialogic approach as "one that takes different points of view seriously, encouraging students to talk in an exploratory way that supports development of understanding" (p. 4-81). Thus, the decisions that teachers make are crucial, especially those that are taken during the whole class discussions, a particular form of classroom work used in exploratory teaching (Ponte, 2005). It is important that different points of view be considered and that students be encouraged to explain and justify their reasoning and solutions, thus developing, with the teacher's support, their understanding (Ruthven, Hofmann, & Mercer, 2011). In order to have a fruitful whole class discussion, Stein, Engle, Smith, and Hughes

(2008) stress the importance of teachers anticipating the way students may think, monitor their work, collect the pertinent information, select the issues to deal with during the discussion, think about the sequence for students' interventions, and establish connections among different solutions during the discussion. This preparation is important, but a good discussion may involve many aspects that cannot be foreseen before and teachers need to be prepared to face them (Cengiz, Kline & Grant, 2011; Ponte & Quaresma, 2016). Ponte, Mata-Pereira and Quaresma (2013) present a framework for analysing a discussion where they distinguish four fundamental kinds of actions related to mathematical aspects: (i) Inviting, seeking to initiate a discussion; (ii) Supporting/Guiding to lead students to present information, making questions or observations; (iii) Informing/Suggesting, introducing information, giving suggestions, validating students' responses; and (iv) Challenging to encourage students to produce new representations, interpret a statement, establish connections, or formulate a reasoning or an evaluation.

Quantitative reasoning involves reasoning about relationships between quantities. It "is the analysis of a situation into a quantitative structure — a network of quantities and quantitative relationships" (Thompson, 1993, p. 165). The relationships, in this kind of reasoning, are established between quantities. Numbers and numeric relationships are of secondary importance. Thompson (1993) connects the notion of quantity to the idea of measure, clarifying that quantities when measured have numerical value, but the reasoning does not depend on their measures. For example, we can think about the heights of two persons determining who is taller than the other without having to know the actual values. "Quantities are attributes of objects or phenomena that are measurable" (Smith & Thompson, 2008, p. 101), but "what is important is relationships among quantities" (Thompson, 1993, p. 165). A quantitative operation is the comparison between two quantities to find the excess of one relative to the other and the quantitative difference is the result of the quantitative operation of comparing two quantities additively. According to Thompson (1993), numerical difference, as the result of subtracting, is not synonymous to quantitative difference, which is an item in a relational structure. A quantitative difference is not always evaluated by subtraction and subtraction can be used to compute quantities that are not quantitative differences. For Smith and Thompson (2008), quantitative reasoning implies "to focus on quantities and how they relate in situations" (p. 121). Therefore, teachers' actions should lead to discussions of quantities, not numbers. "The goal is to get students to describe situations as they see them" (p. 122). Teachers "should imagine how their students might describe the situation differently and what conceptual difficulties might be lodged in their descriptions" (Smith & Thompson, 2008, p. 121). In whole class discussion teachers should invite students to share their solutions, guide and support them through questions focused on situations description, and challenge them to explain their reasoning. Thus, the discourse used by teacher is a key aspect accounting for students' understanding of quantitative relationships.

Methodology

This study follows a qualitative approach framed in an interpretative paradigm (Bogdan & Biklen, 1994). It is focused on the educational processes and the meanings of the study participants. The project adopts the modality of teaching experiment conceived with the purpose of developing in students the flexibility of calculation and the quantitative reasoning.

The data were collected from two classes in a public primary school in Lisboa, with 26 students each – a 2nd grade class (7-8 years old) in November 2015 and a 3rd grade class (8-9 years old) in October 2015. The Project team defined a sequence of tasks involving addition and subtraction problems. The task sequence was previously discussed and analysed with the two classroom teachers having been made minor adjustments.

Both teachers are female expert teachers with a large experience of teaching. The two teachers are considered good teachers of mathematics and both try to develop an exploratory teaching in their classes. However, teachers are quite different in what concern their relationship with mathematics. Catarina (2nd grade teacher) likes mathematics and did a specialization in mathematics education. Catia (3rd grade teacher) always had trouble with mathematics that she wanted to overcome, that's why she wanted to participate in this project. The two teachers are friends, work together, and they had been invited, at the beginning of the project, to collaborate with the project team members and both accepted.

The names of students and teachers have been changed to ensure confidentiality. The data collection was done through participant observation, complemented with field notes and videotaping of the classes, including whole class discussion. The students' productions were also collected. In the part of the exploration of the tasks, the videotaping focused on two pairs of students in each class, selected by habitually verbalize their reasoning among themselves.

The task was the sixth of a sequence. In the previous lessons and in the first part of this lesson the students had solved other tasks where they had to make comparisons in gains and losses problems. The task solved by 2nd and 3rd grade students was the same, but the magnitude of numbers was adapted accordingly. Figure 1 shows how the task was presented to the 2nd grade students.

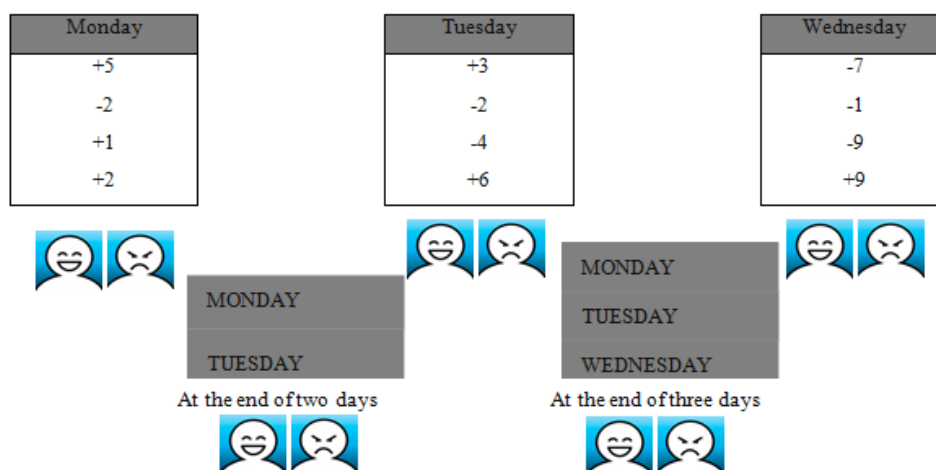


Figure 1: The task solved by 2nd grade students.

For the 3rd graders the numbers were: Monday: +50, -20, +10, +20; Tuesday: +30, -40, +60; Wednesday: -74, -94, +90. The students should select a smiley or a sad face circling it depending on their sense of whether there would be more gains or more losses at the end of the day(s). The students should also find the quantity representing how many marbles they won or lost at the end of the day, that is, the final quantitative difference. They did not work with the notation of negative

numbers before this task. The analysis focuses on whole class discussions and especially on categories of action coming from Ponte et al. (2013) and Smith and Thompson (2008) frameworks: *the inviting action* seeks to get students to share their solutions; *the guiding action* seeks to get students to describe the situation, modelling it and focusing on quantities; *the challenging action* seeks to encourage students to explain the quantitative relationships.

Results

2nd grade: The teacher introduced the task saying to students “This new proposal has only the gains and losses in different games, let's see what happens at the end of each day”. First, students solved the task in pairs, being videotaped the pairs Paulo and João and Luís and Lúcia.

Paulo seemed to deal easily with the quantitative difference whether it was an excess or a deficit. This facility is evident in the way the task was carried out by Paulo and João pair. They did it very quickly, in two minutes, although João followed Paulo's notes (he wrote "+8" after 3 days, and rectified later to "+1", when looking at Paulo's sheet). As the researcher got closer, Paulo explained that he looked at the nulls ("nulls", for example, -2, +2), having calculated only the remaining ones.

Luís and Lúcia approached the task using a different strategy. They gathered the gains, they joined the losses, and only then did the comparison. In the situation related to Wednesday, the students surrounded the sad face, understanding that at the end of the day, the player lost marbles. But they were stuck in the numerical record because it was a loss. With the researcher's guidance, when she came close, they wrote "-17; +9".

In the whole class discussion, the most presented strategy was to add, on the one hand, the gains, and on the other, the losses, and then determine the difference. For example, for Tuesday, the teacher invited the Marta and Mónica pair to go to the blackboard and record their results. They wrote quickly: " $+9 - 6 = +3$ " and drew a 'smile'.

Mónica: We did +9 which is what won +6+3.

Teacher: And the other 6?

Mónica: Out of what he lost: the 4 and the 2.

Teacher: How do you get 3? (addressing the class)

Students: The difference between 9 and 6 is 3 to win.

First, the teacher *guided* students through a question that led students to focus on losses (quantitative difference). Then *challenged* students to explain the results: “How do you get 3?”. Here the students seemed to dominate the situation and the teacher's questioning led students to explaining their quantitative reasoning.

But the teacher *challenged* the class, trying to have another explanation:

Teacher: Is there another explanation?

Paulo: Yes, the +6, -4 and -2 annul themselves. (The teacher records on the blackboard: $+6-4-2=0$)

Paulo: And only +3 left over.

This idea of 'annulling' was used by other students in the following situations, correctly by many. Paulo had the opportunity to present his explanation when the teacher asked for other explanation.

3rd grade: The teacher introduced the task saying that now they had to look just for gains and losses and should find what happened at the end of each day. Then, students started to explore the task in pairs, and showed difficulties confounding the notion of quantitative difference with the absolute amount of marbles, increasing these difficulties for Wednesday.

At the whole class discussion, the teacher *invited* Vítor to present his solution for Monday:

Vítor: So, he had 50 on Monday...

Teacher: Won...

Vítor: 50, but then he lost 20; then he won 30 and won. He was happy. (...)

Teacher: Does everyone agree?

Students: Yes.

Teacher: How many marbles did he win?

Students: 60. [And Vítor puts the sign +, +60]

Vítor began by confusing the notion of quantitative difference with the absolute amount of marbles: when he says "he had 50", he seems to assume that the player began the game of marbles with 50 marbles. But, the teacher *guided* him, focusing on quantitative difference when she referred to "won" and after Vítor took a discourse centered on the gains and the losses describing the situation.

Concerning Tuesday, Rui explained his solution on the blackboard.

Rui: Then, on Tuesday, he won 30 more, he had 60. Then on Tuesday he lost 40. (...)

Teacher: In general, he won more or lost?

Rui: He won. (...)

Teacher: So, overall, how much did he win?

Rui: I think he got 60. [Rui records $30-40+60=60$ on the blackboard] (...)

Student A: It's wrong! It's fifty because 30 minus 40...

Alexandre: From 30, you cannot take out 40.

Again, there is a conceptual confusion between the quantitative difference and the absolute amount of marbles: "he had 60"; "he got 60"; "From 30, you cannot take out 40". The teacher tried to focus on global balance "In general, he won more or lost?" and "So, overall, how much did he win?" [*guiding* action]. Rui, confronted with the reactions of his peers, corrected his solution adding the gains ($30+60=90$) and subtracting the lost marbles ($90-40=50$). At the end, Rui circled the result 50, concluding that the player won 50 marbles on Tuesday.

Then, the teacher *invited* Alexandre to present his solution for Wednesday:

Alexandre: I saw we had 94.

Teacher: So, let's start at the beginning ... on Wednesday, in the first game, what happened in the first game, Alexandre?

Alexandre: He lost 74.

Teacher: He lost 74, and after, in the second?"

Alexandre: And after he lost 94.

Teacher: And after he lost 94 ... and after he won ...

Alexandre: He won 90.

Teacher: 90. And then, with the data we have there what did you do? Tell us.

The dialogue began by focusing on the modelling of the situation, with the teacher *guiding* about what was going on the three games on Wednesday. Alexandre continued explaining his solution: " $94+90=184-74=110$ ", justifying the addition of 94 and 90 because "94 is the largest number and 90 was the second largest number". Alexandre tried to have an explanation for the computations, not for the situation. Again, the teacher tried to *guide* him, seeking to focus his reasoning on losses:

Teacher: But why did you take out 74 now?

Alexandre: Because I had already spent these numbers; it was just missing 74, I went to take them out.

Teacher: You were to take out that because it was missing ... but Alexandre, in fact, he lost 74 and lost 94... (...) What does this mean?

Alexandre: He got less than zero.

Teacher: I can only loose what I have...

On Wednesday, the balance between gains and losses is negative: after the three games, the player lost 78 marbles. The quantitative difference in this situation requires ignoring the absolute number of marbles and Alexandre was not able to deal with a negative point of departure ("He got less than zero"). Alexandre seems to consider 94 as the initial number. The teacher revealed difficulty *to guide* the student's reasoning referring to the assumed need of knowing the initial number of marbles ("I can only loose what I have..."). In this situation, Jaime offered himself to explain. He added all the numbers (" $74+94+90=258$ ") and went to the blackboard to explain why he added 74 plus 94:

Jaime: Because 74 and 94 were what you had before.

Teacher: 74 and 94 were the marbles he had, because he can only loose, we can only loose what we have, isn't it? I cannot loose 74 marbles if I had nothing. (...) My friends, the 74 and 94 what were they?

Jaime: That's what they had.

Teacher: They only lost because they had them.

The students confused the quantitative differences with the initial number of marbles. Again, the teacher tried to *guide* students focusing them in the modelling of the situations. However, the teacher could not reorganize her discourse when the situation became more complex. Catia had difficulty to *challenge* students in a way that they could understand the quantitative relationship

without knowing the absolute amounts of marbles. An impasse was created, and, in the meantime, the class had to end, it was time to break.

Discussion

The results show that the two classes reacted differently to the task. 2nd graders seem to do it without any difficulty. In contrast, 3rd graders had difficulties, namely the fact that they did not have the initial number of marbles. These difficulties increased for Wednesday where the losses are greater than the gains, as no student did it correctly. From the beginning of the solution, while the 2nd grade students added the losses and gains, 3rd grade students started to operate with numbers, apparently not thinking about the meaning of their procedures. This was not usual in the class because, as stated earlier, both classes used to do exploratory work in mathematics.

Catia, the 3rd grade teacher, started to guide students focusing on the modelling of the situation, but confronted with the students' answers, her questioning could not overcome the students' conceptual understanding of quantitative relationships. Although she had solved the task before, trying to anticipate students' responses, she was not able to deal with it when the situation became more difficult (on Wednesday). She began guiding the students to the description of the situation, as advocated by Smith and Thompson (2008), but at the end she got lost as her students. She repeated a statement which did not help as it was focused on the absolute numbers involved ("we can only lose what we have") and this did not help students to solve the situation because it probably reinforced the confusion between the relative change of losses (-74-94) and the absolute amount of marbles ("what you had before"). She seems to have not understood students' difficulty and was not able to retell the situation (Franke et al., 2007). On the contrary, the 2nd grade teacher, Catarina, was always at ease with the situation, challenging her students in different ways, much more focused on the quantities in presence.

Final remarks

As Ruthven et al. (2011) state, the questions posed by teachers are fundamental aspects of classroom communication. Catarina and Catia have a similar inviting action seeking to initiate the discussion and to get students to present and share their solutions. However, they differed regarding ways of communication concerning this specific task, with Catarina's questions being more focused on the quantities and how these are related to the situation (Smith & Thompson, 2008). So, how teachers develop the process of communication in classroom has a crucial role in students overcoming the challenging situation related to quantitative difference. Thus, 2nd grade students were able to focus on quantitative difference, looking at gains and losses, and dealing, naturally and comprehensively with negative number notation. On the contrary, the 3rd grade students were not able to disconnect from arithmetic operations and the need to consider an initial number and they were unable to focus on quantitative reasoning. So, it could be argued that the 2nd grade students were able to conceive a quantitative difference independently of numerical information about quantities (Smith & Thompson, 2008). For us, this situation is intrinsically linked to teachers' actions and more specifically to the ways teachers guided and challenged students through questioning so that they latter could advance in their quantitative reasoning process. This study allows to emphasize the important role of teacher's actions of guiding and challenging in

promoting students' ability to reason with quantitative relationships, even when the initial values are unknown.

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