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In this paper we present and illustrate a framework for analyzing the relation between teachers' actions and students' meaning making in mathematics. We adopt a pragmatic perspective on learning, and a methodological approach using already analyzed material, to see whether and how the framework of epistemological move analysis can contribute when analyzing the relation between teachers' actions and students' meaning making. The results suggest that epistemological move analysis can be used to identify teachers' purpose in students meaning making in mathematics, by analyzing students' responses. Further, it makes it possible to identify what earlier knowledge students use, and how they use it, to re-actualize mathematical objects, relations and concepts.

Keywords: Mathematics teaching, meaning making, pragmatism, epistemological move analysis.

Introduction

The teacher plays a major role in making classroom talk the arena for learning mathematics and teachers' interactive strategies are central to how students become engaged in math-talks in a classroom practice (Walshaw & Anthony, 2008). To succeed in eliciting and extending students' mathematics, a teacher should encourage the mathematical activities of analyzing, making connections, and generalizing (Fraivillig, Murphy, & Fuson, 1999). It has been proven important that a teacher stimulates students to struggle with important mathematics by challenging them with non-routine tasks and trying to find out more efficient solution methods (Hiebert & Grouws, 2007). However, providing successful math-talks in practice can be challenging. Teachers tend to funneling (Bauersfeld, 1998) students' responses towards the answer they want, blocking them from being fully engaged in math-talk (Brodie, 2011). Also, rather than addressing problems in terms of the underlying concepts, students address problems mainly in terms of what they think that the teacher is expecting (Millar, Leach, & Osborne, 2001).

Talk-based teaching of mathematics is a complex teaching practice, calling for the need of theoretical constructs that take serious the social and interactive nature of teaching and learning (Lerman, 2000). However, concerns have been raised that a focus on social and interactive issues in mathematics teaching may steer away learning from content (Lerman, 2006). For instance, analyses of practices or activity systems common in sociocultural traditions tend to focus on what is done in a general objective (Bakker, Ben-Zvi, & Makar, 2017), which leaves little room for fine-grained analysis of how students make sense of mathematical content.

In the present paper we use a pragmatic perspective on learning (Dewey, 1929/1958) as a starting point, where meaning making is considered to emerge "in the process of doing and undergoing the consequences of action" (Rudsberg, Öhman & Östman, 2013, p. 600). Based on this perspective the aim of this paper is to present and illustrate a framework that enables detailed investigations of teachers' role in directing students' meaning making processes of mathematics in talk-based teaching. The methodological approach in this paper is to use already analyzed material to see whether and how the framework can contribute when analyzing the relation between teachers' actions and students' meaning making. We aim to clarify and investigate both social and individual aspects of the meaning making process in mathematics, in a descriptive manner.

Theoretical background

A pragmatic perspective on meaning making: The first question to be addressed is how to understand learning and meaning making in classroom practice. Often the visibility of such processes is seen as a central problem (Östman & Öhman, 2010), for example the problem of not knowing what the students have in mind. The problem of visibility is often connected to a view of learning as something mental, something that takes place in the student's head hidden from the observer (Rudsberg & Öhman, 2013). However, here we take Dewey´s theory of action (Dewey, 1929/1958)[1] as a starting point in which meaning making and learning are understood in terms of people's actions; something they do, in relation to the environment and the specific situation at hand. This can be seen as a way to dissolve dualisms, such as, for example, between the concepts of inner mind and reality (Garrison, 2001). Dewey (1929/1958) describes life as a process of constant change that contains an active phase, doing, and a passive phase, undergoing the consequences of action. Accordingly, meanings are seen as the way that humans respond to the environment and they are practical in the sense that "we use them as *means* to *consequences*" (Garrison, 2001, p. 284). Dewey (1929/1958) states that meaning "is not indeed a psychic existence; it is primarily a property of behaviour" (p. 179). Communication is seen as a coordinating activity in which the participants make something in common (Garrison, 2001). In this way, "meaning making emerges in the process of doing and undergoing the consequences of action" (Rudsberg et al., 2013, p 600) and in this process experiences continually changes. Continuity can be seen as the process where students re-actualise earlier knowledge in order to make meaning in a new situation. In the process of re-actualization earlier knowledge also transforms i.e., earlier knowledge is not perceived as fixed entities. Meaning making can thus be understood as a practical social process (Garrison 1995). Hence, in this pragmatic perspective meaning making and learning is possible to observe *in* students' coordinating activities. Learning takes place when students develop a new or more developed and specific repertoire of coordinating activities (Semetsky, 2008).

Epistemological move analysis: Epistemological move analysis (EMA) is based on a pragmatic understanding of epistemology, where it is seen as both a part and a result of human practices (Wickman & Östman, 2002). This means that what counts as relevant knowledge is different in different practices and the knowledge that is considered to be true and valid is a discursive

[1] For a more thorough elaboration of Dewey' theory of action and its use, see, for example, Östman and Öhman (2010).

construction (ibid.). EMA focuses on the relation between the epistemological moves that the teachers make and the students' meaning making (Lidar, Lundqvist & Östman, 2006). An *epistemological move* can be defined as the way a teacher's utterance affects a student's meaning making in an educational situation, that is, the way a teacher's actions facilitate a certain meaning making. In other words, how a teacher's answer to, and/or evaluation of, a student's utterance affects the student's meaning making. EMA has been used in educational practices of, for example, science education (Lidar et al., 2006). From previous research, we can delineate at least five different types of epistemological moves: *confirming*, *re-constructing*, *instructional*, *generative*, *re-orienting* (ibid.)[2].

As described above EMA enables investigations of the relation between the teacher's actions and students' meaning making in mathematics. In order to investigate students' meaning making EMA includes Practical Epistemology Analysis (PEA) (Lidar, Lundqcist & Östman, 2006). That is, EMA can be seen as a development of PEA with a specific focus on the role of teachers' actions. EMA aims at giving account of both the content and the processes of students' meaning making in relation to the teacher's moves. In the analysis of students' meaning making four central components are used; *encounter*, *gap*, *stand fast* and *relation* (Wickman & Östman, 2002). Teaching takes place in situations where students *encounter* their environment and interact with, for example, other students and the teacher. The interaction is made in relation to the purpose of the activity. That is, the first step of the analysis is to clarify what the student encounter and the purpose of the activity. In education, students sometimes do not immediately know how to proceed, i.e., the students cannot proceed by habit and a *gap* occurs. In teaching situations, a gap becomes visible when for example the teacher asks a question or when the students hesitate. In order for the interaction to proceed the gap needs to be filled, by, for example, responding to a question. This is done in the process of re-actualization. In the process of re-actualization the students bridges the gap by creating *relations* between their earlier knowledge, what *stands fast*, and the problem at hand. That is, the students' earlier knowledge becomes actualized in a new situation. The earlier knowledge becomes visible in the educational practise as the students use knowledge without hesitation and no explanations are needed. The relations created by the students constitute the content of the individual's meaning making.

In our analysis we focus on the relation between the teacher's actions and students' meaning making. The teachers' actions, for example, giving instructions, asking questions or making comments, can be seen as guidelines that direct the students' meaning making in a certain way. These actions made by the teachers indicate both what counts as knowledge and appropriate ways of acquiring this knowledge in the specific educational practice. Here it is important to notice that we categorize teachers' epistemological moves in terms of the function they have for the students' meaning making. On this account, we have analyzed whole events in order to clarify the differences

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[2] The moves identified in this study will be described in the analysis; for a description of earlier identified epistemological moves see, for example, Lidar et al. (2006) and Rudsberg and Öhman (2010).

between students' utterances before and after the teacher's actions. An event consists of a sequence consisting of three turns, including (1) a student's actions, (2) teachers' actions that encourage a student's action, and (3) a student's actions after the teacher's actions (Klaar & Öhman, 2014).

Illustration of EMA in mathematics education

To illustrate epistemological move analysis we use two examples from Blanton's and Kaput's paper focusing on classroom practices in algebraic reasoning (Blanton & Kaput, 2005). These examples are rich in mathematical content, given that the characterization is built on features of algebraic reasoning. Blanton and Kaput aim to "understand those types of instructional practice that indicated a generative and self-sustained capacity to build on students' algebraic reasoning" (ibid, p. 416). From this study we have chosen examples from audio-taped classroom talks. To identify the teacher's purpose of these classroom talks, the authors have studied notes by and interviews with the teacher. By using epistemological move analysis we can study the relationship between the purpose that the teacher expresses and the purpose visible in how the student responds to his/her moves. Blanton's and Kaput's paper includes several examples of principles for task-design and indications on what kind of question the teacher should ask to promote algebraic reasoning. However, their intention is "not to address how June's practice affected student achievement; this would require a more detailed look at how her actions played out in the classroom and how students were involved" (ibid, p. 435). In this paper we aim to employ such a detailed look by analyzing how the teacher's epistemological moves direct and support students' meaning making. This way we can clarify students' meaning making concerning both process and knowledge content in the actual situation. Important to notice, however, is that we do not investigate whether or not this change is of long term.

Algebraic treatment of number: In reviewing homework of addition tasks, the teacher shifts focus to properties of even and odd numbers and, during this sequence, there is an *encounter* established between the teacher (June) and two students (Tony and Jenna). In the first part of this encounter a scheme of even and odd numbers is used. The teacher's purpose is that the students should find an algebraic principle of parity in adding odd and even numbers. This aim becomes visible when the teacher uses large numbers to guide Tony to use an algebraic principle instead of computation that specifies even and odd numbers. The transcript (ibid.) used in this illustration is the following: "June challenged a student's use of arithmetic strategy to deduce that 5+7 was even:

1	June:	How did you get that?
2	Tony:	I added 5 and 7 and then I looked over there [pointing to a visible list of even and odd numbers on the wall] and saw that it was even.
3	June:	What about 45678+85631? Odd or even?
4	Jenna:	Odd.
5	June:	Why?
6	Jenna:	Because 8 and 1 is even and odd, and even and odd is odd." (p. 422).

Teacher "spontaneously shifted the focus from computing sums to determining if the sum of two numbers would be even or odd. When students responded by first computing the sum to determine if it was even or odd, June began to use numbers that were sufficiently large so that students could not compute. Instead, they were forced to attend to the structure in the inscriptions themselves." (ibid, p. 417). The *gap* in this encounter is between the use of addition in order to determine if the sum is even or odd and the ability to use an algebraic principle in order to determine if the sum will be even or odd. The gap becomes visible when the teacher (line 3) uses large numbers.

The students are able to make arguments on the parity of sums of odd and even numbers without using computations. The *stand fast* can be regarded as computation of specific numbers and using the list on the wall to determine odd and even numbers. This is *re-actualized* into the ability to use numbers as placeholders, or variables, for *any* odd or even number. This change is made because of the teacher's epistemological moves in line 3 and 5. Together these moves enabled the student to change the way of determining if a sum of two positive integers will be even or odd. Hence, the student has made new mathematical meaning about the structure of the inscription itself in terms of an algebraic principle of even and odd numbers. This way the teacher's epistemological moves in line 3 and 5 are *generative move* as they have the function of generate students learning of algebraic principles and relations. The teacher's purpose becomes visible in line 5 since the response by the student in line 6 is built on an algebraic principle rather than on computation.

Finding a mathematical formula: In working with the concept of area, the teacher introduced an activity in which students should determine how many two-colored counters were needed to cover a large purple square. In this sequence we can conclude that it is an *encounter* between the teacher and four students (June, Mari, Zolan, Stephanie, Kevin) and also one more student (unidentified in the audio-taped recording), who may or may not be one of the other four students. The teacher's purpose is that the students reach an understanding of how to use length and width to determine the area of rectangular planar objects, i.e., finding a formula for the area. This purpose becomes visible in line 23 as it generates the concepts of length and width in line 24. The transcript (ibid.) used in this illustration is as follows: "June had the following conversation with her students:

13	June:	Do I know how big the square is?
14	Mari:	No.
15	June:	What do you see [referring to the large square which students had covered with tiles]?
16	Zolan:	Four columns, four rows.
17	June:	[June then covered a desk with the large purple squares like those she had given students.] So, what would the area of the desk be?
18	Stephanie:	Twenty-four big squares.
19	June:	What if we found the area of this table [pointing to a large table in the room]?

One student suggested using a ruler. Kevin proposed to examine how many purple squares are in a row and in a column.

20	Zolan:	Count how many [purple squares] are in the bottom row.
21	June:	One, two, three, four, … eighteen. [June counts out the number of purple squares in the bottom row.] And how many 18s do I need?
22	Kevin:	Seven!
23	June:	What's the best way to find area?

24 Kevin: You measure this way and that way [indicating length and width] and multiply.

25 June: What do you call “this way” and “that way”?

26 Student: Length and width!” (p. 428-429).

The purpose in this encounter is located between the use of square tiles for covering rectangular planar objects and finding a formula for area of such objects. In line 13 the teacher asks the students if they know the size of the large purple square after the students have covered it with tiles. However, the teacher’s first question (line 13) does not have any function in terms of meaning making, since the student cannot bridge the gap that the question creates. Indeed, the teacher’s question (line 13) is an action, but it is not an action that qualifies as an epistemological move. A *gap* between square tiles and the size of a planar object becomes visible when the teacher in line 15 asks the students to look at the square and explain what they see. This is an *instructional move* since the student’s answer is describing what he (Zolan) can see and not mentioning anything on area. This means that the gap lingers. In the third question (line 17) the teacher explicitly asks the students to find the area of the desk. This is a *re-orienting move* since the students now are focusing on the total amount of squares rather than numbers of columns and rows. Here it *stands fast* for the student how to cover a square object with square tiles (line 18). This earlier knowledge is re-actualized in order to make new meaning to view area as a grid of square tiles. Hereby the students are able to fill the gap.

When the teacher explicitly asks the students to determine the area of another planar object (line 19), the gap changes. The new *gap* is between using columns and rows and finding the formula for area. Related to the change of gap the student also actualizes other earlier knowledge that becomes visible when the teacher asks the students to determine the area of yet a new object (line 19). Here the knowledge that *stands fast* can be regarded as knowing how to use columns and rows to cover a rectangular planar object. This is re-actualized when the student starts to investigate the grid in a new way by counting the numbers of squares in the bottom row (line 20). In this way the teacher’s action functions as a *re-orienting move.*

Finally, the teacher makes two *re-constructing moves* (line 21 and 23). The first becomes visible when the student (line 22) shows awareness of the importance in comparing the total number of square tiles with the numbers of columns and rows. Thereafter the student (line 24) is able to relate number of columns and rows to create a formula for area. Finally, the students are able to fill the gap by finding a formula, in terms of length and width, for determining area of planar objects.

Concluding remarks

The analysis shows how teacher’s epistemological moves during math-talk is crucial for students’ meaning making processes, both concerning the procedure (how) and the mathematical content (what). For example, in the first illustration the *generative move* deepens the student’s mathematical knowledge in terms of an ability to build on algebraic principle rather than computation when determining even and odd numbers. We can conclude that the teacher enables the students to generalize (Fraivillig et al., 1999), which is done by challenging them to find a more general and efficient solution to the problem at hand (Hiebert & Grouws, 2007). By using EMA it was possible to identify how the students' earlier knowledge in this specific situation was *re-actualized* into the ability to use numbers as placeholders for *any* odd or even number. More explicitly, this re-actualization is made because of the teacher's epistemological move (line 3), made visible by the student's answer in line 6, where the latter uses numbers 1 and 8 as placeholders for odd and even numbers. By this re-actualization the student fills the gap that becomes visible by the teacher's question in line 3. In this way the student has made new meaning about an algebraic principle of odd and even numbers. Here we stress that the contribution in using EMA is to point out what actually takes the meaning making a step forward, rather than analyzing certain approaches in forehand intended to promote specific motives. The strength in EMA is in the situational, rather than long term. However, in combining EMA with well-known strategies promoting algebraic reasoning, such as the framework used by Blanton & Kaput, sustainable learning perspectives may be closely related to situational and contextual perspectives.

This can be compared with our second illustration. In the first part of the transcript the teacher poses different questions with the purpose of enabling for the students to understand a formula for the area of rectangular objects. However, in the first part of the sequence the students answer the teacher's questions without re-actualizing any earlier knowledge about area. Instead they answer the question more in terms of what they think that the teacher are expecting (Millar et al., 2001). Important to notice here is also that the teacher's purpose, to understand how to determine area, isn't visible in the students' actions until later in the sequence (line 19-20). At the end of the sequence (line 24-26), after the re-constructing move, which also clarifies the teacher's purpose, the students re-actualize earlier knowledge that enables them to establish a formula for area. By using EMA we can identify that the teacher's purpose is not initially clear to the students. The students are not able to see how the general properties for area correlate to the teacher's purpose. The students try to answer the question (line 13) but are not able to fill the gap until the teacher changes character of the questions (line 23). This illustration can be seen as an example of funneling (Bauersfeld, 1998), with the consequence of students being partly blocked from being fully engaged in the talk (Brodie, 2011).

To summarize, using EMA, we *can* analyze the relation between teachers' actions and students' meaning making concerning process (how) and content (what) in mathematics. This way, EMA can contribute to the research on social and interactive processes in mathematics (Lerman, 2006). Further, EMA makes it possible to identify the role of teachers' purpose in students' meaning making in mathematics. However, it should be stressed that meaning making is seen as situational. We believe that the concept of re-actualization should get more attention in studying the role of teachers' actions for students' meaning making in mathematics. Re-actualization should not be misinterpreted as transfer or application, but rather as the process describing *what* stands fast in a certain encounter and *how* the students use knowledge that stands fast to create *new* meaning of mathematical objects, principles or concepts. Based on the present analysis, we claim that EMA offers analytical tools to enhance our understanding of the relationship between content and process in student's meaning-making in mathematics and the roles teachers' actions can have in providing directions for meaning-making.

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