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Linking the micro and macro context: A sociocultural perspective

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In this paper, I present and discuss findings from a research study the aim of which is to investigate the activity of proving as constituted in a Cypriot classroom for 12 year old students. Through Cultural-Historical Activity Theory (CHAT), the influence of research literature, curriculum prescriptions, the students and critically the teacher are documented. The evolution of objects, in particular the aims of the teacher, and other components in the activity systems are traced. Perceiving the mathematics classroom as a nested activity within educational context levels, this paper considers the role of the broader social context in which this classroom is situated.

Keywords: CHAT, contradictions, proving, mathematics

Theoretical framework

It is now acknowledged that proof and proving should become part of students' experiences throughout their schooling (Hanna, 2000; Stylianides, 2007; Yackel & Hanna, 2003). It is also argued that argumentation, explanation and justification provide a foundation for further work on developing deductive reasoning and the transition to a more formal mathematical study in which proof and proving are central (Yackel & Hanna, 2003). But what is meant by proof and proving? Mathematical argumentation is a discursive activity based on reasoning that supports or disproves an assertion and includes the exploration process, the formulation of hypotheses and conjectures, explaining and justifying the steps towards the outcome and the proof of the statement. Thus, proof is at the core of mathematical argumentation, as a justification, an explanation and a valid argument. Research has responded to the need to conceptualize proof and proving in such a way that it can be applied not only to older students but also to those in elementary school (Stylianides, 2007). The challenge remains however to understand how proof and proving is shaped by the practices in the mathematics classroom. This is in accordance with Herbst and Balacheff (2009), who argue that the focus should not only be on proof as the culminating stage of mathematical activity, but also on the proving process and how this is shaped by the classroom environment. Thus, in understanding how proving is constituted in the classroom, a wider network of ideas is required as these ideas no doubt have an impact on how proof in the narrow sense is constituted.

To address this issue, I refer to pre-proving, that aspect of mathematical reasoning that might nurture proving. What are the roots of proving? The purpose of this study is to investigate proof and proving in the naturalistic setting of the classroom and the way the structuring resources of the classroom's setting shape this process. Instances of students proving statements have been identified in this classroom community but instances where the argument was not in the conceptual reach of the classroom have also been identified. However, this study also points to those aspects of reasoning that appear to have the qualities of proving, even though they may not be proving in themselves. That is, analyses of video-recorded whole class discussions show how processes of explaining and exploring are key sub-systems within the central activity of proving as they provide a key pathway, which often includes defining. Thus, pre-proving refers to those elements that direct

mathematical reasoning towards the ultimate goal of formal proving, namely exploring, explaining, justifying and defining. This study considers mathematical explanation an act of communication, the purpose of which is to clarify aspects of one's mathematical thinking that might not be apparent to others (Yackel & Cobb, 1996). Justification is "the discourse of an individual who aims to establish for somebody else the validity of a statement" (Balacheff, 1988, p. 2). There is insufficient scope in this short paper to consider in detail these various levels and so this specific study focuses on the way the activity of the mathematics classroom (micro context) is influenced and dependent upon wider educational context (macro context). This is in accordance with Balacheff (2009) who argues that among the important pieces in trying to understand the nature and role of proof in a mathematics class is describing the general usage of the word proof in these contexts and the demands this usage imposes in the classroom.

CHAT based underpinnings

As this study is exploring the various forces that impact on the activity of proving, Cultural Historical Activity Theory (CHAT) is being employed as a descriptive and analytical tool alongside collaborative task design (a means of gaining access to the teacher's objectives), to capture the interaction of different levels, such as the actions of teachers, students and the wider field as evidenced in curricula and research documentation. The analysis and discussion in this paper draws upon the following CHAT perspectives: (i) the object of the activity and (ii) the notion of contradictions. Initially, the unit of analysis in CHAT is an activity, a "coherent, stable, relatively long-term endeavor directed to an articulated or identifiable goal or object" (Rochelle, 1998, p. 84). Engeström (1987) introduced the activity system, a general model of human activity that embodies the idea that both individual and social levels interlink at the same time. Jaworski and Potari (2009) argue that the activity system is a micro context within broader macro context levels. Thus, the activity of a mathematics classroom is influenced and dependent upon the structure and organization of the school and the Ministry of Education as wider educational contexts. The object of a collective activity is something that is constantly in transition and under construction. It has both a material entity and is socially constructed and its formation and transformation depends on the motivation and actions of the subject indicating that it proves challenging to define it.

Among the basic principles of CHAT is the notion of contradictions. Contradictions are imbalances, ruptures and problems that occur within and between components of the activity system, between different developmental phases of a single activity, or between different activities. These systemic tensions lead to four levels of contradictions (Engeström, 1987). Contradictions are important because they may lead to transformations and expansions of the system and thus become tools for supporting motivation and learning. That is, contradictions do not serve as points of failure or problems that need to be fixed. "Rather than ending points, contradictions are starting places" (Foot, 2014, p.337). This paper focuses on tertiary contradictions that appear between the object and the culturally advanced form of the central activity, a clash between the micro and macro level. Compared with other studies investigating tertiary contradictions, this study takes a rather different approach in discussing the tertiary contradiction that has emerged within this particular activity system. That is, the collaborative task approach assisted in exposing the teacher's object. Even though introducing a new mediational tool resulted in new actions being brought into the activity,

this did not affect the object of proving as a cultural historical activity system. Thus, when elaborating on tertiary contradictions, this discussion focuses on a possible clash between the micro and macro level of this activity system, due to a differentiated object.

Data collection and analysis

This study was conducted in a Year 6 classroom in a public primary school in Cyprus. This mainstream school is considered to be a dynamic school; it actively encourages teachers and students in engaging at a deeper level with the educational experience. Apart from the researcher, the participants were the teacher, a Deputy Principal at the school who endorses the integration of technology in teaching mathematics, and voluntarily agreed to take part in the research, and 22 students (11–12 years old) of mixed abilities. Even though using computers was part of the classroom's routine, the students were not familiar with Dynamic Geometry Environments (DGEs). The data collection process was undertaken in three phases. Phase I aimed at identifying the system level and the teacher level, by employing documentary analysis and semi-structured interview. The system level, which remained the same throughout the study, in the broader sense, refers to the policy statements, curriculum, textbooks, research about proof and proving. The official documentation was analyzed so as to collect information concerning the role of proving in primary education, the objectives for teaching and learning geometry, the geometrical tasks illustrated as important for developing geometric thinking and understanding, the approaches the ICT offers in facilitating the teaching and learning of school geometry. The teacher level refers to the teacher's attitudes and perceptions concerning the role of proof in the curriculum and in the mathematics classroom, compared with what the teacher actually does in the everyday mathematics classroom. The interview with the teacher aimed at exploring the teacher's beliefs and views regarding the nature of mathematics, the nature of teaching mathematics and the nature of learning mathematics. The main research focus of Phase II was to map the current situation of the classroom. The data collection process included video data from the classroom observations and field notes from the informal discussions with the teacher. My involvement in the classroom could be described as moderate participation. In Phase III, the researcher collaborated with the teacher to design DGE-based tasks as a means to gain access to the teacher's objectives. The tasks were the research vehicle, the window for generating data rather than any kind of curriculum intervention. The research instruments were classroom observation, informal discussions with the teacher and the DGE-based tasks. In Phase III, I had an active role in the classroom. My involvement was related with answering questions related with the tools the DGE provided, which the students had to use in order to explore the tasks, and asking probes. Each phase of data collection was distinct as it corresponded to specific purposes. At the same time, themes of interest, emerging from the ongoing analysis of each phase, also informed the design, implementation and analysis of the subsequent phases. The content of the curriculum covered during the classroom observations was the area of triangles, and the circumference and the area of circle.

The overall process of analysis of the collected data was one of progressive focusing. According to Stake (1981, p.1), progressive focusing is “accomplished in multiple stages: first observation of the site, then further inquiry, beginning to focus on relevant issues, and then seeking to explain”. The systematization of the classroom data led to the evolution of two broad activities: (i) the activity of

exploration including the exploration of mathematical situations, exploration for supporting mathematical connections (between the content of mathematics, with which the students are engaged, with parts of mathematics that they would be taught in secondary school or that were taught either recently or in the past, as well as between classes of problems) and exploration of the DGE and (ii) the activity of explanation which focuses on clarifying aspects of one's mathematical thinking to others, and sometimes justifying for them the validity of a statement. These activities were then interpreted through the lens of CHAT, by generating the activity systems of both exploration and explanation. Analysis of the classroom data revealed that the activity of explanation unfolded and expanded around mathematical definitions and defining as activity. What is the connection between definitions and explanation? Definitions are conventions that require no explanation. However, the teacher wanted reference to the attributes that involved properties. That is, the move from a definition involving only perception to a definition that involved properties needed explaining. The situation of the classroom regarding proving activity was further scrutinized by contrasting the outcome of the activity with the social context in which it emerged. Instances of both congruence and diversion existed between the micro and macro level.

What is the object of developing proving in the classroom?

It has been illustrated that pre-proving activity is closely connected with exploration and explanation. Correspondingly, the object of developing proving in the classroom is related with these notions. The object has multiple manifestations for the participants engaged in the activity. Exploration is related with the pre-proving activity when information is revealed through the immediate feedback students get from the manipulation of objects. For instance, on a blank DGE window, the teacher asked the students to find the area of triangles. The students had the opportunity to explore this mathematical situation on a DGE and decide for themselves which tools should be utilized that would assist them in finding the area of the triangles.

Additionally, explanation entails a process where mathematical definitions are being formulated. The students cannot rely only on perception as a definition in this particular classroom is considered more what a concept really is rather than a description of how a concept is used. For instance, the question "What is the altitude in a triangle?" directed the classroom towards formulating the definition of the altitude in a triangle. Explanation also entails a process where the sociomathematical norms are being negotiated. For instance, the first lesson where the students were introduced to the area of mathematics related with circle, was initiated by a question.

Teacher: What is circle?

Student: It is a shape that does not have sides or angles.

Teacher: I draw a circle according to this definition. (*The teacher draws a non-regular shape with curved lines.*)

Student: This is not a circle.

Teacher: We said that in mathematics our definitions must be accurate.

Students are expected to use precise mathematical language when communicating their ideas as well as when writing coherent geometrical explanations, clarifying aspects of their mathematical

thinking to others, as well as justifying for them the validity of a statement. For instance, following the classroom discussion on defining circle, the teacher asked students to determine whether several shapes illustrated on the whiteboard were circles.

Teacher: Is this a circle?

Student: No.

Teacher: I do not accept your answer. Why?

Student: There ... on the right ... the other shapes are not circles because their center does not have the same distance from their circumference.

As proofs begin with an accepted set of definitions and axioms, it can be argued that ultimately all proofs depend on the underlying definitions and the earlier results derived from these definitions. Thus, understanding and explaining these definitions is a prerequisite when approaching a proof. For instance, after the class reached a conclusion regarding the mathematical formula for the circumference of circle and made hypotheses concerning the mathematical formula for the area of circle, the teacher gave each pair a circle divided in either 8 or 10 pizza pieces, commenting that they could use the pizza slices to explore the area of circle.

Student: We finished. Can we tell you? Radius times half the circumference. It's a rectangle thus the length is the radius and the width is half the circumference because it's half.

In addition, making forward connections provides more information and knowledge about the axiomatic system in which the classroom community is working. Forward connections also strengthen the formulated definitions. For instance, after the class explored the number of altitudes in a triangle, the teacher made the following comment:

Teacher: This is what I was trying to achieve. All the altitudes pass ... this is not part of our curriculum but part of the mathematics curriculum of secondary school, but it's good for you to know because it helps you. A triangle has three altitudes. From each vertex I can construct an altitude to the opposite side.

Consideration of the aforementioned manifestations of the object leads to the conclusion that the object of the central system of pre-proving activity is related with exploration that leads to explaining and justifying for a specific part of the mathematics curriculum. Nevertheless, the object is simultaneously hindered due to the dichotomies, tensions and conflicts. At a first glance, this object seems to be clear and distinct. However, this object is multifaceted. The teacher on one hand understands the importance of providing enjoyable exploring opportunities that keep students' motivation and interest to engage with the problem. As a result, the teacher provides opportunities that can be approached by the students in their own way. On the other hand, students, through the exploration of these opportunities are expected to reach those conclusions regarding triangles and circles as pre-determined by the teacher. The two poles of the object lead to a constant struggle in the teacher's everyday practice. The teacher, due to this multifaceted object, is faced with the play paradox (Hoyle & Noss, 1992) as well as the planning paradox (Ainley, Pratt, & Hansen; 2006). As a result, teacher would at times decide to close down an exploration opportunity. For instance,

after exploring the circumference and area of circle, the teacher, asked the students to find a relationship that related the circumference and the area of the circle. Soon after that she asked them to prove mathematically that the ratio area/circumference of a circle is $r/2$. In a similar way, the object of the activity of explaining as part of pre-proving is multifaceted. The object for the teacher is twofold: explaining mathematical procedures and explaining related with 'proving'. On one hand, the teacher's object is related with engaging students in formulating definitions (of concepts and formulas) in the same way that mathematicians do. In order for these definitions to become operable for the students, they need to focus on the properties required. Thus, this process includes a continuous interplay between the concept image and the concept definition, promoting the characteristics of definitions and making the distinction between 'ordinary' and mathematical definitions. Even though the above facilitate the justification of statements, a tension within the object arises. That is, ensuring that the classroom engages in the construction of stipulated definitions and that these definitions are not just descriptive for the students seems to be competing with moving to justification based on these definitions. Furthermore, even though the teacher is embracing this object, she is simultaneously faced with the play and planning paradoxes, influencing the way she intervenes while this process of explaining and justifying develops in the classroom. If the students' argumentation leads to a discussion that diverts from the teacher's object, the teacher may decide not to take advantage the opportunity that arises, for further engaging students in explaining and justifying.

Identifying points of contradiction

Tertiary contradictions appear between the object of activity in a central activity and the 'culturally more advanced' activities. Analysis of the micro activity system as a classroom which is nested within the system level such as the institutional level in which the school is part of, as well as the cultural-historical level which is involved with the available research literature results into identification of a tertiary contradiction due to a differentiated object. The two poles of the object of the central activity of the classroom related with pre-proving activity will unavoidably clash with the object of pre-proving activity as identified in the system level. Initially, a contradiction between the classroom level and the institutional level resides in the fact that there is no clear identification of an object related to proving. That is, analysis of the official documentation indicates a general object of mathematical activity that is not necessarily in accordance with the object of the teacher related to pre-proving activity. To be more precise, the information collected from the official documentation points to low level of expectation with regards to exploration and investigation in problem solving in Year 6. Analysis of the report of the official documentation shows that proof and proving is not being acknowledged as a key criterion, nor mentioned in the mathematics curriculum. Furthermore, explaining and justifying points to an explanation being given by providing the mathematical operations used to find the answer and the justification being provided by using the definition. Adding to the above, there is no formal requirement regarding definitions. Definitions as approached by the official documentation are descriptive and extracted. This is not in accordance with the teacher's practice where definitions play a vital role. One may argue that a consideration of the educational objectives, as pre-determined by the mathematics curriculum, leads to the conclusion that the outcome of the teacher's practice is the one intended by these objectives.

However, in order for this to be achieved, the pre-proving activity is narrowed down. Thus, for instance, providing answers based on definitions and properties of shapes clashes with providing explanations based on the conceptual aspects of the definitions and the shapes. In a similar way, this tertiary contradiction concerns the cultural-historical level as well. Even though at a first glance the teacher's objects seem to be in line with the established research literature related with proving, the dilemmas the teacher needs to confront, as well as the ambiguity of the notion of proving existing at the institutional level, clash with the cultural-historical level.

Discussion and conclusions

A consideration of the above rationally points to the inference that the advanced form of the central activity object is not yet the dominant form of the activity. Thus, it can be argued that a first step towards a unification of these activities should be the resolution of the tension that exists within the macro system. Would providing a mathematics curriculum, which defines its object concerning proving and defining activity by incorporating crucial elements from the research literature, lead to a desired outcome?

The discussion regarding tertiary contradictions reveals the value of this concept in understanding systems of activity. By identifying the manifestation of contradictions through the materialized tensions, a holistic view of the phenomenon under investigation emerges. It is accepted that not all emergent contradictions can be resolved simultaneously. While a resolution exists for some contradictions, others are suppressed. That is, the contradiction on the teacher's object is continually present, surfaces in the teacher's everyday practice in various forms and is foundational to the other levels of contradiction. However, since this contradiction remains, the discussion should be centered on the means that the teacher can turn to for a possible and fruitful resolution of the contradictions that emerge in the other levels. Elaboration of the emergent tertiary contradictions leads to asking whether a possible balance between the macro level would be an aid in the resolution of the tensions manifested as contradictions in the micro level. Due to the way the aforementioned forces impact on the classroom activity, providing a straightforward answer is not an easy task. Undoubtedly, as it has been exemplified, proof and proving might be encouraged in all school levels. This indicates that exemplification of the role of proof, explanation, exploration and definitions might be included in the mathematics curriculum and the relevant curriculum material. Perhaps, a clear connection between the aforementioned would relieve, to an extent, the teacher from paradoxes. That is, knowing that the above aspects of mathematical reasoning might not be necessarily competing with each other and may be the way for a resolution of the play and planning paradoxes, as the purpose and utility underlining the task design would not clash with the object of the central activity system (Ainley et al., 2006).

Adding to the above, the fact that the official documentation is implemented in the classroom by the teacher points once again to the crucial role of the teacher. Specifically, the above findings further highlight the role of the teacher's knowledge about proof in mathematics teaching. Keeping in mind the findings of this study related with definition construction as part of pre-proving activity, it is important to consider essentially that the types, the characteristics and functions of mathematical definitions should be taken into account when understanding and describing the mathematical

knowledge for teaching when engaging students in proving activity. Would this element of knowledge enable mathematics teaching to support desirable student learning outcomes in the domain of proof and in mathematics more broadly?

References

- Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal*, 32(1), 23–38.
- Balacheff, N. (1988). *A study of pupils' proving processes at the junior high school level*. Unpublished paper presented at the Joint International Conference 66th NCTM and UCSMO Project: Chicago.
- Balacheff, N. (2009). Bridging knowing and proving in mathematics: A didactical perspective. In G. Hanna, H. N. Jahnke & H. Pulte (Eds.), *Explanation and proof in mathematics. Philosophical and educational perspectives* (pp.115–135). New York, NY: Springer.
- Engeström, Y. (1987). *Learning by expanding: an activity-theoretical approach to developmental research*. Helsinki, Finland: Orienta-Konsultit.
- Foot, K. A. (2014). Cultural-Historical Activity Theory: Exploring a theory to inform practice and research. *Journal of Human Behavior in the social Environment*, 24(3), 329–347.
- Hanna, G. (2000). Proof, explanation and exploration: an overview. *Educational Studies in Mathematics*, 44(1-2), 5–23.
- Herbst, P., & Balacheff, N. (2009). Proving and knowing in public: The nature of proof in a classroom. In D. Stylianou, M. Blanton & E. Knuth (Eds.), *Teaching and learning proof across the grades: A K-16 Perspective* (pp. 40–63). New York, NY: Routledge.
- Hoyles, C., & Noss, R. (1992). A pedagogy for mathematical microworlds. *Educational Studies in Mathematics*, 23(1), 31–57.
- Jaworski, B., & Potari, D. (2009). Bridging the macro- and micro-divide: using an activity theory model to capture sociocultural complexity in mathematics teaching and its development. *Educational Studies in Mathematics*, 72(2), 219–236.
- Rochelle, J. (1998). Activity theory: A foundation for designing learning technology? *The Journal of the Learning Sciences*, 7(2), 241–255.
- Stake, R. E. (1981). The art of progressive focusing. In 65th Annual Meeting of the American Educational Research Association (pp. 1–8). Los Angeles, CA: AERA.
- Stylianides, A. J. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education*, 38(3), 289–321.
- Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458–477.

Yackel, E. & Hanna, G. (2003). Reasoning and proof. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research Companion to NCTM's Principles and Standards for School Mathematics*, (pp. 227–236). VA: NCTM.