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## ► To cite this version:

Edyta Nowińska. Assessing how teachers promote students' metacognition when teaching mathematical concepts and methods. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02430150

**HAL Id: hal-02430150**

**<https://hal.science/hal-02430150>**

Submitted on 7 Jan 2020

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# Assessing how teachers promote students' metacognition when teaching mathematical concepts and methods

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*An important aspect of teaching mathematics is designing learning environments and guiding students' learning activities in such a way that learners can develop or improve their mathematical competencies. Promoting students' metacognition in teaching mathematics is regarded as having an important role in achieving this goal. Unfortunately, rarely anything is known about how metacognition can be effectively fostered in regular mathematics classes. The paper presents an analysis and an assessment of metacognitive activities practiced in a regular mathematics lesson. It also describes the rating system used for this assessment, and exemplifies how this tool can be used to analyze the extent to which teacher's practices are conducive to foster students' metacognition, as well as to describe the need for a further improvement of these activities.*

*Keywords: Metacognition, instructional quality, classroom discourse.*

## About the role of metacognition in teaching and learning mathematics

In mathematics education, metacognition is usually understood as knowledge about cognition and regulation of cognition (Veenman, Van Hout-Wolters, & Afflerbach, 2006; Willson and Clarke, 2004). Most definitions of metacognition differentiate between *metacognitive knowledge* (e.g. the person's knowledge about cognitive tasks in mathematics, about strategies to cope with these tasks, and about one's own competencies related to these tasks and strategies), *metacognitive skills* (e.g. the person's procedural knowledge for regulating one's own problem solving and learning activities) and *the execution of such skills in form of metacognitive activities* (for an overview see Veenman et al., 2006). Such differentiation is important for theoretical considerations concerning the meaning of metacognition, whereas in concrete situations it is hardly possible to distinguish between these different components.

Metacognition has been ascribed an essential role in regulating students' cognitive processes in *problem solving*, but also in the *learning of mathematics in general*, in particular when constructing and organizing knowledge (cf. Schraw & Moshman, 1995; Wilson & Clarke, 2004), as well as in a *self-regulated use* of mathematics in different contexts in order to achieve some goals (Boekaerts, 1999). Thus, from a teachers' perspective, promoting students' metacognition can be considered as a *mean* for effective teaching – for engaging students in cognitive processes needed to understand the mathematical concepts and methods to be learned. But it must be also considered as a *goal* of teaching – as an essential aspect of students' mathematical competencies to be improved through teaching.

Much research has been done on modelling, analyzing and promoting metacognition in solving mathematical problems. Many paradigms adopted to this context in the past research were based on the assumption that the way students learn to solve problems is “to first acquire the mathematical knowledge needed, then acquire the problem-solving strategies that will help them to decide which

already known procedure to deploy, then acquire the metacognitive strategies that will trigger the appropriate use of problem-solving strategies (...)" (Lesh & Zwojewski, 2007, p. 793). Such paradigms not only separate problem solving from concept development. They also separate metacognition from teaching mathematical concepts and reduce promotion of metacognition to teaching a list of simple rules (e.g., make a plan, draw a picture, mark important words, control your solution, evaluate the result). I consider this separations as one possible reason for the unimpressive results (ibid.) from this kind of research, because it seems hardly possible to acquire adequate metacognitive knowledge for solving problems without being engaged in metacognitive processes when constructing substantial meta-mathematical knowledge, meaning knowledge about relevant mathematical concepts, methods, and reasoning strategies (cf. Veenman et al., 2006). And therefore I see the need for shifting the focus of research on metacognition from teaching problem solving to regular teaching situations in which the learners have to learn mathematical concepts and methods.

### **An operationalization of metacognition in teaching and learning mathematics**

Since rarely anything is known about how metacognition can be effectively fostered in class when teaching and learning new concepts and methods (cf. Wilson & Clarke, 2004), a deep analysis of metacognitive practices in regular classes can be the first step for research aiming at enhancing students' metacognition. For this, an operationalization of the construct "metacognition" for the context of teaching and learning mathematics is needed. This has been done by the research group Cognitive Mathematics at the University of Osnabrueck. Its result is a category system<sup>1</sup> decomposing metacognition in *planning*, *monitoring* and *reflection*. Planning means organizing cognitive activities in order to achieve some goals as well as specifying cognitive challenges to be overcome (e.g., planning the use of mathematical methods and concepts to write a proof or planning the use of a particular strategy or representation to solve an equation). Monitoring means controlling cognitive activities and their results (e.g., controlling the validity of logical statements of arguments and the consistency of an argumentation or the validity and adequacy of tools and methods used in mathematical modelling or the correctness of meaning extracted from a mathematical text). Reflection means thinking about different aspects of cognitive activities involved in learning and understanding mathematics (e.g., analyzing relations between concepts, conceptions, misconceptions and external representations of mathematical concepts, analyzing the process of developing new mathematical concepts, analyzing mathematical concepts, methods and strategies with regard to the kind of problems and contexts for/in which they can be applied).

### **On the role of discourse in promoting metacognition**

Promoting metacognition in mathematics class does not mean teaching only one individual student how to organize, control and evaluate her/his cognition when learning and applying mathematical concepts and methods. It rather means organizing the teaching in such a way that as many as possible students are engaged in metacognitive activities. This can be achieved by establishing a *discursive* discussion culture with rules that force the students to control and regulate their own cognition and comprehension when other students or the teacher explain their ideas, solutions,

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<sup>1</sup> [www.mathematik.uni-osnabrueck.de/fileadmin/didaktik/Projekte\\_KM/Kategoriensystem\\_EN.pdf](http://www.mathematik.uni-osnabrueck.de/fileadmin/didaktik/Projekte_KM/Kategoriensystem_EN.pdf)

conceptions or difficulties in understanding the discussed mathematical ideas. Such discursive culture implies that the teacher and the students precisely control what the classmates say, reflect on differences between various conceptions and reasoning strategies mentioned in the class, and use this reflection to regulate their own understanding. However, this can be practiced in an effective and beneficial way, only if the teacher and the students use class discussions to precisely explain their cognition and if they aim for a coherent discussion. Such discursive aspects of class discussion seem to be inevitable for promotion of metacognition in regular mathematics classes.

To analyze such kind of discursive practices in regular mathematics classes, the category system decomposing metacognition in planning, monitoring, and reflection (see footnote 1) also includes two categories for analyzing and coding the precision and coherence of class discussions: *discursivity* and *negative discursivity* (both are not in the focus of this paper; for details see Nowińska, 2016b).

## **Research question**

To deepen our understanding on how metacognition is practiced in regular mathematics lessons, 24 lessons videotaped in six classes have been analyzed and assessed with the rating system described in the next section. One research question in this explorative study was: Which implications can be drawn from these assessments to improve the instructional quality with regard to metacognitive activities? This paper presents some results from this research. It reveals and explores an alarming fact concerning metacognitive practices in a regular mathematical instruction, and explains the need for a further improvement of the metacognitive-discursive quality. In doing so, it contributes to the discussion about how metacognition can be effectively implemented into mathematical instructions.

In the following, to begin with, the tool used for analyzing and assessing metacognitive activities is presented. Afterwards, metacognitive activities in one class are analyzed with this tool in order to make the alarming aspect more “visible”.

## **The rating system for assessing metacognitive-discursive instructional quality**

To assess the extent to which metacognitive activities are used to elaborate mathematical concepts, methods, and students’ cognition related to them, one needs a reliable evaluation method which also takes into consideration the *discursive* aspects of the class discussion. Such a method has been developed at the University of Osnabrueck, and evaluated in cooperation with the German Institute for International Educational Research (DIPF) in Frankfurt. It is based on the application of two tools explained in detail in Nowińska (2016a). The first tool is the afore-mentioned category system for coding metacognitive and discursive activities of teachers and students in their utterances in public class discussions (PCD). The second is a rating system for an evaluation of *metacognitive-discursive* instructional quality (MDQ). This tool was presented at the Third ERME Topic Conference (Nowińska, 2016b), and the evaluation results concerning the reliability of the ratings at CERME10 (Nowińska & Praetorius, 2017). The wording “metacognitive-discursive” has been chosen to stress that metacognitive activities are analyzed together with discursive aspects of class discussion.

The rating system is a set of seven rating scales for assessing different aspects of the instructional quality of these activities. In the two-step rating procedure, the rater first watches the video and reads the transcript; thereby she/he interprets each verbal student's and teacher's utterance and codes metacognitive and discursive activities using the category system. In the second step, the rater assesses the quality of these activities with regard to seven rating dimensions (quality aspects). Thereby she/he uses the video and the transcript with all codes for metacognitive and discursive activities set by her/him in the first step. Each rating dimension is given by an item called guiding question (*GQ*) and by a nominal rating scale consisting of several answering categories<sup>2</sup>. The answering categories describe in detail how the relevant quality aspects are reflected in the class discussion. Different answering categories describe qualitatively different situations. Their order on the rating scale is based on the increasing quality of the class discussion with regard to the particular quality aspects. The rater has to choose the answering category that best describes the situation given in a class. For reasons of space, in the following, only three of seven guiding questions are explained.

The *first GQ* regards the extent to which learners practice metacognition in PCD in an autonomous and elaborated way, and focus it on the mathematical content (e.g. concepts, conceptions, methods) to be learned. There are four answering categories to it. The first category says that metacognitive activities are carried out solely by the teacher, and/or no effort is made to foster students' metacognitive activities in order to elaborate the mathematical content and students' understanding of it. The fourth category says that the learners are autonomous in practicing metacognition and thereby they make efforts to elaborate and understand the mathematical content.

The *second GQ* focuses on justifications combined with metacognitive activities, and on the extent to which they are used to elaborate on the mathematical content and on students' understanding of it. It asks for the extent to which students combine their metacognitive activities with elaborated justifications in an autonomous way, and also for the extent to which such justifications and coherent global argumentations of the whole class seem to be important in the culture established in the class (and consequently fragmentary justifications have to be corrected). There are four answering categories graduated in an analogue way as these to the first *GQ*.

The *third GQ* puts the focus on a hypothesized learning effectiveness of metacognitive and discursive activities of students and the teacher. The rater has to assess the extent to which the metacognition she/he observed in the videotaped PCD contributes to construct or deepen students' meta-mathematical knowledge or to foster their meta-mathematical skills related to the mathematical content of the lesson (tasks, questions, tools, methods, reasoning, concepts, conceptions). The first answering category describes a PCD without any constructive use of metacognitive and discursive activities; the second says that only very few learners make efforts to deepen their meta-mathematical knowledge or to reflect on their meta-mathematical skills, but their metacognitive activities are not elaborated by the classmates or the teacher. The third and last

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<sup>2</sup> [https://www.mathematik-cms.uni-osnabrueck.de/fileadmin/didaktik/Projekte\\_KM/GuidingQuestions.pdf](https://www.mathematik-cms.uni-osnabrueck.de/fileadmin/didaktik/Projekte_KM/GuidingQuestions.pdf)

answering category says that metacognitive aspects of the mathematical content are discussed in a constructive and elaborated way.

## Analyzing and assessing metacognition in mathematical instruction

The lesson analyzed in this paper was videotaped in one class in grade 7 in a German secondary school (Gymnasium) during the introduction to linear equations. Four lessons in this class were videotaped and analyzed with the rating system, and all of them feature the alarming aspect described in this paper. The term ‘alarming aspect’ is used here to emphasize some striking characteristic of the instructional quality of metacognitive and discursive activities of the teacher and of the students in lessons planned for an introduction of new mathematical concepts and methods. Due to the small number of lessons, the case of the lesson analyzed in this paper has to be used with caution. But since similar aspects were also observed in other classes, there seems to be a tendency for some similar teaching practices also in the case of other teachers/classes.

### The instructional context of the introduction of a new mathematical concept

To motivate the introduction of the term ‘equation’, the teacher uses two mathematical problems. One of them is: “The three major oceans (Pacific, Atlantic, and Indian Ocean) together have a water surface of 320 million square kilometers. The Pacific Ocean is twice as large as the Atlantic, and the Indian Ocean is 10 million square kilometers smaller than the Atlantic Ocean. The surface  $x$  (in million square kilometers) of the Atlantic is unknown.” The teacher makes no comments on the concept of equation to be learned in this lesson.

Solving this problem requires the use of metacognitive activities of planning a solution, controlling the solution steps and the result, and reflecting on the text and the solution process. Since solving the problem should motivate the introduction of equation and of a method for solving equations, students’ reflection about the formal representation of an equation and about the regulation of their own cognitive activities needed to write and solve an equation would be required to construct *metacognitive mathematical knowledge* about the new mathematical content.

Most of the learners solved the problem at home. In the class, Sabrina presents her solution (Figure 1):

x: Atlantik	$x + y + z = 320 \text{ Mio. km}^2$	$x = 82,5 \text{ Mio. km}^2$
y: Pazifik	$x + 2x + x - 10 \text{ Mio. km}^2$	$y = 165,0 \text{ Mio. km}^2$
z: Indischer Ozean	$4x - 10 \text{ Mio. km}^2 = 320 \text{ Mio. km}^2$	$z = 72,5 \text{ Mio. km}^2$
	$4x = 330 \text{ Mio. km}^2$	$\frac{320 \text{ Mio. km}^2}{4}$
	$330 : 4 = 82,5 \text{ Mio. km}^2$	

**Figure 1: Problem solution**

The transition from the equation in line 3 to this in line 4 raises many questions. A few students try to find out why one is allowed to change the equation in line 3 to the equation in line 4 (for reasons of space, the utterances are presented here in a shortened and more precise version):

Johana: I do not understand how you get from line three to line four.

Sabrina: Here, I did not know what to do, and then I asked my mother and she simply said, if you have to subtract something from  $x$  or from the variables, then you add that here to the result of it to be able then to continue the calculation.

Jonas: (...)  $4 \cdot x - 10$  million that is equal to 320 million. And then four times  $x$  without minus 10 million, it is clear that there are then 10 million more, because then you do not make the subtraction. That's why it's 330 million. (...)

Anna: (...) we want to have  $x$  on the left side alone. (...) and then you can only get the minus 10 million..., if you want to get rid of it, you just have to put 10 million on both sides. So on the left you have then  $4x$  minus 10 million plus 10 million. One can then... yes you can take this away. Then there is only  $4x$ , and on the other side 320 million plus 10 million. (...)

### **An analysis and assessment of metacognitive activities**

Johana **reflects** (R6b) on her understanding of the solution, whereas Jonas and Anna **reflect** (R4) why one is allowed to change the equation in line 3 to the equation in line 4. (The meaning of codes R6b and R4 is explained in the category system linked in footnote 1.). Jonas analyzes the effect of the subtraction by 10 million on the value of  $4x$  and describes a relation between  $4x - 10$  and  $4x$ . He uses this relation to find the value of  $4x$  if it is known that  $4x - 10$  equals 320 million. Anna describes a clear goal of changing the equation (“we want to have  $x$  on the left side alone”) and suggests an arithmetic operation to change the term on the left side, but she does not explain why the terms on both sides of the equation have to be changed and can be changed as suggested by her.

The teacher arranges the class discussion in such a way that the students discuss their solutions without his guidance. This situation encourages the learners to take the responsibility for *solving the task* and to regulate their own cognition, thus to practice metacognitive activities such as planning, controlling and reflection. The students make use of this opportunity. They seem to be willing to plan, control and reflect their mathematical activities. In the sense of the *first guiding question*, this scene shows a high quality of metacognitive activities (answering category 4). High quality means that many metacognitive activities can be observed in students’ utterances and that these activities indicate students’ efforts to understand the solution.

The class discussion is led by a small group of students, who practice their metacognitive activities in an elaborated way by giving justifications related to mathematics and to their own understanding. They elaborated on the solution steps presented by Sabrina and on their own understanding of these steps. In the sense of the *second guiding question*, the lesson seems to indicate a high quality of metacognitive activities with justifications (answering category 4 to guiding question 2), meaning that giving elaborate metacognitive comments on the mathematical activities seems to be a desired, well established rule of the classroom culture in the class. But on the other hand, following this rule does not mean that the global class argumentation is coherent and focused on a precise elaboration of students’ understanding of the mathematical content. Consequently, this indicates a low quality of metacognitive activities with justifications (answering category 2 instead of 4 must be chosen).

Can this kind of metacognitive activities support students’ understanding of the new mathematical concept? Can it help to construct or deepen students’ meta-mathematical knowledge or to foster their meta-mathematical skills related to the mathematical content of the lesson? The three solution ideas as described by Sabrina, Jonas and Anna in the transcript differ significantly from each other. Unfortunately, the teacher makes no effort to elaborate on these approaches and to initiate co-

constructive processes with regard to the meaning and adequacy of the new formal representation and of transforming it to find a solution. In the whole class discussion there are no activities indicating that the students reflect on each other ideas in order to understand how they differ from their own ideas, and whether all of them are correct and based on a valid mathematical argumentation. *Only very few students* try to explain the meaning of the equation and seem to have an idea of how to justify the transformations of the equation (answer 2 to guiding question 3). But since their ideas are not elaborated and related to each other, the process of solving the equation is not effectively used to co-construct metacognitive mathematical knowledge about equations, and to make this knowledge well comprehensible for *all* students. Thus, despite lots of students' metacognitive activities, there are no visible effects concerning the construction and acquisition of metacognitive mathematical knowledge or fostering mathematical skills at the level of the whole class discussion. In the sense of the *third guiding question*, the metacognitiv-discursive quality of this lesson must be assessed as low (answering category 1 to guiding question 3), meaning that the extent to which metacognitive activities are used to foster students' understanding is not satisfying.

## Discussion

Training programs for fostering students' metacognition in learning mathematics concentrate on problem solving. But it seems hardly possible to acquire adequate metacognitive knowledge for solving problems – or even more general speaking, for a self-regulated use of mathematics in order to achieve a particular goal – without being engaged in metacognitive processes when constructing substantial mathematics-specific knowledge, such as knowledge about relevant concepts, methods, and strategies for thinking and reasoning (cf. Veenman et al., 2006). The lesson described in this paper shows that engaging students (in this particular class) in such metacognitive processes is not in the teacher's focus when solving a mathematical problem designed to introduce a new mathematical concept (equation). The teacher supports students' metacognition in the sense that he encourages them to plan, control and evaluate the solution among themselves. But he neither supports students' reflection on the new mathematical concept used in this process nor tries to explain or relate to each other the different conceptions described by the students to explain one step in solving an equation.

This is an alarming feature of the instructional quality of mathematics lessons. Mathematics teachers are expected to foster students' metacognition in order to deepen their understanding of mathematics and to enhance their skills in regulating their own cognition when using mathematics in different contexts. This alarming feature shows how promotion of metacognition in regular mathematics lessons is reduced to engaging the students in self-regulated processes of solving mathematical problems and tasks without taking care on the discursive quality of their mathematical discussion, and on their efforts in elaborating mathematical concepts and methods.

The rating system used to analyze the lesson can also be used to describe the need for a further improvement of instructional quality with regard to fostering students' metacognition. The high assessments given to the lesson with regard to the first and second guiding questions indicate that the students are able to execute metacognitive activities and willing to justify their answers and conceptions related to solving a linear equation. Unfortunately, the teacher does not use this



potential to change the class discussion to a *discourse about the new mathematical concept*. In different words, the teacher does not make any efforts to shift the focus of the discussion from discussing the local steps of solving the given problem to a coherent discussion about the new mathematical representation (a formal representation of an equation) worked out in the process of solving this problem. This is, regrettably, the case in all four lessons videotaped in this class. This observation let us assume that improving the instructional quality with regard to students' metacognition requires changes in the discursive quality of the class discussion.

But this means more than just changing some social rules of discussion or increasing the number of students involved in the discussion. It rather means that the teacher and the students have to precisely control what the classmates say, explain differences between various conceptions and reasoning strategies mentioned in the class, and use this reflection to regulate their own understanding. It seems hardly possible to achieve this change without a close cooperation between teachers and researchers, and without engaging teachers in reflection on their practice.

The one lesson described in this paper does not allow us to generalize our assessment to all lessons of the particular teacher in this particular class. But, since the assessments given to all four lessons videotaped in this class are quite stable between lessons, it can be assumed that the problem described here is a general problem in this class. A transcript from the lesson following this one discussed in this paper is published in Nowińska (2016b). It shows that the students reflect on the sense of applying equivalence transformations to solve linear equations, and on their own difficulties in understanding the solution process. The teacher does not use these metacognitive activities to elaborate on the mathematical content and on students' understanding of it. He even seems to ignore students' reflections and justifications concerning their difficulties in understanding the mathematical content.

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