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The King's birthday, potentials for developing mathematics teaching

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In this paper, we have analysed a video in which a small group of students were asked to find out when the Norwegian King was born given his 80 years of age today on his birthday Feb 21st 2017. We have identified potentials for initiating shifts in discourse in which children's ways of solving a problem could become an explicit object of reflection. We suggest that revoicing as a talk move to encourage collective reflection, did not work as suggested in the research literature. Additionally, working on a number line could have been a joint activity of making what was previously done in action, to an object of reflection.

Keywords: Reflective discourse, collective reflection, empty number line, revoicing.

Background and context

The lesson, from which this episode is taken, is part of a Norwegian national project, which investigates if decreased student/teacher ratio is beneficial for students' learning. In the project, one dedicated teacher takes a group of students, typically 4–6 into a separate classroom to teach specific subjects and methods. The teacher is by the researchers in this project, acknowledged as a “skilled teacher” with more than twenty years teaching experience. According to the research team, the episode investigated here, is a typical example of this teacher's teaching practice.

Five year-four children (two boys and three girls) were seated around a table, and the teacher was standing up front by the flip over. The episode lasted for about twenty minutes. The teacher presented a task (problem) that the children were asked to solve:

I am going to give you a task now, []. You will get a sheet on which you may draw. Just close your books for now. The task is to find out when the [Norwegian] King was born. He is 80 years today, so you shall try to find out in what year he was born. [] The year is 2017, and 80 years backwards.

After having presented the problem, the teacher said they were allowed to collaborate. He emphasized that the task “is quite challenging” and that he would give them some time to solve the problem. One of the children, a girl, here called Tove, addressed the teacher and the other children several times throughout the whole episode. As a response to her, the teacher said: “I will give you a tip, not to trust anybody else, but only to think for yourself and trust yourself”.

When having started working on the task, the children both aloud and inaudible, suggested solutions. However, the teacher interfered after about 2 minutes saying: “*remember what we have been saying, it is not the answer that is going to impress me. It is the way you have got the answer, that will impress me the most*”. He thus emphasized the process rather than the product. The children continuously volunteered to come up with an answer. However, the teacher told them to wait yet another minute.

We watched the video clip several times. Focusing on the rehearsal of students' proposed solutions of the problem, we noticed that the teacher frequently revoiced students' explanations and made

claims about their thinking: “So you have been thinking like this,...”, was a repetitive comment from the teacher on students’ proposed solutions.

Against this backdrop, we address the role of classroom discourse in examining potentials for supporting mathematical sense making within a group of children. We draw on the concepts of *reflective discourse* and *collective reflection* (Cobb, Boufi, McClain, & Whitenack, 1997) in searching for potentials for development of possibilities for mathematical learning in this episode (Cobb, Stephan, McClain, & Gravemeijer, 2011) .

Theoretical and methodological perspectives

Reflective discourse and collective reflection

Cobb et al. (1997) focused on classroom discourse, which appeared to support the development of students’ conceptual activity. In this work, they considered students’ conceptual activity as a source of mathematical ways of knowing, and meaningful mathematical activity as creation and manipulation of mathematical objects. They introduced the term “reflective discourse which is characterized by repeated shifts such that what the students and teacher do in action subsequently becomes an explicit object of discussion” (Cobb et al., 1997, p.1). They emphasized the teacher’s role, because the teacher is in the position to judge the significance of the children’s mathematical activity, and thus initiate shifts in discourse. One such shift may occur in further probing children for more ways of solving a task. Another shift occurs when the children’s different solutions of a mathematical problem may emerge as an explicit object of discourse. This activity involves comparing different solutions and a discussion of how they may relate to each other together with a corresponding reflection. This reflection does not occur simultaneously for all children since all participate in a common discourse, but it occurs as a collective reflection. Pragmatically, analysis of “reflective discourse” can clarify how teachers may support students’ mathematical development and potentials for such, and the term “collective reflection” refers to the joint activity of establishing what was previously done in action, as an object of reflection. Students’ participation in the discourse “collective reflection” may constitute conditions for possibilities of learning (Cobb et al., 1997; Cobb et al., 2011). The role of students is crucial in these activities. If no child is able to respond to the teacher’s initiation of shift in discourse, no shift will occur. The individual child’s reflective activity may contribute to developing a discourse with potential for mathematical sense making.

Cobb et al. (1997) argue that the teacher must initiate shifts in discourse so what the children have done in action, becomes an explicit object of reflection. In our data, the activity was solving a problem and proposing solutions. Our research problem was to investigate how students’ access to mathematics was supported, and to identify potentials for initiating shifts in discourse where children’s ways of solving the problem could become an explicit object of reflection and a collective reflection could take place.

The issue of revoicing

Chapin, O’Connor, & Anderson (2013) presented four suggestions for using whole-class discussions in problem solving: to understand the problem; to explain one solution method, to extend students’ knowledge and to compare solution methods. We consider the latter crucial with

regard to collective reflection in our study. In order to encourage whole-class discussions, Chapin et al. (2013) suggested talk moves which may help individual students clarify and share their thoughts: wait time; turn and talk; stop and jot; will you share that with the class and revoicing (so you are saying...). They suggested revoicing as a talk move if it seemed as if what the student was trying to say made things more confusing if the student carried on. As a goal for the teacher is to improve all students' mathematical thinking, the teacher cannot give up on an unclear explanation from one student. Revoicing was also suggested if a student says something insightful, which might move the class or group forward. Revoicing what a student is saying, will give that student a chance to clarify and the other students a chance to hear it again, may be in a clearer version. We wanted to further investigate revoicing as a talk move, and we discuss how a group discussion on comparing solution methods could become a discourse of collective reflection.

Kazemi and Hintz (2014) build on the talk moves of Chapin et al. (2013) and emphasise revoicing as "repeat some or all of what the student has said, then ask the student to respond and verify whether or not the revoicing is correct. Revoicing can be used to clarify, amplify, or highlight an idea" (p. 21).

Since the mathematical activity involved multidigit subtraction, using both standard algorithm and mental strategies, we have also drawn on Plunkett (1979) as well as on theories about the use of empty number line (Beishuizen, 2003; Gravemeijer, Bruin-Muurling, Kraemer, & Van Stiphout, 2016).

In addressing our research problem, we posed the following research questions: (i) What potentials for facilitating productive mathematical discussions did we see? (ii) How did revoicing as a talk move influence the group's discourse in supporting students' access to mathematics?

Presentation of the episode and analysis

The purpose of our analysis was to identify potentials for development of reflective discourse and collective reflection. We discuss four students' presentations in light of the frameworks presented above. The teacher frequently revoiced students' explanations and thoughts. In the literature (Chapin et al., 2013; Kazemi & Hintz, 2014), revoicing is suggested as a talk move to encourage whole-class discussions in order to extend students' knowledge and to compare solution methods. We have analysed the part of the episode in which the students presented their solutions.

Tove was the first student (after eagerly volunteering) who was asked by the teacher: "Can you tell us about the way to figure out the answer?"

Tove: I started with seventeen minus eighty. I removed seventeen from eighty so I could figure out what to remove from 2000

Teacher: What answer did you get when you removed 17 from 80?

Tove: I got 63

Teacher: What do you want with 63? What are you going to do with these 63?

Tove: I will remove 63 from 2000 to find the answer.

At this point, the teacher intervened Tove's explanation, while he was drawing on the board:



Figure 1: Teacher's drawing to illustrate Tove's thinking

Teacher: Right. Look at this. Tove, look at this. We are here in 2017, and now we are traveling 80 years back in time. She started by thinking... First, she wanted to go to the year 2000, and then go from 2000 to the year the King was born. Right? Therefore, you started with 80 minus 17, and got 63. She removed 17 from 80. Then you got 63. Then she wanted to find what 2000, from here, and jumping 63 years backwards from year 2000. That is how she thought. You tried to do 2000 minus 63, but this is a really difficult equation. Because... You have not really learned this, with such large numbers. Did you manage to finish it, too?

Alternating between addressing Tove directly and as a third person, the teacher made claims about her thinking in saying “She started by thinking...,” and “That’s how she thought”. He revoiced her explanation, probably in order to clarify for the group. He underpinned and highlighted with an illustration (figure 1). The empty number line which he drew, indicated a linear or sequential way of thinking (Beishuizen, 2003). However, studying the continuation of Tove’s explanation, there was a move away from a linear thinking towards the use of the standard algorithm, as the teacher also did:

Teacher: So you tried to do 2000 minus 63, but this is a really difficult equation

Tove: Yeah. I removed a thousand and exchanged it for 10 hundreds, and then I exchanged a hundred for tens, and a ten for ones. (Removed 3 from 7. 3 from 10. Yeah, 3 from 10. Then I had 7 left, so I removed... Umm... 9... Umm, 6... [] 9 minus 6. Then I had 3. But I still didn't have any to be removed here, and I didn't have 10 here, since I removed one. So I have 9 here. Then I removed one thousand, so I have one left.

While sitting, Tove explained orally how she had subtracted 63 from 2000 through the use of standard algorithm for subtraction. She was not only referring to digits, but to “ones”, “tens”, “hundreds” and “thousands”. Again, the teacher intervened:

Teacher: I must say that I am extremely impressed. You managed to figure out by yourself without having learned it before. I still want to show her way first, then you can explain afterwards. I just want to show her way of thought.

At this point, the teacher had left his earlier sequential thinking and use of empty number line. He re-voiced what Tove had explained (orally), claimed her thoughts (I just want to show her way of thought) and used the flip over to explain and show the standard algorithm in detail.

The teacher monitored the conversation, and there was no communication between the students. The other four students in the group were not invited to participate. Instead of asking if there were

questions or comments to Tove's explanation, he "wanted to show her way of thought", without knowing if there was a need for that. As an alternative initiation of the whole-class discussion, (Kleve & Solem, 2014), the other students could have been invited to ask clarifying questions (Chapin, et al., 2013). Although having emphasized that it was not the answer as such, that was going to impress him, he neither challenged Tove's thinking, nor encouraged students to ask clarifying questions.

After having spent some time on Tove's way of solving the problem, three more students were invited to present their strategies. Lars, Vera and Kari. The teacher had obviously noticed that Lars had written down the right answer without any written calculations very quickly. Inviting Lars to explain he said: "I am eager to know how you have been thinking to get the answer, how did you think?" This initiated a shift discussed by Cobb et al. (1997). Lars responded to the invitation and walked up front.

Lars: I thought like this. We remove 17 so there's only 2000 left (writing 2000 on the flip over).

T: Yeah.

Lars: Then do minus 80 (writing 80 underneath).

T: Ah, you thought like that!

Lars: Afterwards we get 1920. That is simple...

Teacher: Can you write 1920, so we have it written down? I just want to repeat what you did so the others will understand what you meant. If you can move aside for a bit. What I understood is that you did not bother about the 17 at first. You just went straight back, and thought that: "What if we are in the year 2000 now?" If we then move 80 years backwards, we will land on 1920. Now I am eager to know, what did you do next?

Lars: I added 17, which is simple. Just 20 plus 17.

Teacher: Fantastic!

Lars: And that is... Nineteen...

Teacher: 1920 plus 17. Then you got? Nineteen?

Lars: Thirty...Thirty-seven.

Teacher: Did anyone understand this clever way?

As an answer to the teacher's question, a fifth student, said:

Per: Yeah. When he wrote 1920, I thought it looked like... I thought it looked good, in a way. Seemed like a good way to think.

Per's comment indicated an individual reflective activity, which was not followed up.

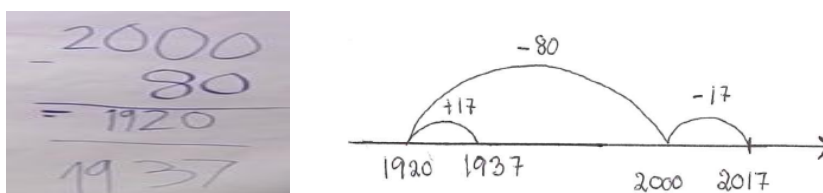


Figure 2: Lars' presentation and our illustration of it with an empty number line

Again, the teacher's use of revoicing and making claims about a student's thinking became evident. In this case, the teacher intervened, saying, "I just want to repeat what you did so the others will understand what you meant". A reason for revoicing might have been that the teacher found Lars' presentation being "clever". According to Chapin et al. (2013), revoicing is recommended when a student says something insightful, because there may be a potential then to move the class forward.

At this point we see another possibility for moving the class forward through a potential shift in discourse. Comparing Tove's and Lars' solutions and making that into an object of reflection could have created conditions for possibilities of learning (Cobb et al., 1997). What were the similarities between the two ways of solving the problem? How did Tove's solution differ from that of Lars'?

Following and contrasting this, Vera now volunteered to present saying: "I actually did not really do it that complicated". Hence, a comparison of methods was initiated by a student, and a potential shift in discourse was identified. Vera went to the board and subtracted 80 from 2017, using the standard algorithm. The teacher praised Vera's method and compared it to that of Lars:

You actually chose to use the algorithm in subtracting from 2017. It demands good knowledge to set that up. You worked it out elegantly and nicely and got the correct answer. However, his [Lars'] way of doing it was a method that is easy to use in mental calculation. [To Lars]: Really smart, that is why you spent so little time.

Here the teacher emphasized why a mental strategy may be smart compared to the use of the standard algorithm (Plunkett, 1979). The teacher elaborated the shift in discourse, initiated by Vera, further. He referred to mental calculation and speed, and a potential occurred to move the class forward. However, a discussion, which could have involved a collective reflection on the differences, did not take place. In figure 2, we have illustrated Lars' mental strategy, his sequential way of thinking, on a number line. Such an illustration could have been useful in comparing the two methods (Beishuizen, 2003; Gravemeijer et al., 2016).

The last student to present was Kari. She was invited to present her solution on the flip over:

Kari: First I did: Nineteen... [Writes 1900]. And then I did... Plus... 20 since the King is 80 years old. Then I added 17. [Writes 1937]

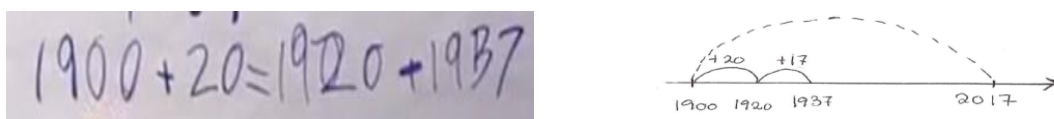


Figure 3: Kari's presentation and our illustration of it with an empty number line

Teacher: Right. You started with the year 1900. Now you are challenging me, I am actually not entirely sure how you have been thinking. Why did you start at 1900 and why did you add 20?

Kari: It is since the King became 80 years old,...

Student: [Whispers] 8 and 2 are “ten-buddies”

Kari: and 8 and 2 are “ten-buddies” so then I added 20.

Teacher: Ah, I see, because you knew that there are 80 years between 1920 and 2000?

Kari: Yeah

Teacher: I see! So you started here and tried to come up to the year where you had 80 years from there, up to the year in which we are now? I have to say, that is pretty smart.

The teacher seemed to have a different approach to Kari’s presentation than to the others. Kari had used a mental strategy, which for the teacher seemed “difficult to catch” (Plunkett, 1979, p. 3). In response, he asked clarifying questions rather than revoicing Kari’s presentation. Although a student “rescued” Kari in answering why she added 20 (“ten-buddies”), the teacher neither utilized it as a potential to involve the other students in further discussion nor to ask clarifying questions.

Discussion

Potentials for facilitating productive mathematical discussions

In the episode there were shifts in discourse, for example when the teacher was further probing for more ways of solving the task. However, these shifts were not followed up by a collective reflection. Potentials for other shifts in the discourse, which could lead to collective reflection, have been identified. We pointed out a possible shift in discourse in suggesting to compare Tove’s and Lars’ solutions and make that an object of reflection. Per’s comment on Lars’ way of solving the problem indicated an individual reflective activity, which was a potential for collective reflection. In her presentation, Vera initiated a shift in discourse. The teacher followed this up in comparing Vera’s use of the standard algorithm with Lars’ informal strategy, which he valued. Here the students could have been invited to participate in a collective reflection on the methods, which would have created possibilities for learning. With regard to Kari’s use of informal strategies, the teacher was challenged, and did not utilize the other students’ help (“ten-buddies”) for a further discussion.

Tove and Vera used standard algorithm for subtraction while Lars and Kari displayed a sequential way of thinking. Although the teacher started by representing Tove’s explanation with an empty number line, sequential thinking, he left it, preferring Tove’s use of the standard algorithm. He never went back to the number line when Lars and Kari both demonstrated a sequential way of thinking. We have illustrated Lars’ and Kari’s ways of thinking on empty number lines, figure 2 and figure 3.

Mental strategies, as Lars and Kari used, are difficult to explain (Plunkett, 1979). Representations are useful in making mathematics accessible for students, for sharing different strategies and ideas,

hence facilitating productive mathematical discussions (Duval, 2006). Working on a number line (see figure 2 and figure 3) could have been a joint activity of making what was previously done in action to an object of reflection and thus create conditions for possibilities of learning (Cobb et al., 1997). This could serve as a tool for Vera, who found Lars' method complicated, to see connections (similarities and differences) between the different strategies.

“Revoicing” or revoicing: A pitfall or potential for developing productive mathematical discussions?

In the analysis above, we have pointed out instances where the teacher revoiced students' contributions, but in a way that students were not encouraged to respond or contribute and take part in a discussion. Rather, the revoicing discussed in our analysis served as an interruption. When Per commented on Lars' method, he was not encouraged to explain further. Contrary to what is recommended in the research literature, revoicing as a talk move was neither used to clarify or amplify ideas, nor to extend students' knowledge and compare strategies. Our findings suggest that, instead of being a mean for shift into a reflective discourse, “revoicing” became a pitfall for developing a productive mathematical discussion.

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