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# Effects of a Scaffolding Model for small groups in mathematics

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*We investigated whether teachers' use of the Small-Group Scaffolding model (SGS model), when supporting heterogeneous student-groups in a collaborative setting in math lessons, raised students' mathematical level more than teacher support without use of the model. Participants were eight teachers and 266 students of two schools working collaboratively in 73 small heterogeneous student-groups on the topic of Early Algebra. The five teachers in the experimental condition were taught to scaffold the groups in their classes according to the SGS model (give support adapted to student-groups' needs). The three teachers in the control condition were not given prior instruction of how to support the students. Analysis of a pre- and post-test on mathematical level raising showed that more mathematical level raising occurred in the Scaffolding condition than in the control condition. We concluded that using the SGS model can lead to more mathematical level raising.*

*Keywords: scaffolding, mathematics, collaborative learning, mathematical level raising, teacher-student interaction.*

## Background

According to Freudenthal (1991) and Van Hiele (1986), the learning of mathematics occurs in discrete steps, implying the existence of levels of mathematical thinking. Van Hiele (1986) described four levels: 1) Visual level: the forms of mathematical objects are the object of study, 2) Descriptive level: the properties of mathematical objects are the object of study, 3) Theoretical level: relations between the properties of mathematical objects are the object of study, and 4) Formal logical level: relations between theorems are the object of study. Freudenthal (1991) built upon this theory and stated that the levels are more relative than discrete, meaning that level raising occurs every time a mathematical activity (performed at a lower level) consciously becomes the object of reflection (at a higher level). Freudenthal (1971) defines a mathematical activity as an activity of organizing subject matter, which can be matter from reality or mathematical matter, according to mathematical patterns or new ideas. For example, at a lower level, patterns of the number of tables and chairs in a table setting can be studied. At a higher level, these patterns themselves become the object of reflection when one tries to create a formula with which the number of chairs in a particular table setting can be calculated when the number of corresponding tables is given.

According to Freudenthal (1978), when students collaborate in a heterogeneous small group (a small group of students of mixed ability) on one task, there will often be at least one student who experiences an "Aha moment" (jumping to a higher level) when understanding the subject matter. A typical higher-level activity will follow for that student, such as reflecting on how he/she mastered the subject matter and explaining to other students in the group what he/she just learned. In other

words, discussions between students in a heterogeneous group while performing mathematical activities (mathematical discussions) enhance mathematical level raising.

A reoccurring problem with attaining mathematical level raising when learning Early Algebra in secondary education is that students find it difficult to make a shift from studying patterns at a lower level to understanding formulae at a higher level (Kieran, 1992; Sfard & Linchevski, 1994; Van Stiphout, Drijvers, & Gravemeijer, 2011). Janvier (1987) addressed this problem by defining Algebra representations, situation, graph, table and formulae as four ways to describe the relation between two variables in a formula. He calls the ability to switch between the representations a translation skill. According to Janvier (1987), translation skills are best learned when they are taught in a pair-wise manner. For example, the translation skill modeling is learned best when students first learn how to construct a situation from a formula, followed by learning how to construct a formula given a situation, or vice versa. In this study, all translation skills involving the representation formulae are considered to contribute to mathematical level raising. These translation skills are: Parameter recognition (formulae to situation), Computing (formulae to table), Sketching (formulae to graph), Modeling (situation to formulae), Fitting (table to formulae) and Curve fitting (graph to formulae).

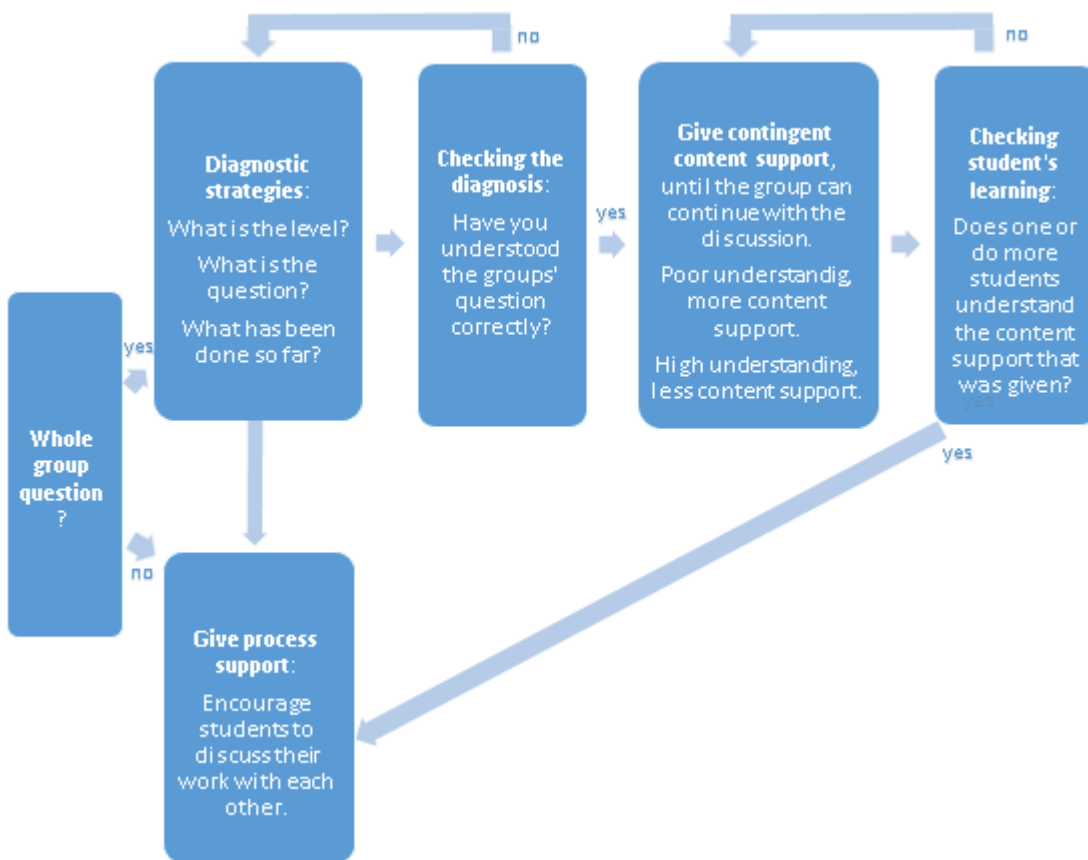
Collaborative learning is considered an important means to improve students' learning of mathematics (Webb, 1982). Several studies have shown that mathematical discussions in small heterogeneous student-groups contribute significantly to mathematical reasoning and level raising (Freudenthal, 1991; Dekker & Elshout Mohr, 1998, 2004). Dekker and Elshout Mohr, (1998) developed a 'Process model' (PM), which promotes and operationalizes mathematical discussions. This model distinguishes three types of learning activities: key activities, which help mathematical level raising; regulating activities, which regulate key activities and mental activities that occur in students' minds. They found that stimulating mathematical discussions by the teacher resulted in more mathematical level raising compared to providing content support (Dekker & Elshout Mohr, 2004). However, teachers find it difficult to support mathematical discussions in small-group learning (Webb, 2009).

Content support adapted to the level of the student's understanding (contingency) is considered one of the key features of scaffolding (Van de Pol, Volman, & Beishuizen, 2010). The other two key features of scaffolding are diminishing content support over time (phasing out) and returning responsibility for learning processes to the learner. Scaffolding in the context of group work seems to enhance learning outcomes in general (Van de Pol, Volman & Beishuizen, 2010; Webb, 2009). Van de Pol, Volman and Beishuizen (2012) developed a model to describe scaffolding interactions, the Contingent Teaching Model (CTM), in which teacher support is adapted to the level of individual students. It consists of four steps: applying diagnostic strategies, checking the diagnosis, applying intervention strategies, and checking students' learning. In the CTM individual students are scaffolded in a small-group setting.

In the context of collaborative learning teachers often scaffold individual students within small groups. How teachers can scaffold small groups at group level and what the effect is on students' mathematical learning, has not yet been investigated. A combination of stimulating discussions and

providing contingent content support adapted to the level of the group might result in more mathematical level raising. To help teachers support small heterogeneous student-groups we integrated the CTM (Van de Pol, et al., 2012) and the PM (Dekker & Elshout Mohr, 2004) into a scaffolding model for small student groups (Calor, Dekker, Van Drie, & Volman, 2019). We call this model the Small-Group Scaffolding model (SGS model).

In Figure 1 a flowchart representation of the SGS model is shown. The model consists of six teaching steps: evaluating if the question is a question of the whole group, applying diagnostic strategies, checking the diagnosis, giving contingent content support, checking students' learning, and giving process support (stimulating mathematical discussions).



**Figure 1: A Flowchart of the Small-Group Scaffolding model (Calor et al., 2019)**

Following the Small-Group Scaffolding model, teachers should first determine if the question is a question of an individual in the group or if it is a question of the whole group. In case the question is a question of the whole group, the teachers should diagnose the group's current level of understanding in the second step, by for example asking students what they have done so far and asking the students to show or tell their work to the teacher. It is possible that one or more students come up with an answer to the question during this step. In that case, the teacher should encourage the students to explain the answer to each other. If none of the students come up with an answer during step two, teachers can check if their diagnosis is correct in the third step. In the fourth step, the teachers should give contingent support to the group (increase support when the students show poor understanding and decrease support when students show high understanding) until one or more

students understands the question. In step 5, the teacher should check if at least one of the students understood the content support. If this is the case, teachers should stop giving content support (phasing out, the second key feature of scaffolding) and return responsibility for learning back to the group (third key feature of scaffolding) (Van de Pol et al., 2010). Returning responsibility for learning to the group can be done by encouraging students to explain and show their work to each other and to criticize each other's work, i.e. give process support, in the last step. In case the outcome of the determination of the question in the first step was that the question was not a question from the whole group, but rather a question of one or more individuals in the group, teachers should return responsibility for learning back to the group by giving process support.

In this study, we combined Fruedenthal's, Janvier's, and sociocultural perspectives as theoretical lenses to study mathematical lessons.

### **Research question**

Does the use of the Small-Group Scaffolding model raise seventh grade students' mathematical level in Early Algebra?

### **Hypothesis**

We hypothesize that the mathematical level of students will be raised more in the Scaffolding condition than in the control condition.

### **Method**

In a quasi-experimental design, we compared the mathematical level raising of students who were taught by teachers who were trained to give help to students according to the SGS model with those of students who were taught by teachers who gave help without prior instruction. In both conditions students worked on an Early Algebra unit that was especially designed to invoke mathematical discussions and raise the level of mathematical knowledge (Calor, Dekker, Van Drie, Zijlstra, & Volman, 2018).

Participants were eight teachers (7 males, 1 female) and their 266 seventh grade students of two schools in Amsterdam, the Netherlands (5 teachers and 169 students of two schools in the Scaffolding condition and 3 teachers and 97 students of one school in the control condition). Two of the eight teachers (one in the Scaffolding condition and one in the control condition) worked with an Early Algebra unit in a collaborative setting in a previous study (Calor et al., 2018). Other than that, the teachers had no experience with collaborative learning. Teachers were matched into pairs according to age and teaching experience. Of every pair one teacher was randomly assigned to a condition. Every teacher taught their own seventh grade class. Teachers in the Scaffolding-condition were trained on the job so that they could be engaged in inquiry into the SGS model and reflection on its use and to provide an opportunity for them to make connections between their learning and classroom practice (Borko, Jacobs, & Koellner, 2010). Prior to the lesson series, teachers in the Scaffolding condition were trained in a two-hour training to give help to students according to the SGS model. In addition, before and after every lesson the use of the model was discussed individually with the teachers and questions were answered. The aim of the research was only known to the teachers in the Scaffolding condition. The teachers in the control condition were

instructed to let the students work together on the assignments and give their usual support. All students worked in small heterogeneous groups (46 Scaffolding and 27 control condition). Small student-groups consisted of students of mixed ability according to track advice related to the students by their primary school administration. Ideally groups consisted of four students, but due to class size restrictions 24 groups of three students (13 Scaffolding and 11 control condition) and one group of two students were formed (Scaffolding condition). Students' age varied between 12 and 15 years. Groups in the Scaffolding condition were instructed by the teacher to explain and show/tell their work to each other, ask each other for explanations, and to criticize each other's work. Teachers in both conditions implemented the lessons as intended (teachers in the Scaffolding condition used the SGS model to support students, whereas teachers in the control condition gave support as they normally would). Implementation check was performed by the first author during and after lessons (check with teacher and watch videotape of lessons).

In both conditions, students worked on a lesson series of 12 lessons of 60 minutes on the topic of Early Algebra for five weeks. We replaced five of these lessons, with lessons that explicitly aim to invoke mathematical discussions and raise the level of mathematical knowledge (Calor et al., 2018). During these five lessons, students worked collaboratively on the same assignment. Every lesson started with 10 minutes introduction by the teacher, followed by students working collaboratively for 50 minutes. During the remaining lessons students sat together in the same small heterogeneous groups and worked on regular assignments from the textbook.

Students' mathematical levels in Early Algebra were measured by means of a test. A pre-test was administered in the lesson prior to the intervention, and the same test was administered as a post-test in the lesson after the intervention. The test aims to measure the mathematical levels of students based on the translation skills from Janvier (1987). The highest level implies being able to translate from representation formulae to representations situation, tables and graphs and vice versa. The lowest level implies not being able to translate from and to the representation formulae at all.

The test consists of six questions and 17 sub-questions, of which 10 were used to measure mathematical levels. They consist of questions involving all translation skills of the representation formulae (Janvier, 1987). The other seven questions were discarded (not included in the scores) during determination of level raising. They consist of basic primary school arithmetic questions so that students would be able to answer at least some of the questions when the test was administered as a pre-test. Scores of 0, 1 and 2 were assigned; 0 meaning low mathematical level, 1 meaning medium mathematical level, and 2 meaning high mathematical level. The sub-questions of the test corresponded to the switches between the Janvier representations involving formulae (Janvier, 1987). The first sub-question only involved a less abstract representation of a formula; therefore, the score for this question was limited to 1 (medium mathematical level). The sum of the scores amounts to a maximal number of 19 points in total that students could score for the test. The levels that were assigned to the sub-questions were discussed with a second coder (the second author of this article). Students worked individually on the test for 60 minutes. An example of a basic primary school arithmetic question is: "Out of one package pancake mix you can bake six pancakes. How many pancakes can you bake with 12 packages of pancake mix?" An example of a question that measures mathematical level is: "Shane wants to get in shape, therefore he goes swimming. With

the following (word) formula you can calculate Shane's costs. Number of times swimming  $\times 3 + 30 =$  costs. (Here, 3 stands for €3, the cost for swimming once at a swimming pool for a member and 30 stands for €30, the cost of a yearly membership to a swimming pool.) Draw a graph for this formula." A score of 2 (high mathematical level) was assigned when the graph was drawn correctly (linear relationship in the graph was depicted correctly); a score of 1 (medium mathematical level) was assigned when small errors occurred in the graph (e.g. graph starts at (0,0) rather than (0,30)); a score of 0 (low mathematical level) was assigned when the graph made no sense (e.g. when only names of axes were written down). Another example of a question that measures mathematical level raising is: "Eric saves money to buy a smartphone. Every month he saves €27,-. He has currently no money in his savings account. Give the formula with which the amount of money on Eric's savings account can be calculated". A score of 2 (high mathematical level) was assigned when the formula was correct; a score of 1 (medium mathematical level) was assigned when the formula contained small errors or when the linear relationship was explained in words; a score of 0 (low mathematical level) was assigned when the formula made no sense (e.g. when only 27 was written down).

Interrater reliability for the pre- and post-test between two coders, determined in a prior study, was excellent (ICC 0.95) (Calor et al., 2018).

A multilevel model for repeated observations on fixed occasions with an unrestricted covariance matrix (Snijders & Bosker, 2012) was used to test for differential growth between the Scaffolding condition and the control condition. The fixed occasions are the measurements (pre- and post-test) nested in individual students. The measurements (pre- and post-test) are the first level, the individual students are the second level, the small groups are the third level, and the classes are the fourth level. This multilevel model (Snijder & Bosker, 2012) enables us to test the growth in the Scaffolding condition against the growth in the control condition.

## Results

We present the means of the pre- and post-test on mathematical level for the Scaffolding condition and control condition in Table 1.

	Scaffolding condition	Control condition
	M (SD)	M (SD)
Pre-test	2.04 (3.32)	1.43 (2.66)
Post-test	11.93 (5.88)	9.22 (6.44)

**Table 1: Means and Standard Deviations Pre- and Post-test on Mathematical Level, Scaffolding condition (N=169) and Control condition (N=97), Maximum Score of 19**

Table 1 shows that the students' mathematical level improved a fair amount in both conditions. For the Scaffolding condition (SC), the mean score increased from 2.04 to 11.93, and for the Control condition (CC), it increased from 1.43 to 9.22.

A multilevel model for repeated observations on fixed occasions with an unrestricted covariance matrix (Snijders & Bosker, 2012) was used to test for differential growth between the SC and the CC.

We report on the model for three levels (with measurements (pre-test post-test) nested in individual students and classes) since there was no significant improvement in model fit for four levels (measurements, individuals, groups, classes).

The expected outcome for the CC was pre-test = 1.40 and post-test =  $1.40 + 9.20$ . For the SC the expected outcome was pre-test =  $1.40 + 0.692$  and post-test =  $1.40 + 0.692 + 9.20 + 2.116$ . Thus, compared to CC the growth in the SPC is 2.116 larger. This differential growth is significant ( $p = 0.003$ ).

## **Discussion/conclusion**

In this study, we investigated whether the mathematical level of students who were taught by teachers who were trained to give help according to the Small-Group Scaffolding model was raised more than the mathematical level of students in a control condition.

With respect to our hypothesis, multilevel analyses showed that the mathematical level of students in the Scaffolding condition was raised significantly more than the mathematical level of students in the control condition.

This study was conducted at two schools, which limits the generalizability of our findings. In addition, the conditions were observed at different schools. Consequently, results might be affected by qualitative differences between the schools. For example, the average pre-test score for the Scaffolding condition was higher than that of the control condition. This could be indicative of a higher aptitude, which in turn might be favorable for the growth in the Scaffolding condition. However, this is not very likely since there are no large descriptive differences between secondary schools in the Dutch education system (e.g. track advice given in primary schools determines the track students follow in seventh grade).

In a follow-up study, we will report elaborately on the scaffolding behavior of the teachers in both conditions. We will also focus on the quality of mathematical discussions in the groups, since reflection during mathematical discussions may have a positive effect on mathematical level raising (Freudenthal, 1978).

We conclude that the Small-Group Scaffolding model is a useful extension of the Contingent Teaching Model (Van de Pol et al., 2012). In addition, we conclude that using the SGS model for Early Algebra can raise the mathematical level of students (ability to switch back and forth between representation formulae and other algebra representations (Janvier, 1987)). The SGS model can therefore be useful to help teachers support small heterogeneous student-groups.

Future research should focus on the investigation of what kind of help the teachers in the conditions gave to the students and whether or not the quality of the mathematical discussions mediates mathematical level raising. In addition, this study should be replicated on a larger scale to find more robust findings.



## References

- Calor, S. M., Dekker, R., Van Drie, J. P., Zijlstra, B. J. H., & Volman, M. L. L. (2018). *Let's discuss math: Effects of shift-problem lessons on mathematical discussions and level raising in early algebra*. Manuscript submitted for publication.
- Calor, S.M., Dekker, R., Van Drie, J.P., & Volman, M.L.L. (2019). *Scaffolding small groups at group-level: Improving the scaffolding behavior of mathematics teachers during mathematical discussions*. Manuscript in preparation.
- Dekker, R., & Elshout-Mohr, M. (1998). A process model for interaction and mathematical level raising. *Educational Studies in Mathematics*, 35(3), 303–314.
- Dekker, R., & Elshout-Mohr, M. (2004). Teacher interventions aimed at mathematical level raising during collaborative learning. *Educational Studies in Mathematics*, 56(1), 39–65.
- Freudenthal, H. (1971). Geometry between the devil and the deep sea. *Educational Studies in Mathematics*, 3(3), 413–435.
- Freudenthal, H. (1978). *Weeding and Sowing: Preface to a Science of Mathematics Education*. Dordrecht, the Netherlands: Reidel.
- Freudenthal, H. (1991). *Revisiting Mathematics Education – China Lectures*. Dordrecht, the Netherlands: Kluwer Academic Publisher.
- Janvier, C. (1987). Translation process in mathematics education. In C. Janvier (Ed.), *Problems of representation in mathematics learning and problem solving* (pp. 27–31). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.). *Handbook of research on mathematics teaching and learning* (pp. 390–419). New York, NY: Macmillan.
- Sfard, A., & Linchevski, L. (1994). The gains and pitfalls of reification – the case of algebra. *Educational Studies in Mathematics*, 26, 191–228.
- Snijders, T. A. B., & Bosker, R. J. (2012). *Multilevel Analysis: An Introduction to Basic and Advanced Multilevel Modeling* (2<sup>nd</sup> ed.). London, England: Sage Publishers.
- Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher-student interaction: a decade of research. *Educational Psychology Review*, 22(3), 271–297.
- Van de Pol, J., Volman, M., & Beishuizen, J. (2012). Promoting teacher scaffolding in small-group work: A contingency perspective. *Teaching and Teacher Education*, 28(2), 193–205.
- Van Hiele, P. M. (1986). *Structure and insight*. Orlando, FL: Academic Press.
- Van Stiphout, I., Drijvers, P., & Gravemeijer, K. (2011). The development of students' algebraic proficiency. *International Electronic Journal of Mathematics Education*, 8(2), 62–80.
- Webb, N. M. (1982). Student interaction and learning in small groups. *Review of Educational Research*, 52(3), 421–445.

Webb, N. (2009). The teacher's role in promoting collaborative dialogue in the classroom. *British Journal of Educational Psychology*, 79, 1–28.