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Teacher responses to public apparent student error: A critical confluence of mathematics and equitable teaching practice

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This paper uses analysis of shared video data to discuss teacher responses to public student apparent error. Drawing upon conceptions of student thinking and theories of agency, authority, and mathematical identities, we argue that certain ways of responding to public student apparent error have the potential to elevate student thinking and empower students with mathematical agency, authority, and standing. We also suggest that such teaching practices may be a high-leverage site to develop equitable mathematics teaching.

Keywords: Mathematical knowledge for teaching, teaching practices, positioning, agency, authority.

Introduction

Maria's 7th grade class is beginning to learn arithmetic with integers. She is called to the board to show her solution to the subtraction problem, $38 - 52$. Maria writes and says the following:

3 8 "8 take away 2 is 6"
-5 2 "3 take away 5 is -2"
-2 6

Is Maria right? What is there to understand here about place value, with negative numbers? How might the teacher respond to Maria? And what effect will this have on her?

These questions, about this and about countless other similar scenarios, are the object of our study. We are concerned here with the interactions of the mathematics, the student's thinking and standing, and the impact of the teacher response on all three. We are not proposing that there is a "right" teacher response, but we do argue that the response is consequential for the individual student's learning and standing, as well as for that of the collective. Moreover, much knowledge and insight goes into understanding the range of possible responses and their consequences. In particular, what mathematical knowledge for teaching, what knowledge of student thinking, and what cultural and psychological sensitivities might be involved? So, our research question is:

What kinds of teacher responses to public student apparent error can help cultivate students' mathematical agency, authority, and standing?

Student mathematical productions are rarely flawless. Overt error may distract teacher attention from otherwise robust reasoning. A student's lack of technical vocabulary may cloud insightful conceptual expression. Such apparent flaws happen with all students, but they are understandably

common among students who are educationally disadvantaged, because of race, ethnicity, poverty, language, etc. For this reason, we feel that our research question matters for equitable instruction.

The title refers to a core task of teaching. Three terms there – “public,” “apparent error,” and “equitable” – deserve comment. “Public” indicates that this is about collective learning and norm setting, more than bilateral teacher-student transactions. The student work and teacher response communicate to the whole class, not just to Maria. “Apparent error” draws attention to the teacher’s perception without evaluating the student’s thinking. Rougée (2017) has studied teacher responses to “apparent student error,” but her study explored the teacher stress and anxiety this can precipitate. As indicated above, we believe that our focus is a high leverage site for “equitable teaching practices.” The latter are often described in terms of inclusive classroom culture and participation structures. Our interest focuses further on how equitable practices intersect with content.

Our method is to first examine records of practice (video and other artifacts), to generate hypotheses about teacher responses to public student apparent error that can either cultivate or degrade students’ mathematical agency, authority, and standing. Robust conclusions of this kind will ultimately require more longitudinal data.

We will here explore the above task of teaching in two cases, one involving 4th graders identifying fractions on the number line, the other showing another group of 4th graders figuring out a difficult subtraction problem before having learned a general subtraction algorithm. These examples will help to open the analytical space behind our question.

Conceptual frameworks

Teacher responses to public student apparent error, the focal phenomenon of our study, is a site of confluence of three research traditions – 1) mathematical knowledge for teaching, 2) student mathematical thinking, and 3) agency, authority, and identity – each with its own theoretical frame and perspective. It is our task to coordinate, if not integrate, these traditions for our work. To understand the “work of teaching” we use the “Instructional Triangle” (IΔ) of Cohen, Raudenbush, and Ball (2003), as later elaborated by Ball (2018).

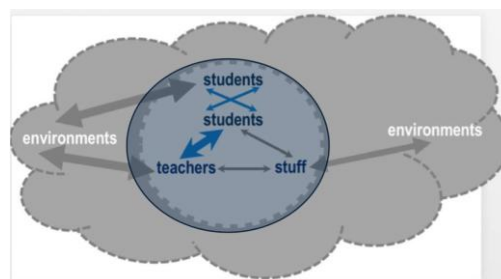


Figure 1. Revised instructional triangle (Ball, 2018)

The “stuff” in our case is “mathematics.” The inner circle represents instruction, where mathematical knowledge for teaching primarily resides. Student mathematical thinking spills into the environment. Agency, authority, and standing constitute an arena in which the instruction may attempt to disrupt inequitable environmental influences. Much mathematics education research is situated at a vertex, or along an edge of the IΔ. Our question spreads across the entire diagram.

Mathematical knowledge for teaching

For decades, a core problem in research on mathematics teaching has been what knowledge is needed to teach well. Researchers have approached this problem in different ways. Some have produced lists of what teachers should know. Others have investigated what knowledge teachers have, and yet others have studied knowledge teachers use. Ball (2017) suggested flipping the question by focusing instead on what mathematics the work of teaching entails (see also Ball et al., 2008). This shift was not new. Already two decades ago, it was elaborated in several publications (e.g., Ball, 1999; Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001). The present paper follows this line of research and targets the challenges that are embedded in a particular mathematical task of teaching. Eliciting, interpreting, and responding productively to student thinking, all entail deep and flexible mathematical knowledge.

(Interpreting and responding to) student thinking

As we indicated above, mathematical thinking, by experts as well as students, is rarely flawless. It is typically a mix of ideas, some intuitive or sketchy, some more formalized, some of it sound, some of it not yet fully formed, some of it incorrect. Simply calling this complex package “error” is to ignore the substantial mathematical value there to be discerned. The phrase “apparent error” is our way of keeping open this broader range of possibilities.

When a child publicly presents a problem solution, or a mathematical explanation or comment, what tasks of teaching does this present? One is to *understand* the child’s thinking. This may draw on several knowledge resources: Of the mathematical terrain and the diverse ways it can be represented; of typical student (mis)conceptions and representations of the ideas; and of the child’s identity and background, and the culturally diverse forms of children’s communication (language, gesture, etc.). Once the child’s thinking is understood, what can the teacher do to help make this understanding shared by the other children? And if the teacher does not at first understand, what can she do to probe the child’s thinking to gain such understanding? The above are all challenging tasks of teaching.

Once the teacher possesses an adequate understanding of the child’s thinking, what is the next move? In some common practice, the teacher will first notice (perhaps even before fully understanding the child’s thinking) the mathematical flaws, and either announce them, or simply pass on, without comment, to another student in search of a more “acceptable” response. This can be characterized as a *deficit teacher response*. The opposite kind of response, sometimes called assigning competence (e.g., Cohen & Lotan, 1995), or *asset oriented response*, is to offer some detailed public appreciation of the positive mathematical qualities of the child’s thinking, without prematurely announcing that the mathematical work is done. Sometimes the teacher might even invite other students to comment on “what was good” about the child’s contribution. Again, enactment of this kind of move calls upon a fine-tuned understanding of the mathematics as well as of the student thinking.

Agency, authority, identity

Teaching is relational work, challenging in its complexity. We highlight three aspects of this complexity: agency, authority and identity.

Agency is the capacity, freedom and autonomy of an actor to act in a given environment. The environment of concern here is the mathematics classroom. Agency may either be encouraged and supported, or constrained and suppressed, for example through the influence of such factors as race, social class, religion, gender, ethnicity, perceived ability, customs, etc.

Authority is characterized by Benne (1970, p. 393) as a social relationship between the bearer (of the authority, a person or group), the subject(s) (of the authority), and the field (context). The bearer receives willing obedience from the subjects while helping provide for their need of advice, leadership, guidance, or direction in the field. This is an example of “reasoned power,” but distinct from more general forms of power, which may be based on domination and coercion. Benne distinguishes three kinds of authority: Expert (as in the doctor-patient relation); Rule-based (as in say the playing of baseball); and “Anthropogogical” authority (partly exemplified by the doctor-medical student relationship, an apprenticeship into practice). In rule-based authority, the rules that govern behavior represent an implicit consensual authority of the community of players, who are thus individually subjects, and collectively bearers.

Authority in a mathematics classroom has at least two aspects. One, common to all classrooms, is about behavioral norms, based on socio-cultural norms. Another, more pertinent here, is about *disciplinary authority*, about how to certify the authenticity of a piece of mathematical knowledge. Education reforms over the past half-century have moved in the direction of giving students more disciplinary agency and authority. How can that be interpreted in the above framework? For Benne, the ‘bearer’ of authority is a human agent, a person or group of persons; so how can we speak of “the authority of the discipline?” Amit and Fried (2005) suggest that mathematics, with its “fixed set of rules,” supports a kind of rule-based authority. We think instead of the discipline as a growing (inanimate) body of knowledge and methods, but that it is the cumulative product of (centuries of) human effort. How do new concepts or methods gain entry into (the current form of) this corpus? There are disciplinary norms for such membership, but human judgment (peer review) is needed to decide if these norms have been adequately met. Thus, in our view, disciplinary authority is mediated through those (human) agents empowered to certify proposed disciplinary claims.

So conceived, it now makes sense to speak of the distribution of disciplinary agency and authority in a mathematics classroom. In the traditional transmission model of instruction, the teacher alone bears expert authority to which the student subjects are expected to conform. In contrast, distributing agency among students means that they can participate in the construction of new knowledge and methods, finding new solution strategies, noticing patterns, speculating, conjecturing, proposing evidence. Distributing authority means that students themselves will judge the adequacy of claims of their peers, and find ways of resolving disagreement. This is somewhat captured in the third mathematical practice standard of the Common Core (2010): Construct viable arguments (agency) and critique the reasoning of others (authority).

This broad distribution of mathematical agency and authority among students, as co-producers (with the teacher) of knowledge, resembles what Benne (1970) calls anthropological authority.

(Mathematical) identity refers to how students consider themselves as mathematical doers, thinkers, and learners. Aguirre, Mayfield-Ingram, and Martin (2013) define mathematical identity as “the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives” (p. 14). Positive mathematical identity involves willingness to take risks to engage in discourse and to see one’s self as capable and worthy of being heard (Berry, 2018). This is related to *productive disposition*, the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy (NRC, 2001). A premise of our work is that positive mathematical identity can be cultivated and supported by equitable teaching.

Case 1: Fractions on the number line

This diverse class of 9 to 10-year old children has learned how to name what fraction of (the area of) a rectangle is shaded. The method is to first assure that the (*whole*) rectangle is divided into a number (*d*) of *equal* size parts, to then count the number (*n*) of those equal parts that are shaded, and finally to name the fraction as n/d . The lesson segment in the video shows the start of their learning how fractions correspond to points on the number line. This is based on linear measure, in contrast with the area measure of shaded rectangles; on the number line one is counting lengths of intervals, and the unit of measure (the *whole*) is taken to be the interval from 0 to 1. We see the students presented with the following task (Figure 2).

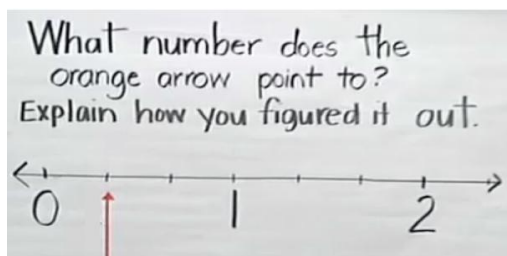


Figure 2. Naming fractions on the number line.

The correct answer is $1/3$, but the student notebooks show a variety of answers. The teacher asks who would like to come up and show their work; she calls on Aniyah. The teacher reminds the class that they should “listen closely and see what you think about her reasoning and her answer.” Aniyah puts “ $1/7$ ” next to the orange arrow (an “evident error”), at which point Toni asks, in surprise, “Did she say one-seventh?!” Aniyah responds, “Yeah, because there are seven equal parts, . . .” as she counts, with her spread thumb and index finger, the seven *intervals* (not hash marks) visible on the displayed (portion of the) number line. Here we see the “public student apparent error” to which the teacher responds, not to Aniyah, but to the class: “Before you agree or disagree, I want you to ask questions if there’s something you don’t understand about what she did.” Toni asks, “Why did you pick one-seventh?!” The teacher, affirming “That’s a very good question,” also says, “Let’s listen to her answer.” Aniyah says, “. . . because there’s seven equal parts.” Lakeya asks, “If you start at the zero, how did you get one-seventh?” She may have been thinking about counting hashmarks instead

of intervals. Dante then struggles to formulate a more speculative, but difficult to understand question, perhaps to the effect that, “If you moved the orange arrow to where the one is, would you still know it was one-seventh to put it where the orange line is now?”

Several things are noteworthy about the teacher response to Aniyah’s “apparent error.” Though Aniyah’s “one-seventh” is wrong, it is so far not challenged or corrected either by the teacher or the students, though, at the teacher’s invitation, the children pose questions to understand *Aniyah’s thinking*, which is made central to the instruction. The mathematical issue is resolved by the end of the lesson (beyond the video segment), but we here see the teacher publicly sheltering *Aniyah’s standing*, as author of well articulated mathematical reasoning, which is flawed only in not using the standard unit (interval) as the whole. Further the teacher confers on Aniyah’s peers the authority to question, and eventually evaluate, Aniyah’s solution, free of teacher judgment. In these ways, the students are given/assigned remarkable collective agency and authority to develop the mathematics.

Case 2: Subtracting before you know how

In this case, the teacher presents a group of five 4th graders with the following problem: “The king celebrates his 80th birthday today” (this was in 2017). “In what year was he born?” This amounts to calculating $2017 - 80$. The teacher has not yet taught these students how to subtract large numbers by using a standard algorithm, though some may know this already. The students show four different approaches. One starts by taking away 17 from 80 to get 63, and then subtracts 63 from 2000 by using a standard algorithm. Another student uses the standard algorithm directly to subtract 80 from 2017. In both these cases, the teacher explains their thinking to the other students, and provides a lot of praise. Two other students come up with more unexpected approaches to solving the problem. Below, we take a closer look at one of these student’s method.

When Brian (pseudonym) is called to the board to present his solution, the teacher comments that he noticed how Brian finished the problem quickly. He continues, “So I’m very excited to hear what you were thinking!” Brian explains that he just got rid of the 17, so that you only have 2000. He writes down $2000 - 80$. Brian then states, “So, then we have 1920, which is pretty easy!” When Brian has written down this partial answer, the teacher interrupts, “So, now I would just like to repeat what you did. I just want to repeat what you did, so that I understand your thinking.” The teacher asks Brian to move a little bit to the side, before he continues, “I understood from what you were saying, Brian, what you did that I understood, was that, to begin with, you didn’t want to deal with the 17. You just jumped back to (points to 2000), and thought: What if we are in 2000 now? And then, when we go 80 years back, we get to 1920. And, now I’m eager to know, what did you do next?” Brian jumps up to continue his explanation, “And then I just added 17, which is pretty easy, simply 20 plus 17.” The teacher proclaims that this is “simply fantastic,” before Brian moves on to write down the answer on the board. When Brian has finished explaining, the teacher asks, “Did anyone understand this clever way?” Anna responds that she understood it, and that she thought it looked like a good way of thinking about the problem. The teacher then continues to rephrase what Brian did when he decided to “jump back 17” and start on 2000, and how “he already then knew that he had jumped back 17 years too long (...) So, when he got to 1920, he had to jump 17 years forward again, to correct it.”

One of the students early displayed an incorrect answer and seemed otherwise distracted at times. He was the only student not called to present his work. His was the only appearance of apparent error. For each of the others, at issue was the uncertainty of their solution strategies, for which they were each praised in varying degrees.

Concluding discussion

The choices made by the teachers in these two cases differ in several respects. We will here highlight and discuss two particular differences in the teachers' responses. The first relates to how the teachers assign competence to students. In case 1, we notice only one instance of explicit teacher affirmation or praise, whereas the teacher in the second case offers a lot of praise to the students. We argue that both teacher responses can be described as asset oriented, and we posit that assigning competence does not have to include direct praise or public teacher statements. The teacher in case 1 assigned competence to Aniyah by inviting her to be the teacher. She positioned Aniyah as a recognized contributor, with standing, in the mathematical discussion, and she conferred on Aniyah's peers the authority to analyze and critique Aniyah's work. In the case 2, the teacher assigned competence to Brian by re-voicing his thinking and praising his contribution. This leads to the second highlighted difference, relating to the distribution of mathematical agency and authority, which we next discuss.

The work of mathematics teaching demands a mathematical knowledge that exceeds ability to solve a problem by using the standard algorithm. The teacher in case 2 chose not to confine attention to the two solutions using variants of the standard algorithm, thus disrupting the rule-based authority that is common in many mathematics classrooms. Instead the teacher highlights an unexpected student solution strategy, which he has to interpret and evaluate on the fly and assure that all of the students understand. By bringing Brian to the board, and expressing excitement and eagerness to understand Brian's thinking, the teacher publicly affirms Brian's agency and standing. It is interesting to notice how the teacher, in the middle of Brian's explanation, decides to interrupt to repeat what Brian did and make sure he understands Brian's thinking. It is likely that the teacher in fact understood well what Brian had done, but, by saying "so that I understand your thinking," the teacher was nominally putting himself on the same level as the other students, thus modeling the kind of inquiry needed to understand another student's thinking. In case 1 above, the teacher assigns this work to the students, whereas the teacher in case 2 does not do so. He himself is modeling, by example, how such inquiry is done. This effort is further reinforced when, after Brian has finished his explanation, the teacher again decides to re-voice Brian's explanation, to emphasize the crux of his thinking. Throughout this episode, the teacher demonstrates, and so models, attentive listening, and a pressing need to understand Brian's mathematical thinking. Brian is positioned as a mathematical authority, but one whose thinking needs to be publicly analyzed and sanctioned by the teacher, so that Brian's knowledge is not only validated and praised, but becomes collective, thus hoping to support all of the students' understanding. In case 1, Aniyah also gains mathematical authority, but only after public resolution of the integrity of her thinking by her peers, who are assigned this responsibility.

We have presented two examples of teacher responses to “public student apparent errors.” We described, in each case, teacher moves that might have potential to cultivate the mathematical agency and authority of the students. In both cases, student agency is well supported, but in case 2, more of the mathematical authority appears to still reside with the teacher

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