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### ► **To cite this version:**

Rukiye Ayan, Mine Isiksal-Bostan, Michelle Stephan. A math teacher's participation in a classroom design research: teaching of ratio and proportion. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02430031

**HAL Id: hal-02430031**

**<https://hal.science/hal-02430031>**

Submitted on 7 Jan 2020

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# A math teacher's participation in a classroom design research: teaching of ratio and proportion

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*The purpose of this study is to explore how a mathematics teacher contributed to a classroom design research and how this contribution promoted student learning and the establishment of students' mathematical practices of linking composite units and iterating linked composites, which is the basis for proportional reasoning. In this classroom design research study, a classroom learning trajectory and related instructional sequence were formulated based on the theory of Realistic Mathematics Education and implemented by the teacher. Findings showed that the teacher contributed to the design research in each phase, and her contributions promoted students' mathematical practices regarding linking composite units and iterating composite units.*

*Keywords: Realistic mathematics education, proportional reasoning, classroom design research.*

## Introduction

Proportional reasoning lies at the heart of many mathematical structures, especially those included in the primary and middle school mathematics curricula (Lesh, Post, & Behr, 1988). However, several studies reported students' difficulties and misconceptions in proportional reasoning. These difficulties are said to have stemmed from the fact that proportional reasoning instruction is superficial and limited since it is conventionally referred to as solving missing-value problems with procedural algorithms, such as cross multiplication (Lesh et al., 1988). Thus, it is clear that there is a need for improved instruction for proportional reasoning.

Simon (1995) was a pioneer in proposing the development and use of Hypothetical Learning Trajectories (HLT) for improving instruction in such a way that lessons are designed in line with related research findings on student thinking and learning. Stephan (2015) introduced the construct of a classroom learning trajectory, which is referred to as anticipated classroom mathematical practices that might evolve over the course of an instructional sequence. Using Stephan's approach, we conducted a classroom design research project, in which we formulated a classroom learning trajectory and related instructional sequence for ratio and proportion for seventh grade based on the theory of Realistic Mathematics Education (RME) since working in realistic contexts can be an instructional aid for meaningful learning and reinventing mathematical ideas (Gravemeijer, 1994). While the larger study concentrates on these issues, we particularly focus on the teacher's role in classroom design research for the big ideas of linking composite units and iterating linked composites in this paper. The research questions addressed are:

1. How does a seventh-grade math teacher participate in the preparation/design phase of a classroom design research study? How does this contribution promote the emerging and

establishment of mathematical practices regarding linking composite units and iterating linked composites?

2. How does a seventh-grade math teacher support the emerging and establishment of mathematical practices regarding linking composite units and iterating linked composites through the implementation of the instructional sequence?

## Methods

Design research studies have gained importance in the last two decades and have been conducted for various goals in educational research area from designing and examining innovations, such as activities, institutions, interventions, or curricula (Design-Based Research Collective, 2003). This study is a classroom design research in which an instructional sequence is formulated, tested, and revised by a research team including teachers and university members (Stephan, 2015). A classroom design research cycle entails three phases as design, implementation, and analysis (Stephan, 2015).

This classroom design research study was conducted in a seventh-grade classroom in a public school located in a rural area in the capital city of Turkey. Hence, most of the students had low socioeconomic statuses. The students had been instructed on calculating the ratio of two quantities and finding different values of the quantities that are directly proportional in sixth grade. They had not been instructed on ratio and proportion in seventh grade prior to the study. The teacher was female with 10 years of experience in teaching in seventh grade. She was selected since she adopted student-centered teaching practices and was willing to cooperate with the researchers in every part of the study. Even though she had not been trained specifically before the study, she was informed about the purposes of the study and the literature on proportional reasoning, RME, and HLT.

## Instructional Sequence

The approach that undergirds the design of the proportional reasoning instructional sequence is RME. The roots of RME are based on the idea of mathematics as a human activity. Freudenthal (1968) stated that people need to see mathematics not “as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics” (p. 7). Within the context of RME studies, students are guided to reinvent mathematical ideas through organizing realistic contexts that are didactically rich (Gravemeijer, 1994). In addition, students are encouraged to create and reason with models and mental imagery associated with the physical tools, inscriptions, and tasks they employ.

In the instructional sequence, which was developed for the students in the USA, the instruction begins with a story about a bad dream in which aliens were chasing the teacher and a bar of food was enough to satisfy three aliens. In this way, students were encouraged to link together one food bar with three aliens and knew that if this relationship was broken, there would be an alien attack. As students solve problems, they are encouraged to organize their pictures or numbers, and eventually a ratio table is introduced as an efficient way to keep track of how the two quantities may scale up or down. For the purposes of this paper, we will not describe the remainder of the instructional sequence because we focus our analysis only on the first part, *linking composite units*

and iterating linked composites. For more information of the sequence and the materials themselves, see <https://cstem.uncc.edu/sites/cstem.uncc.edu/files/media/Ratio%20T%20Manual.pdf>

A small part of the HLT that is related to this study can be seen in Table 1 below. The Big Idea column describes the learning goal for that portion of the instruction, and the Tools/Imagery column outlines the specific inscriptions and/or notations that are intended to support the corresponding learning goal. The third column, Possible Topics of Discourse, is meant to guide teachers in the types of questions or mathematical conversations that are important for that section of the sequence. The HLT for the entire proportional reasoning sequence can be reached via the same link.

Big idea	Tools/Imagery	Possible Topics of Discourse
Linking composite units	Connecting pictures of aliens to food bars	If the rule is 1 food bar feeds 3 aliens, the rule can't be broken if we add more food bars
Iterating linked composites	Informal symbolizing (e.g., tables, two columns of numbers, pictures of aliens and food bars)	How students keep track of two quantities while making them bigger

**Table 1: A small part of the HLT related to linking and iterating**

### **Data collection**

Cobb, Confrey, Lehrer, and Schauble (2003) stress that the units of analysis in a design research are the key elements of a learning environment and these are:

the tasks or problems that students are asked to solve, the kinds of discourses that are encouraged, the norms of participation that are established, the tools and related means provided, and the practical means by which classroom teachers can orchestrate relations among these elements. (p. 9)

Therefore, in this study, it is crucial to analyze the teacher's contribution to each phase of the study and how her contribution had an impact on the emerging and establishment of students' mathematical practices of linking and iterating. The related data were collected through the videotapes of class sessions, audiotapes of teacher interviews and research team meetings, and analysis of student work.

### **Data analysis**

The teacher's contribution to the design research was documented by an analysis of the written transcripts of her interviews and of the design team meetings. These qualitative data were analyzed by an interpretative framework and a related analytic approach that focuses on the "meanings made both by the social actors and by the researcher" (Miles & Huberman, 1994, p. 8). Students' mathematical practices were analyzed using Stephan and Rasmussen's (2002) adaptation of the Toulmin's (1969) argumentation model. After classroom argumentation process was coded as claim, data, or warrant, the ideas that emerged were noted. The criteria by Stephan and Rasmussen were used in order to see what mathematical ideas became taken-as-shared (or established) by the

classroom community. For the last part of the analysis, these taken-as-shared ideas were organized around common mathematical activities, which were later named as classroom mathematical practices. Seven mathematical practices were obtained in the larger study; however, we focus only on the first mathematical practice in this study.

## **Findings**

Findings are presented by describing the teacher's contribution to each phase of the study with specific examples. In addition, the first classroom mathematical practice regarding linking composite units and iterating linked composites is explained with details in order to reveal how the teacher's contribution to the design phase was realized in the classroom and had an impact on the establishment of students' mathematical practice of linking composite units and iterating composite units.

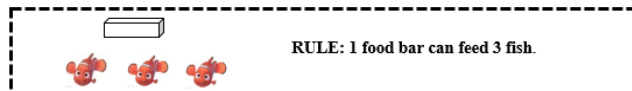
### **Teacher's contribution in the preparation/design phase**

The initial phase of the classroom design research is mostly about reviewing literature, finding a realistic and engaging context, and hypothesizing a classroom learning trajectory. Literature review was conducted by the design team on teaching and learning of ratio, proportion, and related concepts and instructional activities and tools. In addition, the teacher was interviewed regarding her experiences on the learning and teaching of related topics and curriculum. Even though an already developed and tested trajectory was available, a number of adaptations and modifications were needed for Turkish students. For instance, the context was changed from alien-food bars to fish-food bars in order to make it more experientially real for Turkish students. Moreover, pictures were added to some of the questions, and quantities in some of the problems were changed in order to improve the sequence and the HLT. While deciding on these changes, the design team was engaged in anticipatory thought experiments in which they tried to imagine possible classroom mathematical discourse, which were helpful in shaping the design and coming up with conjectures about the teaching and learning of the topic. Even though the teacher had an active role in all of this process, we represent below a number of substantial teacher suggestions in order to reveal the unique value of her contribution to this phase. To begin with, the teacher suggested that pictures of food bars and/or fish were included for some problems in order to visually support students' processes of linking units and iterating composite units. In addition, she suggested that some values in the questions should be altered in a way that it was possible to make connections between questions. For instance, the question "how many fish can be fed with 5 food bars?" was followed by "how many fish can be fed with 9 food bars?" and "how many fish can be fed with 10 food bars?" in order to support the use of build-up strategies.

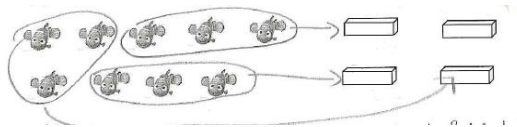
### **Teacher's contribution to the implementation phase**

The teacher launched the trajectory with the adapted version of the bad dream story in which her fish were making noise and attacking her since they were hungry. The teacher, then, asked students if they had pets and how they fed their pets. Students suggested that pets had to be fed with a certain amount of food each day: if they were underfed they would be hungry and if they were overfed they would get sick or even die from overfeeding. This was an on-action (unplanned) instructional move that laid the ground for students to make sense of why the rule could not be broken.

In the first problem in the sequence, seven fish and four food bars were given, and the rule was one food bar for three fish. The question asked whether or not there were enough food bars for those fish. The sequence started with simple whole number ratio situation with pictures of both food bars and fish in order to help students link the composites concretely. Even though those questions were easy for the students, starting with those questions made it possible for the teacher to capitalize on taking three fish as a unit and linking this unit with one food bar. This was also helpful for students to group the three fish and link them with a food bar with arrows as given in Figure 1 below.

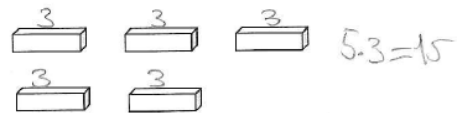
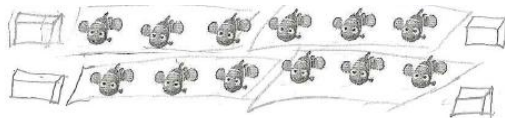


According to the rule in the box, is there enough food for the fish?



**Figure 1: Linking units by with arrows**

In the later problems, pictures of either food bars or fish were provided as suggested by the teacher. Whereas some students drew the pictures as in Figure 2a, some of them just wrote the corresponding number of fish numerically as in Figure 2b. Therefore, it was inferred that the teacher's suggestion was helpful for students to make pictorial, numerical, and mental linking and

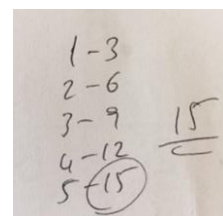


iteration.

**Figure 2a: Linking units by grouping**

**Figure 2b: Linking units with numerical values**

After a few problems through which this link was established, scaling up problems were posed (e.g. How many fish can be fed with 5 food bars? with 10 food bars?). Pictures were not included in the problems in order to help students create mental images of composite units and the link. In their solutions, a number of students drew pictures of food bars and/or fish, and some used numerical build up strategies as illustrated in the figures 3a, 3b, and 3c below. This gave us evidence that the students reasoned with models and mental imagery, and associated those with the physical situation.



**Figure 3a: Drawing pictures and linking with numbers**

**Figure 3b: Drawing pictures and linking with numbers**

**Figure 3c: Numerical build-up**

After finding the answer for the number of fish that can be fed with nine food bars as 27, a student answered the following question that asked the number of food bars for feeding 10 fish immediately as 30 by building up on nine and 27. He explained his reasoning as “We found that nine food bars could feed 27 fish in the previous question. So, I added three to 27 and found that 10 food bars can feed 30 fish since the number of fish goes up by three...since one food bar can feed three fish.” Therefore, the teacher’s suggestion regarding changing numerical values helped students make connections and improve their understanding of the link between the number of food bars and fish.

Later on, after all the students explained their answers with pictorial and numerical strategies, the teacher took the opportunity to introduce a long ratio table in order to keep track of students’ iterations, organize information, and make calculations more easily. By using long ratio tables, students were able to make such interpretations as “When the number of food bars goes up (or down) by one, the number of fish goes up (or down) by three”. In the following instances, students started to use abbreviated build-up strategies where they did not add values one by one; instead, they made interpretations of the type “when the number of food bars is doubled (or tripled, quadrupled etc.) the number of fish is also doubled (tripled, quadrupled etc.)” as in Figure 4 below.

13. Yukarıdaki kurala göre yem ve balık arasındaki ilişkiyi gösteren tabloyu doldurunuz.

Yem (Kutu)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Balık	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45

*Handwritten annotations: "3x" above the table, "2x" above the table, "3x" below the table, "2x" below the table.*

**Figure 4: Representation with abbreviated build up strategies**

After the one-to-three relationship was established among the classroom community, the instructional sequence was continued with different rules (e.g. 2-4, 2-3). When the rule was two food bars for four fish, a couple students used the given rule, whereas most of the students discovered that they could change the rule to one food bar for two fish. Below is a conversation in which students discussed about changing the rule when the question asked the number of food bars needed for feeding 12 fish:

Student 1: Six food bars are needed for 12 fish [claim] because I divided 12 by two. [data]

Teacher: Why did you divide 12 by two? [challenge]

Student 1: Because four fish can be fed with two food bars. [warrant]

Student 2: Then you have to divide by four, right? [challenge]

Teacher: Why do you think you have to divide by four? [challenge]

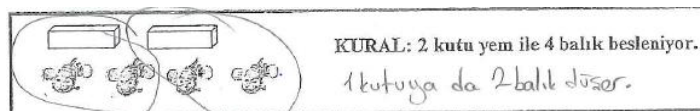
Student 2: (Draws 12 circles to represent 12 fish and groups each four fish by a bigger circle) I grouped [the fish] by fours and obtained three groups [of four fish]. I know that I

need two food bars for each group. So, I multiplied three by two [data] and obtained six. [claim]

Student 3: We changed the rule. If two food bars feed four fish, then one food bar feeds two fish. [warrant] So we divided each value by two. [data]

Teacher: What does everyone think about this?

Student 4: It is easily seen with the pictures (Linking one food bar with two fish as in Figure 5). It is easily seen that one food bar feeds two fish. It is easier to use this rule. We can group fish by twos after we change the rule. [warrant]



**Figure 5: Changing the rule while preserving the link between the number of food bars and fish**

As deduced from the figure and the dialogue, students changed the rule while preserving the link and the invariant relationship between the number of food bars and fish, which was an evidence of the strength of the link between food bars and fish. On the other hand, even though she did not use the term, Student 4 also referred to the concept of unit rate and how using unit rate makes calculations easier. Therefore, it could be deduced that including the pictures of fish and food bars as suggested by the teacher helped students to make sense of the ideas of equivalent ratios and unit rate.

Following the above activities, problems with a non-integer ratio (i.e. two food bars for three fish) were included in order to strengthen students' linking and iterating processes. At some point in the instruction, the teacher needed to communicate with the first researcher (participant observer in the classroom). While the students were filling in the table using the rule two food bars for three fish, a student asked if she had to fill in the table by building up by ones or by twos and threes. The teacher communicated with the researcher, and they decided that the tables had to be filled by building up by twos and threes. The teacher asked the student which way made more sense for her. The student replied that filling in the tables by twos and threes was meaningful since there would be decimal values while building up by ones and decimal values for the number of fish would not make sense.

### **Teacher's contribution to the analysis/revision phase**

Design research includes ongoing and retrospective analysis. As part of the ongoing analysis process, the teacher participated in the daily/weekly design team meetings that focused on how students were engaged with the tasks and the required revisions, which helped the teacher see whether or not the intended learning goals and practices emerged. The retrospective part of the analysis was conducted by the university members since they were trained in this area.

### **Discussion and Conclusion**

The power of design research is that it blends theory, research, and practice. It is reported that teachers can benefit from classroom design research in such a way that they would be up-to-date research-wise and experienced in developing/adapting tasks that would help students learn



meaningfully (Stephan, 2015). The findings of our study have further shown that the teacher could contribute to classroom design research aiming at developing an instructional sequence and classroom learning trajectory for linking composites and iterating linked composites.

The teacher contributed to the study and the establishment of students' mathematical practices regarding linking units and iterating composite units by implementing the activities and the HLT in the pre-planned ways. The findings obtained by Toulmin analysis related to students' participation behaviors of claiming and providing data/warrants for their claims and the documentation of mathematical practices showed that she contributed to the establishment of the mathematical practice of linking units and iterating linked composites. What is more, the teacher made a few on-action instructional decisions and communicated with the researcher in the implementation phase. Apart from implementing the instruction, the teacher contributed to the design research in the design/development phase by making suggestions on the necessary adaptations to the tasks and the HLT. However, it might not be obvious how substantially the teacher contributed to the design phase in this study. It should be noted that the analysis included the teacher's contribution only in the first part of the HLT and the instructional sequence, which were the anchor that laid the ground for the rest of the activities. Since it was the anchor activity it was well developed, and the teacher did not need to make big modifications. It is anticipated that the teacher's contribution to the design phase would be more substantial in the rest of the activities. Still, we claim that teachers can be valuable sources in designing instruction since teaching plans should be informed by students' mathematics (Steffe, 1991) and teachers are supposed to know their students. Lastly, the teacher contributed to the analysis phase by participating in the daily/weekly design team meetings that focused on enhancing student learning and required revisions. Engaging teachers more in classroom design research that focuses on a variety of mathematical topics could give more information about the contributions of the teachers to each phase of the studies.

### **Acknowledgment**

This research is part of the research project 217K430, funded by the Scientific and Technological Research Council of Turkey.

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