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Capturing Problem Posing landscape in a grade-4 classroom: A pilot study

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In this pilot study, grade-4 students in a Greek primary school are invited to pose problems in three different situations (no feedback, peers' feedback, teacher's suggestions). The aim was to capture the problem posing landscape of the classroom in terms of the complexity of the problems the student pose in these situations. The posed problems were examined in terms (a) of mathematical complexity in the sense of the number of semantic relations they involve, and (b) of linguistic complexity. The findings give evidence that as the students become progressively aware of the initial problem's attributes, they start to pose more complex problems by increasing both the number of problems that involve at least 2 semantic relations and the number of linguistic instances.

Keywords: Problem posing, primary school, mathematical complexity.

Introduction

Problem posing has been recognized as an important intellectual activity in school mathematics and integral part of a balanced mathematics curriculum (Hansen & Hana, 2015). The truth is however, that in the school setting students are merely asked to solve problems rather than pose (Stoyanova, 2003), and therefore they often face difficulties in posing mathematical problems (Silver & Cai, 1996). Even though research findings support that students are capable of posing interesting and important mathematical problems (Cai, Hwang, Jiang & Silber, 2015) most of these problems are mainly cognitive undemanding and textbook-like (Crespo & Sinclair, 2008). This contradiction makes apparent the potential role of an intervention and many research studies (Crespo, 2003; English, 1998) seem to agree that a proper intervention might help students to learn posing problems given that they already have experience in solving problems similar to those they are asked to pose and actually this highlights also the teacher's role in this process.

In this paper, we make a preliminary effort to capture the problem posing landscape in a Greek grade-4 classroom in terms of the complexity (mathematical and/or linguistic) of the generated problems in a variety of situations. The findings will feed the design of a year-long intervention aiming to foster the students' problem posing abilities. Thus, our research question is: How the level of complexity in the problems posed by primary school students varies according to the setting in which the problem posing takes place?

Literature Review and Theoretical Framework

'Are students able to learn to pose problems?' If yes, what is the proper way to achieve that? Brown and Walter (1983) claim that problem posing can be taught. For each problem students can ask what

kind of information the problem gives us (known), what kind of information is unknown and what kinds of restrictions are placed on the answer. Then many new problems can be generated if the solvers remove, change or loosen the restrictions. In essence, solvers are asked to list the attributes of the problem, to take an attribute and ask “What-If-Not” that attribute.

Exploring students’ problem posing performance is not new (English, 1998; Silver & Cai, 1996). Research findings reveal that even primary school students are able to pose good/meaningful mathematical problems. Lowrie (2002) claims that through a supportive and motivating learning environment, grade-1 students increasingly generated more sophisticated problems that were open-ended in nature. Gade and Blomqvist (2015) worked with grade-4 and grade-5 students who posed problems in three different stages (formulating written questions, problem posing in dyads and posing problems to one another) using slips of papers. Through these three stages, remarkable progress was made so as in the problems posed by the students as well as in the way they used the slips of papers. Chen, Van Dooren and Verschaffel (2015) implemented a training program aiming to develop grade-4 Chinese students’ problem-posing abilities. After the implementation, the problems generated from the students were significantly better proving that elementary students can be taught to pose better and meaningful problems. Cifarelli and Sevim (2015) connect the development of the problem posing abilities with successful problem solving. By focusing on episodes of two grade-4 students they explain how both problem posing and solving coevolve during the solution activity and also how the development of problem posing abilities contributed beneficially in the students’ problem-solving abilities, too.

According to the literature the problems posed by students can be more interesting when they have been previously solving similar problems themselves, when the posed problems are addressed to people outside the classroom and when they are prompted by informal instead of formal symbolic contexts (Crespo, 2003; English, 1998). Winograd (1997) working with fifth-grade students found that they were highly motivated to pose problems that their classmates would find interesting or difficult without losing their interest during the process of sharing posed problems. Problem posing is rather redefining the way students learn mathematics since they are actively engaged in the learning process by being encouraged to set questions and generate problems instead of being passive receivers of knowledge (Brown & Walter, 1983).

The problems posed by the students can be evaluated on the basis of three criteria (Silver & Cai, 2005): Quantity, Originality and Complexity. Quantity refers to the number of the generated problems. More specifically, the number of the mathematically correct problems may be indicative of the progress the students accomplished. Originality refers to the number of unusual responses and thus rareness is a way to measure it (they refer to these answers as atypical or original). The third criterion, complexity, can be examined at least from two different perspectives: the mathematical and linguistic complexity. Linguistic complexity focuses on linguistic structures (presence of an assignment, relational and conditional propositions in the statement of the problem). Mathematical complexity is related to the mathematical structure found in the posed problems and one plausible way to measure it is by enumerating distinct semantic relations using a classifications scheme of arithmetic word problems developed by Marshall (1995). This includes five categories of relations: Change, Group, Compare, Restate, and Vary. The Change schema applies when an initial quantity changes over time (increase or decrease). The Group schema relates to a situation where a

number of small quantities are combined into a larger one. The Compare schema involves situations where two things are contrasted with an emphasis on the relation between them (greater, smaller, more, less, etc.). Restate schema involve situations where there is a relation between two variables at a given frame only (e.g., exchange rates). Finally, Vary schema involves a fixed relationship between two variables that persists over time (e.g., relation between euros and cents).

Mathematical complexity has attracted the interest of several researchers on problem posing. According to Silver and Cai (1996) the complexity of the problems posed by the students tend to increase when students generate sequences of chained problems using the results of the simpler problems to pose more complicated ones. Ellerton (1986) comparing the problem posing abilities of high- and low-ability students found that high-ability students generated more complex problems. Finally, it seems that certain interventions influence the quality of the generated problems (Crespo, 2003; English, 1998). English (1997) conducted an intervention to grade-5 students and found that after the program the students posed a greater number of problems that were also more complex.

Design of the study

This study is the first in a series of small pilot studies aiming to capture the landscape of Greek primary students' problem posing abilities. The students were asked to pose problems in three different settings to compare the variation on the complexity of the problems posed in each setting. Apart from the first setting, the following two settings were developed during the implementation taking into consideration the students' answers. Eighteen grade-4 students from a private school participated in this study. The intervention took place in parallel to the normal teaching of mathematics in the sense that an hour per week is dedicated in math tasks aiming to develop students' mathematical thinking. The students had no prior experience on relevant problem posing activities.

In the first phase, the students were given the starting sentence of a potential problem: "Peter has 75 cents..." and they were asked to complete the problem in as many as possible different ways without having to solve their problems afterwards. Silver and Cai (2005) consider this kind of activities appropriate for assessing students' problem posing abilities. The teacher did not give any clue about the operations or the mathematical concepts the students could involve in the produced problems. This session was followed by a whole class discussion and the students had the chance to present their examples. One of the students presented an example that initiated the second phase of the study. In his example he actually dropped an attribute of the problem using a "What-If" approach and produced a problem that attracted the attention of his peers. The attribute was the kind of currency and his question was "What if instead of euros the currency was lev?" (the currency of the adjacent country of Bulgaria).

Based on his suggestion the students were asked to pose new problems in the same spirit using the same "What-if" technique of their classmate even though they did not have similar experience before (2nd phase). No discussion on the notion of listing and dropping attributes took place. Again, a whole discussion followed the session and the students presented their problems.

Finally, during the third phase, certain attributes were explicitly provided to the students by the teacher. More precisely, the teacher asked them to pose problems by considering certain attributes such as the total number of coins, the kind of the coins/currency and the value of coins.

The problem in its incomplete form makes apparent one attribute in an explicit way: the total amount of money. Therefore, students are left to involve and negate more attributes that could potentially be part of the problem such as the number or the kind of coins.

The students' worksheets with the problems they generated during the three phases constituted the data for this study. The written responses were analyzed based on the assessment tools developed by Silver & Cai (2005) and Marshall (1995). According to these tools, the responses are organized in three major categories depending on whether the questions included in the problem were mathematical, nonmathematical or whether there was merely a statement in the problem rather than a question. Then, the mathematical questions were evaluated on their solvability (solvable and nonsolvable). Finally, the solvable ones were examined for indications of mathematical and/or linguistic complexity. We determined how many of the five semantic structural relations (Marshall, 1995) co-existed in each problem. Problems that involved a larger number of semantic relations are considered more mathematically complex compared to those that involve less relations. A comparison followed across all the three phases. All the responses were evaluated independently by the two authors and validity and reliability were established by comparing sets of independent results, clarifying categories until agreement.

Results and Discussion

Phase One

As already mentioned, during the first phase of the study, the students were given the first sentence of a problem and were asked to pose as many as possible problems without feedback neither from their classmates nor their teacher. Each student posed one or two problems. Table 1 summarizes the results of the first phase. All the mathematically complex problems are distributed further according to whether they involved 1, 2 or 3 semantic relations. Moreover, sometimes linguistic and mathematical complexity might co-exist in the same problem and this explains why the partial sums in Table 1 are not in agreement with the total number of solvable problems.

Total number of Responses: 23				
Nonmath Questions: 0	Math Questions: 22			Statements: 1
	Solvable: 19		Nonsolvable: 3	
	Math Complex. 19		Ling Complex. 2	
	1 relation	2 relations	3 relations	
	8	8	3	

Table 1: Results of Phase-1

As it can be seen, in this phase most of the responses (19 out of 23) were solvable problems exhibiting to a certain degree mathematical and/or linguistic complexity. The problems more or less copy the problems the students are familiar with and are included in their mathematics textbooks. Mainly they were problems including 1 or 2 relations that could be solved in one or two steps using more the addition and/or subtraction operations (instead of multiplication or division). One example

with 1 relation was: *Peter has 75 cents. He gave 30c to one poor child and later he gave 7c to another. How much are left?* (1 relation, Change). Another example including 2 relations was: *Peter had 75c. His grandmother gave him 95c but later took 25c from him. His grandfather gave him two more euros but took 125c from him. How many euros will Peter have?* (Change, Vary). This problem combines addition/subtraction to find partial results (Change) but at the same time the concurrent use of cents and euros involves converting cents to euros and vice versa (Vary) which means that multiplication and/or division (depended on how the problem will be solved) are also necessary. Another example of mathematically complex problem which exhibited three semantic relations (change, group and vary) exhibiting at the same time linguistic complexity was: *Peter has 75c. At New Year's Eve his dad gave him 87€ and 46c. In March he bought 8 gums per 27c each. In April he bought an ice cream for 36€. Two weeks ago, he bought a skateboard for 19€ and 29c. a) How much money did he spend on February in order to have 75c now? b) How much € did all the gums cost? and c) How much did he spend for all of them?* The solver has now to interpret the existing information. 'New Year' refers to January and the chain of expenses refer to the period from January to May (the 'two weeks' is related to the time the session took place in school. It was during May). However, February is not mentioned. The initial amount of money is interpreted as the money Peter owns and the end. So, it is a multistep problem involving almost all the four operations and requires thought and interpretation of the information provided by the statement of the task.

There was finally an instance (1 out of 22) the students made a statement. That was: *Peter had 75 cents. He wants to buy a toy for 100€.* It is obvious that the student did not pose any question. During the whole classroom discussion one student posed the question (addressing it rather to the teacher than his classmates): '*What if instead of euros the currency was lev? Would you be able to solve the problem?*'. As already has been mentioned this question initiated the next phase.

Phase Two

During Phase 2 the students were invited to generate new problems in the spirit of the 'What-if' approach exhibited by their classmate in the previous phase. There was not any explicit reference to the list of the problem's attributes and their negation. The results of this phase are summarized in Table 2.

Total number of Responses: 21			
Nonmath Questions: 1	Math Questions: 18		Statements: 2
	Solvable: 16		Nonsolvable: 2
	Math Complex 16		Ling Complex 0
	1 relation 8	2 relations 7	3 relations 1

Table 2: Results of Phase-2

Most of the students' responses were solvable problems (16 out of 21). There was only one problem asking a question that was considered nonmathematical: *Peter has 75 cents. What if he loses all of his money and cannot buy anything?* Furthermore, no linguistic complexity was detected in the

students' responses in this phase.

What is interesting is that almost all the students followed the instruction to use the 'What-If' approach but actually they copied the problem of their classmate in the sense that they just focused and/or negated the attribute of currency. For example: *Peter has 75 cents. What if Peter had a friend from Japan and he wanted to trade yen with 75c?* (restate). Or in another example: *Peter has 75 cents. What if his money was alien money. If an alien cent has the same value as 9€ and wants to buy a wallet which costs 49€, how much change will he get?* (restate and change). In these responses, the students engage the potential solver with currency conversions (which means true statements for a short period of time), and the solution of the posed problem relies heavily on the relationships between the two currencies. During the whole classroom discussion, the teacher shifted the focus on identifying and negating more attributes of the problem and this initiated the third phase of the study.

Phase 3

During this phase of the study, certain problem's attributes were provided to the students such as the number, value and kind of the coins and they were asked to consider them in order to pose new problems. The results of this phase are summarized in Table 3. An example of a 'What-If' approach considering the kind of coins was: *Peter has 75 cents. If he only has 5c coins, how many coins are in his pocket?* (vary). It looks like a common problem. However, the choice of division as the necessary operation to solve the problem is not met frequently in the students' responses. An example of a moderate complex problem in this phase is: *What if there were also 7€ and 16€ coins and Peter has in his pocket sixteen 7€ coins and three 16€ coins. He spent 5.27€ and then lost 3.25€? How many euros are left? How many cents are they?* (Change, Vary, Group). The solver must be familiar with this new element of the weird coins that is not common at our linguistic repertoire and then must relate euros with these new coins and depending on the strategy there will be necessary to use multiplications and/or divisions for the interplay between euros and cents.

Total number of Responses: 32				
Nonmath Questions: 1	Math Questions: 31			Statements: 0
	Solvable: 29		Nonsolvable: 2	
	Math Complex 29		Ling Complex 8	
	1 relation	2 relations	3 relations	
	9	16	4	

Table 3: Results of Phase-3

The students' awareness of the attributes of the problems seems to stimulate both the number and the quality of the posed problems. There was a noticeable increase in the number of the problems (almost 1,5 times more). More precisely, the number of problems that include at least 2 relations was 20 out of 29 problems. This increased number of semantic relations (Silver & Cai, 1996) indicate the presence of mathematical complexity. Furthermore, in this phase, there was a significant number of instances (8 problems) exhibiting a sense of linguistic complexity: *Peter has*

75 cents. ***Is it possible to have only 2c coins?*** The linguistic element in this problem is that its question differs significantly from the ones posed in the first two phases. The word “possible” must be interpreted correctly given that the participants are grade-4 students and a certain language competency is required to negotiate phrases such as “at least”, “possible”, “the more” that are included in mathematical problems.

Conclusion

The purpose of this (first among others that will follow) pilot study was to capture the landscape of problem posing in a Greek grade-4 classroom in order to setup a year-long intervention in the classroom aiming to develop the problem posing abilities of the students. The students were invited to pose their problems in three different setting and the results give evidence that given certain circumstances grade-4 students can pose interesting mathematical problems. Almost all the students were able to pose mathematical questions. Only a couple of statements or nonmathematical questions were identified. Moreover, the complexity and the sophistication of the generated problems tend to increase along with the evolution of the sessions.

During the first phase of the study, students posed without any external influence a series of solvable problems. Most of these solvable problems included one or two semantic relations resembling textbook like problems dealing with the four operations and specifically addition and subtraction which is in accordance with Crespo and Sinclair (2008). There were also a few instances of three relations or linguistic complexity signs that were identified in some of the answers.

In the next phase of the study, a “What-if” technique was used by one of the students who negated the currency attribute of the problem. This was an opportunity to invite students posing new problems under the light of this new approach. The situation was more or less similar to the previous one, and the posed problems were merely mimicking the one posed by their classmate. So, the students did actually drop the attribute of currency not as an action of negating the structural elements of the task but as a way to follow an example that seemed to be successful. No signs of problems’ linguistic complexity were identified.

During the final phase of the study, the students became aware of certain attributes that were highlighted by the teacher. This awareness seemingly helped the students to increase the number of posed problems increasing at the same time the number of the involved semantic relations per problem. Also, more evidence of linguistic complexity emerged.

Given the lack of any relevant experience on problem posing and its strategies the students were able to take advantage of the feedback provided during the whole classroom discussion either from their peers or their teacher. This feedback actually shifted their attention to specific attributes of the task. It is very interesting that the students’ answers determined the design of the next two phases. This study, being a pilot one, feeds the design of a future intervention aiming to create a class environment that will encourage students to raise questions and pose problems. This small-scale study encourages the idea that a systematic teaching on problem posing may develop students’ posing abilities which is in alignment with English (1997).

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