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# Adapting implementation research frameworks for mathematics education

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*Although there have always been investigations in the field of mathematics education that concern implementation research, it is not yet clear how to structure such a research area. In this paper, we take a step towards this. Taking as reference frameworks elaborated in other fields where implementation research is more advanced, we attempt to outline what an implementation research framework in mathematics education could encompass. We illustrate this with an example of implementing the Danish innovation of the mathematics competency framework (KOM).*

*Keywords: Implementation research; innovation; theory of change; value.*

## Introduction

Implementation research has received an increase in attention within mathematics education during the last few years (e.g. Cai et al., 2017; Jankvist, Aguilar, Ärlebäck, and Wæge, 2017). But the discussion of implementation is neither new in educational research in general, nor in mathematics education in particular. Already Fullan and Pomfret (1977) remarked that “implementation is not simply an extension of planning and adoption processes. It is a phenomenon in its own right.” (p. 336). Even if the discussion of implementation has been around for some time, both the work presented and carried out during CERME 10 and the recent discussions in JRME (e.g. Cai et al., 2017) illustrate clearly that there is still a large potential indeed in addressing initiatives within mathematics education from an implementation research point of view. However, it is not at all clear how exactly to do this. Hence, we first need to ask ourselves which existing implementation research frameworks that are applicable to which initiatives in mathematics education as well as how and why? The purpose of this paper is to take some initial steps towards addressing this big question. As a reference point we take implementation frameworks from other research fields to try to sketch an implementation research framework in mathematics education. Then we “test” it on an illustrative example of the decentralized implementations of the Danish mathematical competencies framework—KOM—in selected mathematics programs of the Danish educational system.

## Implementation research frameworks

Within the context of health science, Nilsen (2015) proposed a taxonomy of three aims for the use of theoretical approaches in implementation science, and five categories of theories, models and frameworks. The first aim of theoretical approaches is to *describe and/or guide the process of translating research into practice*. Process models lay out specific steps to implement research into practice, and thus provides practical guidance in planning and carrying out implementation. The second aim concerns *understanding and/or explaining what influences implementation outcomes*. Three different types of frameworks and theories are found to support this aim. The first type is the determinant frameworks that identifies barriers and enablers, which influence the implementation outcomes as well as specifying the relationships between barriers and enablers. The second type is

the classic theories, which are theories that originate from fields outside of implementation science, but may be applied to understand some aspects of implementation. The last type related to this aim are actual implementation theories, which are those developed with the sole purpose of explaining aspects of implementation. The third and last aim concerns *evaluating implementation*. This aim can be supported by the category of evaluation frameworks that may provide a structure for evaluating implementation.

In their review of implementation research within education, Century and Cassata (2016) offer the following (working) definition:

Implementation research, by our working definition, is the systematic inquiry of innovations enacted in controlled settings or in ordinary practice, the factors that influence innovation enactment, and relationships between innovations, influential factors, and outcomes. Thus, frameworks that inform the organization of implementation research address two main concerns—how to conceptualize and describe the innovation itself, and how to identify and organize the contexts, conditions, and characteristics that influence innovation enactment (influential factors). These two fundamental concepts—(a) characteristics of the innovation and (b) influential factors—are basic elements of varied theories of change and a key part of most recent research syntheses or metaframeworks depicting innovations in context. (p. 181)

This definition builds on two other terms, the meaning of which will have to be explained as well; namely those of innovation and change. Century and Cassata (2016) operate with a definition of *innovations* as “programs, interventions, technologies, processes, approaches, methods, strategies, or policies that involve a change (e.g., in behavior or practice) for the individuals (end users) enacting them” (p. 170). More traditionally, an innovation is oftentimes considered to be new ideas creating economic value (Darsø, 2012), or as the middle part of the CIE (Creativity, Innovation and Entrepreneurship) model for value creation and change (Paulsen and Harnow, 2012). However, the dependence on economic bottom line thinking makes this definition difficult to apply to educational situations. Nevertheless, if we instead consider innovation as changes of practices and technologies for the better, then we set ourselves free from the problem of economy as the core goal for innovation. This however leaves open what kind of normative change for the better that we should value in innovation. We should ask what (kind of) value and for whom. The definition of educational innovations provided by Century and Cassata (2016) focuses on *the change for end users* (e.g. teachers and students) and simply views innovations as the object of implementation research. While this is indeed an important perspective, we suggest that the notions of value and stakeholders are critical; an innovation is only an innovation if it creates value for some stakeholders (Krainer, 2014), but the stakeholders do not necessarily have to be the end users. Value can be for the society in general, for educational administrations, school management, etc. We thus define educational innovations as *change in educational practice that are valued by some stakeholders*. However, in order to conceptualize any change, it is necessary to acknowledge that change may take place at different levels (at the individual level, classroom level, school level, etc.), but also that change is mediated by people and institutions who can adopt, modify or even reject the intended innovation implementation. Thus, as pointed out by Century and Cassata (2016), a question that lies at the heart of the implementation research is: “what does it take for people, organizations, and systems to change?” (p. 178).

To address a question like that in the context of mathematics education, we need to draw on a wide range of conceptual and methodological tools from within and outside the field. For example, to try to identify and organize the conditions that influence innovation enactment at the classroom level, it would be important to understand the intentions and individual motives that a mathematics teacher has to adopt or not an educational innovation. Research on curricular implementation in

mathematics education has shed some light on this type of issues (e.g. Remillard and Heck, 2014). If we refer to change beyond the classroom and on a larger scale, say at the institutional level, it could be useful to resort to studies on organizational change from different disciplines such as medicine (Rohrbach, Grana, Sussman and Valente, 2006) and management science (Burnes, 2005), which can provide us with theories and methodological approaches to help us understand how to produce large-scale change behaviors in school systems. Here studies on mathematics education with an institutional perspective could also be useful (e.g. Castela, 2004). On a wider scale, Rogers (1962) has developed a frequently cited theory describing how innovations are spread across a social group (see for example Koichu and Keller, 2017). He distinguishes several groups of adopters; innovators (adopting a new technology instantly), early adopters, early majority, late majority and laggards (who adopts very late). Rogers shows that the distribution of adoption is similar to a normal distribution with the bulk of people in the early and late majority category. Implementation is obviously related to how people adopt. If you aim for mainstream implementation of an innovation, when it is only adopted by innovators, you are bound to fail.

### **Five factors that influence implementations**

The identification of the factors that influence innovation enactment is a fundamental component of implementation research. For years educational researchers have tried to identify the variables that influence the implementation of educational innovations; these variables can be classified in the following five spheres of influence (Century and Cassata, 2016).

*Characteristics of the individual users:* The change that an educational innovation is aimed at generate, it is mediated by the people involved in the implementation process. Hence, it is important to know their individual characteristics. We distinguish between (a) *characteristics of the individual in relation to the innovation* (mathematical background, experience using the materials or resources involved in the innovation, etc.) and (b) *characteristics of the individual that exist independently of the innovation* (willingness to try new teaching methods, attitudes towards new artefacts in the classroom, etc.).

*Organizational and environmental factors:* In the case of an innovation implemented in a mathematics classroom, organizational factors refer, on the one hand, to the characteristics of the setting itself (number of students, characteristics of the physical space, access to material resources, etc.), and on the other hand to the collective beliefs and behaviors of the members of the class (identity, sociomathematical norms, didactic contract, etc.). Environmental factors refer to those outside the organization, but which have an influence on how an innovation is adopted and implemented (economic conditions, educational policies, priorities of government agencies, etc.).

*Attributes of the innovation:* The attributes of the innovation can influence its implementation, however, it is important to distinguish between the *actual* attributes of the innovation (objective characteristics) and the *perceived* attributes of the innovation (subjective characteristics perceived by the user). Of course, the perceived attributes may vary from user to user.

*Implementation support strategies:* It is important that an innovation initiative comes accompanied by an intentional and planned support for the final users and their institutions. Such support strategies can be professional development, specific resources, etc.

*Implementation over time:* Another factor that influences the implementation of an innovation is time. Thus, it becomes relevant to study innovation endurance over time: how can we promote that an innovation, besides being adopted, is preserved over time until it is routinized? It is in this branch of the implementation research where longitudinal studies will become essential to answer questions like the one previously stated.

## **The KOM framework - an “implementation story”**

As an illustrative example of an implementation, we take a historical and chronological look at the Danish mathematics competency framework, referred to as KOM, which was first published in Danish (Niss and Jensen, 2002), and later in an English translation (Niss and Højgaard, 2011). This framework has heavily influenced mathematics education in Denmark, where it has been implemented in primary and lower secondary school through the so-called “Fælles Mål” (common goals) (Undervisningsministeriet, 2014), and in both the technical stream (htx) and business stream (hhx) of upper secondary school—to a lesser extent in the classical stream (stx)—in the mathematics teacher education program as well as in some of the mathematics-related programs in tertiary education. On an international level, KOM’s competencies descriptions were an integral part of the PISA assessment framework for mathematics from approx. year 2000 through 2018. The description has also been influential in mathematics programs in several countries’ (see Niss and Højgaard, in progress). Now, in itself the KOM framework constitutes a normative text, which is not directly translationable/implementable in the various mathematics programs. But KOM’s competencies description, with its eight distinct yet interconnected mathematical competencies, was never meant as a standalone. It must be implemented together with a curriculum for the mathematics program at a given educational level. One distinct feature of the KOM approach is its matrix thinking that links specific competencies to various mathematical areas, i.e. competencies in the rows and mathematical areas, e.g. algebra, geometry, in the columns of a matrix. From an implementation perspective, however, KOM makes up an interesting case.

When published in 2002, KOM was supplemented with an implementation support strategy of meetings and seminars debating specificities of the framework and the value of placing development of mathematical competencies as the key feature. But besides this, the implementation support strategies must be characterized as ad hoc and rather scattered. Still, KOM’s competency description was adopted by a large part of the teacher educators in Denmark. It was implemented in the national standards for compulsory school, partly between 2003 and 2006 and more thoroughly in 2009. Furthermore, it was implemented in the teacher education standards in 2012 and in upper secondary school in 2013 and again in 2017. In 2012 the impact of the competencies framework was evaluated as part of a general evaluation of the national standards (Danmarks Evalueringsinstitut, 2012), showing that teachers were neither using the standards nor the eight competencies to a very large extent. In 2014 the K-9 curriculum was reformed towards an outcome-oriented curriculum, and the competencies were now embedded in this structure (“Fælles Mål”). Furthermore, the number of competencies was changed from eight to six. A transformation that was heavily criticized by the people behind KOM (see e.g. Niss, 2016). As of now, the overall structure of the output-oriented curriculum is being debated again and will be partly rolled back. Nevertheless, for the eight competencies this transformation actually appears to have increased their impact and the outreach of the KOM framework. These days more teachers do seem to know more about the competencies than before; partly due to timing in terms of adoption and partly because the 2010 reform made everyone aware of the curriculum/standards and hence the competencies. As mentioned above, such awareness concerning KOM does not seem to be present at the classical stream of upper secondary school, stx, which is by far the largest of the three upper secondary school programs. Why is this? The answer to this question—we believe—is to do with environmental (and institutional) factors as well as the characteristics of the different individual users.

The characteristics of the individual users in the various institutions are quite different. Danish K-9 mathematics teacher educators are mainly educated at the Danish School of Education, where they are exposed to and thus become accustomed to an educational approach that relies heavily on the competency framework (e.g. Højgaard and Jankvist, 2015). In their future professions at the

university colleges, these teacher educators thus often come to act as enablers for the implementation of KOM, not least in relation to the teachers that they educate. The K-9 mathematics educational system, however, is a big ship to turn since it is comprised of teachers of all ages educated under various different educational paradigms and reforms. Hence, as mentioned above, the prevalence of awareness of the eight competencies was a lengthy process. One indicator of this is the fact that many teachers reacted negatively against the PISA 2012 assessment in mathematics, despite the fact that the 2012 PISA mathematics framework was perfectly aligned with KOM and thus also the national standards of 2009. Yet, as part of these standards, an explicit matrix structure between the competencies and mathematical areas was developed for each grade level. This has played a major role in the implementation of KOM in K-9.

When it comes to the upper secondary stx mathematics teachers, they oftentimes hold a master's degree in mathematics from a university mathematics program. Their teachers at the university typically hold a PhD in some area of mathematics, and hence are not necessarily very well versed in the area of mathematics education research, including the competency approach. This of course serves as a barrier for the implementation of KOM in stx. The reason that the situation is a bit different in the two other streams of upper secondary school, as mentioned above, is most likely to do with the fact that the mathematics teacher population there has a more varied background, e.g. at htx several teachers have a background as engineers. Another identified barrier is to do with the lack a matrix structure in the stx curricular documents. Although these do mention a selection of KOM's eight competencies, then mainly do so on a rhetorical level, i.e. the various competencies are not linked to actual mathematical areas and concepts. To a much larger extent this is done in the curricular documents of both htx and hhx (for a further discussion, see Niss and Højgaard, in preparation).

Implementing the KOM framework has taken time. As described above different stakeholders have adopted the framework to different degrees. Even though all these stakeholder groups are distributed across Rogers' (1962) different adoption-types, it is still possible to distinguish groups that overall are moving faster than others. The teacher educators, for instance, have adopted KOM much faster than the K-9 teachers. However, in the case of the K-9 teachers time seems to work for the implementation of KOM, even though adoption is slow. The 2014 K-9 reform was timewise in a place where the innovators and early adopters was already using the competencies framework. This is to say that the timing was working for the competencies, and the curriculum reform thus acted as a catalyst for the early and late adopters to embrace the framework. In the case of the stx teachers, we are less sure to what extent the adoption and implementation will progress over time. KOM has the learning of mathematics at its core, and hence it should be relevant for this population of teachers as well. Still, the lack of an implemented matrix structure in the curricular documents of stx, which, as described above, is an essential attribute of the innovation, does not hold a lot of promise. Due to the stx curricular documents' use of the competency terms only on a rhetorical level, the actual attributes of KOM and the stx teachers' perceived attributes may simply not be aligned.

## **Discussion and analysis**

Referring back to Nilsen's framework as presented above, the analysis of the implementation of KOM in Denmark mainly addresses the second aim of understanding and explaining what influences implementation outcomes. Here both aspects of what Nilsen refers to as determinant frameworks are in play, as the project identified certain barriers and enablers which influence the implementation outcome. Century and Cassata's (2016) description of factors that influence implementation (see above) was useful alongside Rogers' (1962) focus on stakeholders' adoption. Hence, these implementation research frameworks have provided us with some general perspectives

of where to look and focus our attention, but in themselves they are not detailed enough for cases such as KOM - or any case of mathematics education, we predict. In particular in relation to Nilsen's first and third aim, the description of the implementation process and the evaluation of implementation, one thing that the general frameworks often cannot supply us with is a *theory of change*. Which on one hand can guide the process of implementation and on the other hand is used to evaluate the success of an implementation. The parameters on which to measure change may oftentimes be found within theoretical constructs from mathematics education itself. Let's exemplify.

In mathematics education there are of course many different kinds of innovations, but for the sake of simplicity let us distinguish two. One type concerns that of implementing normative frameworks such as KOM, which in a sense call for a philosophical or cultural change in mathematics programs as to what it means to master the subject. Another type are the grand scale initiatives as for example the Swedish Boost for Mathematics (BM), which offered a further education program to the majority of Swedish mathematics teachers. Although the BM did include a framework, involving e.g. didactical contract, socio-mathematical norms, etc., it was in itself not a framework such as KOM is. This means that when assessing qualitative aspects of the implementation of the innovation, KOM becomes its own theory of change, whereas BM does not. We are not claiming that this necessarily is a problem; only that we should be aware of this when discussing implementations in mathematics education. Another difference is that the implementation of the BM was top-down, whereas many of the local implementations of KOM were more bottom-up. Also, the difference in scale of the two implies that the evaluations of them may be of different nature. While the BM was often evaluated on quantitative terms, e.g. how many schools and teachers participated, etc., KOM to a larger extent was only evaluated on qualitative terms. The qualitative theory of change parameters of both KOM's and BM's evaluative frameworks come from within mathematics education research, while the quantitative parameters of BM's evaluation do not necessarily do so. Nilsen's (2015) framework also includes theories which focus on change from other fields than implementation science, the category of classic theories. However, this category concerns something different than what we call a theory of change, since we regard a theory of change to be something that is locally developed in conjunction with the framework being implemented. The benefits of the close relation between innovation and evaluative framework is that we set us free from binary and simple evaluative categories. In a sense, it is like choosing between relevance and accuracy/circular inference (begging the question/circular argument) in the evaluative frameworks.

## **Concluding remarks**

Century and Cassata (2016) provided us with a definition of implementation research that has been useful in our effort to begin to explore how implementation research in the field of mathematics education might look. However, this conceptualization of implementation research is not exempt from criticism, nor from competing visions in the conceptualization of this area of research. For instance, one could point out the fact that the conceptualization made by Century and Cassata does not take into consideration—at least explicitly—the role of adopters in the implementation process of an innovation. A conceptualization like this runs at risk of positioning innovation as an imposition for the adopters, who may perceive it as an external element and alien to their own didactical ecosystems. To address this issue, we could consider complementing such conceptualization with other already existing theoretical frameworks from the field of mathematics education. For example, a documentational approach could help us to consider the process through which an innovation can be reshaped and transformed by the adopters.

Thus, although implementation frameworks can be useful to focus our attention on key elements of an implementation process and even to conceptualize them, in the case of mathematics education some of these key elements will be of a general nature, while others may be of a more specific nature and connected to mathematics education. Thus, it is feasible to suggest that the implementation research in mathematics education needs to be based on a bricolage of different constructs and theoretical frameworks typical of research in mathematics education, in combination with constructs from implementation of research in other disciplines with a longer research tradition in this regard. However, there is a lot of work to be done to achieve such integration of theoretical frameworks and constructs. A natural step in this effort may be to begin to identify and organize the already accumulated research in the field of mathematics education that addresses aspects of implementation.

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