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# Integrating field interpretations to geological modeling with the potential field method

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## Abstract

Geological structures, such as complex folds, fluvial channels or ore-veins, usually present principal directions of continuity that can be defined locally but vary on a broader scale. Taking into account both these local anisotropies and their variability is key to achieve sound geological 3D modeling. We make the hypothesis here that anisotropy variations can be sampled on the field and incorporated into the model at an early stage of model building. This work develops two strategies to incorporate such observations to implicit modeling using the potential field method: i) adding second-order derivative information to this field; ii) describing the field as a stochastic convolution in which will be given an interpolated anisotropic field.

## 1 Introduction

Over the last decades, new geological modeling methods have proven to be an efficient approach to quickly build complex structural models. They are based on the description of geological interfaces as implicit surfaces and their boolean combination, using Constructive Solid Geometry rules with a geological meaning. Among them, the so-called potential field method (Lajaunie et al., 1997; Calcagno et al., 2008) is based on the co-kriging of field data in order to construct a scalar field  $Z$ . This way, a surface is designed as a particular isovalue of the field in 3D space. Those data can derive from drill-hole data, digital geological maps, structural data, interpreted cross-sections and consist in:

- contact points, sampling a same geological interface  $k$ :

$$Z(\mathbf{u}_j) - Z(\mathbf{u}_{j'}) = 0 \quad \text{for all pair } (j, j') \text{ of any set } J_k \quad (1)$$

- orientation data such as dip directions or tangents, expressed using the gradient of the scalar field:

$$\frac{\partial Z(\mathbf{u}_i)}{\partial x} = G_i^x \quad (2)$$

The interpolation of the scalar field is done thanks to a co-kriging:

$$[Z(\mathbf{u}) - Z(\mathbf{u}_0)]^* = \sum_k \sum_{jj' \in k} \lambda_{k,j} (Z(\mathbf{u}_j) - Z(\mathbf{u}_{j'})) + \sum_i \sum_t^{\dim} \beta_{i,t} G_i^t \quad (3)$$

Though such an approach has provided very satisfactory results, available data is frequently not sufficient to describe complex structures, especially in cases of locally varying anisotropies (Martin, 2019). When facing important uncertainties, experienced users have to introduce *a priori* geological knowledge into the model in the form of additional control points. This workaround is usually time consuming, error prone and user dependent. It may also lead to overconstrained models which no longer leave room to uncertainty analysis and/or quantification. This work will present ways to incorporate geological observations of anisotropy at

an earlier stage of modeling. The first one is based on second-order derivatives data of the scalar field and the second one on a description of the potential as a stochastic convolution of a Gaussian kernel.

## 2 Contribution of second-order derivative information

Just as the gradient is an information of the first-order derivative of the potential field, the second-order derivative can be integrated into the co-kriging equations. They represent the curvature of the field at a point  $u_i$  in the specific direction  $\mathbf{d}$ :

$$\frac{\partial^2 Z(\mathbf{u}_i)}{\partial \mathbf{d}^2} = \lim_{\epsilon \rightarrow 0} \frac{Z(\mathbf{u}_i + \epsilon \mathbf{d}) + Z(\mathbf{u}_i - \epsilon \mathbf{d}) - 2 Z(\mathbf{u}_i)}{\epsilon^2 \|\mathbf{d}\|} \quad (4)$$

It could seem hard to sample in practice but actually special cases of null second-order derivatives are already measured, by fold axis data for example. However, whereas introducing gradients of the potential field requires a twice differentiable covariance function  $K$ , considering covariance between second-order derivatives requires the function to be differentiable four times:

$$\langle \nabla^2 Z(\mathbf{u}), \nabla^2 Z(\mathbf{v}) \rangle = \nabla_{u,u,v,v}^4 K(\mathbf{u}, \mathbf{v}) = \nabla_{u,u,v,v}^4 C(r) \quad \text{where } r = \sqrt{(\mathbf{u} - \mathbf{v})^t A (\mathbf{u} - \mathbf{v})} \quad (5)$$

This way, the cubic covariance traditionally used for the potential method no longer applies. One covariance function which fulfills the requirements and then considered in this work is a radial one, piecewise polynomial, positive definite and compactly supported (Wendland, 1995):

$$C(r) = \frac{1}{3}(1 - r)_+^6(35r^2 + 18r + 3) \quad (6)$$

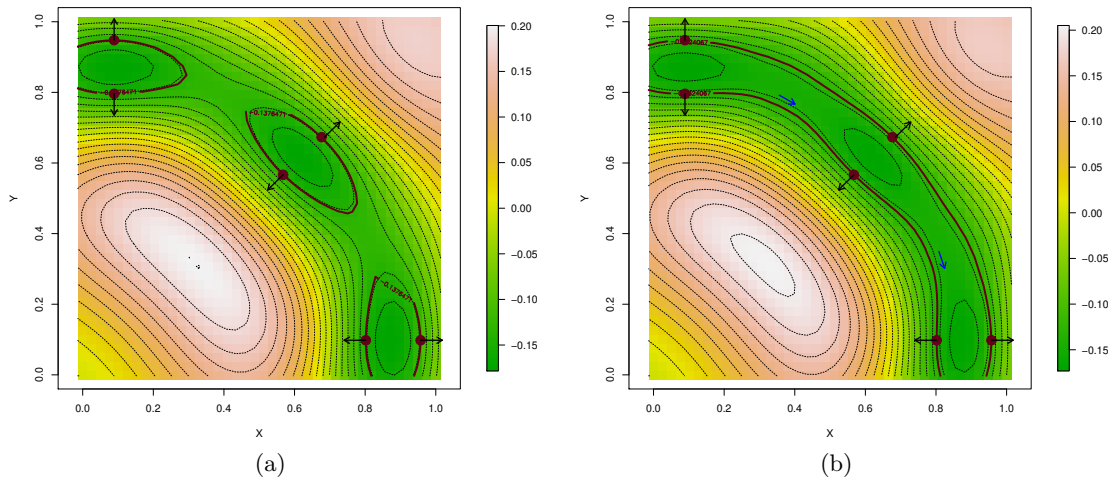


Figure 1: Impact of the second-order derivative data into the potential field method. Red points (6) are contact points (increments) of a same iso-potential. Black arrows (6) are gradient values of the field. The actual result (a) can not reproduce the continuity of the structure. Addition of second-order derivative values, constrained to zero, in this direction ((b), blue arrows) allows the method to follow the interpreted anisotropy.

This approach can integrate fold axis, which are real field data, traditionally described but unused in practice. The contribution of these second-order derivatives values is illustrated on a 2D synthetic example, shown in Figure (1).

### 3 Stochastic convolution by a Gaussian kernel

Another intended approach to integrate anisotropy is to model the potential as a convolution product between a stochastic white noise  $W$  and a deterministic kernel  $\Phi$ , slowly varying with the position of  $u$  (Higdon, 1998; Fouedjio, 2016):

$$Z(\mathbf{u}) = \int_R \Phi_{B_u}(\mathbf{u} - \mathbf{s}) W(d\mathbf{s}) \quad (7)$$

This kernel  $\Phi$  requires a complete definition of the anisotropy since it defines at each point  $u$ , a local anisotropy  $B_u$ , according to 4 (respectively 7) functions in 2D (respectively 3D):

- $\theta(u)$  the main rotation angle of the anisotropy direction in 2D,  $\alpha(u)$ ,  $\beta(u)$  and  $\gamma(u)$  the 3 rotation angles of the main anisotropy direction in 3D
- $a_1(u)$  and  $a_2(u)$ , the scale factors of anisotropy in respectively main and orthogonal direction of anisotropy,  $a_1(u)$ ,  $a_2(u)$  and  $a_3(u)$  in 3D
- $\sigma(u)$ , the standard deviation

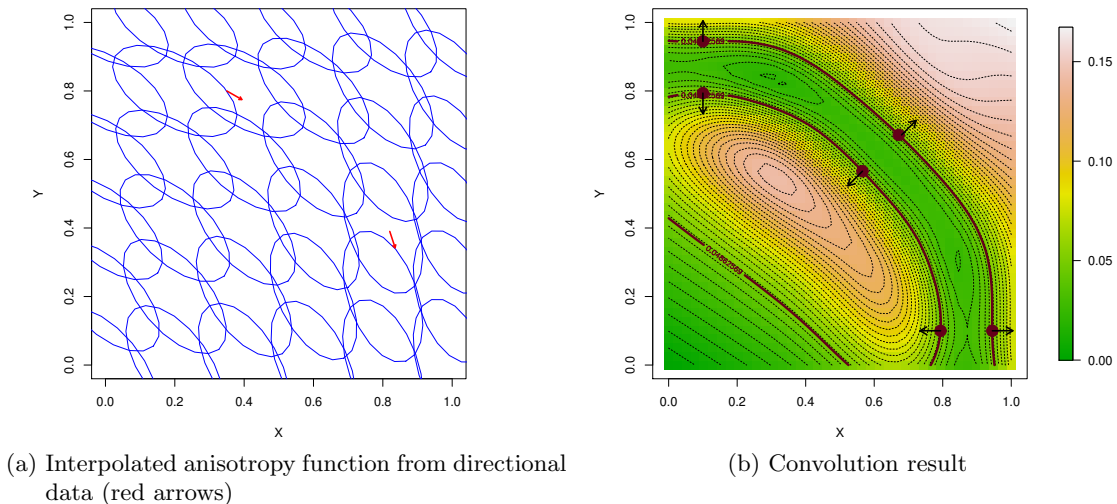


Figure 2: Stochastic convolution method results

A Gaussian kernel has been chosen since its derivative, which are required for derivations of  $Z$ , are explicit. Figure (2) presents a 2D example of stochastic convolution use on the same case than Figure (1). The function of the angle of local anisotropy  $\theta(u)$  is interpolated thanks to a complex kriging (Lajaunie, 1991) of some directional data.

### 4 Conclusion and perspectives

This work presents two different approaches in order to take into account issue of non-stationarity of some structural implicit modeling cases. Both can rely on real field data as well as interpretative descriptions. Whereas the second-order derivative method may appear as a local description of anisotropy, the stochastic convolution method is a more global approach but in return requires an explicit function of the anisotropy angle. Thus one key of this approach is the angle interpolation, performed here with complex kriging in 2D, but which could eventually be achieved in 3D with quaternion interpolation. These methods are not to be opposed, since each

can benefit from the other: it is theoretically feasible - even though it makes the calculations harder - to take into account the second-order derivative data in the convolution method. In future work, these improvements have to be challenged on real 3D datasets which present such variable anisotropy.

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