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# Exploring pre-calculus with augmented reality. A design-based-research approach

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*This paper reports on the development of a learning design addressing pre-calculus concepts using augmented reality-technology. In a first step, both task- and technology design-principles are derived from the research literature and both elements – tasks and technology – will be described and discussed. Within a first design-research cycle, the paper reports on experimental difficulties with the technology used for the first version of the AR-tool and the needs for the development of a second version. By doing so, the paper addresses technological constraints experienced in experimental situations and discusses alternative ways.*

*Keywords: augmented reality technology, pre-calculus concepts, design research*

## Introduction

This paper describes a first cycle within a design-based-research project, which aims at a better understanding of pre-calculus concepts by using augmented reality tools. Specifically, we emphasize the capability of AR to merge dynamic features of a real phenomenon (e.g. a moving object along inclined plane) with those of its mathematical representations, which can be displayed graphically and numerically (e.g. plotting points of graph, ordered pairs in a table of values), as well as the learners' own perspectives with respect to the phenomenon. Within the DBR approach (Gravemeijer, 1994; Prediger et al., 2015) we distinguish between two different layers of design-principles, namely the task and the technology design principles. Hence, this paper aims to discuss the design principles of the tasks and the technological tools of such an environment.

## Exploring pre-calculus concepts

One of the main goals of teaching pre-calculus concepts is to initiate experiences with functional relations. In calculus, students use quantifiable methods to explore certain characteristics of functions. It is essential to allow pre-calculus explorations though, in which such characteristics are not yet experienced as being standardized, formalized and quantified. That means that students need activities, in which they explore pre-calculus concepts in qualitative learning settings. A detailed exactification by introducing the concepts of rate of change, accumulation or derivative is part of a later stage within the learning process.

One way to design such learning activities is the use of real experiments. Inquiry-based enactment of scientific exploration shows how students can be engaged in learning activities, in which they conduct real experiments and - by doing so - develop the mathematical concepts. One key feature of inquiry-based learning approaches is the shift from deductive ways of teaching to more inquiry-based approaches in order to increase the students' interest in mathematics (Rocard et al., 2007, p. 2; Artigue & Blomhøj, 2013). Within such inquiry-based situations students formulate conjectures based on experimental experiences and ask for reasons and try to prove them (Lakatos (1976)

speaks of the quasi-empirical nature of mathematics). Furthermore, the role of physical action and of bodily experiences is highly important for the genesis of mathematical concepts (e.g., Edwards et al., 2009; Radford, 2014; Roth, 2009). For the context of functional reasoning Duval (2006) points out that different representation modes (like graph, table, symbolic expression) are necessary in order to grasp mathematical concepts in general and especially the concept of function.

On the basis of this very dense discussion we formulate the following content-specific design-principles:

- P1: Enable students to conduct their own experiences.
- P2: Use different representations to foster the mathematization.
- P3: Ensure enough room to verbalize the individual conjectures and to interact with other students in order to prove them.

### **The role of augmented reality**

In a mathematical representation of real phenomena, the correspondence between the original phenomenon and the mathematical representation is usually not acquired directly as an effect of a natural similarity (as in analogies and often in diagrams). For example, considering the function that models the fall of a ball, the graph that represents the relation between time and space does not provide a direct, sensory similarity between the actual position of the ball and the shape of the graph; rather, the analogy is between the numerical expression of the respective phenomenon and its graphic representation, which is spatial.

Within the design-research project described in this paper, we use an augmented reality technology to experience and explore the pre-calculus concepts described above. AR technology uses a wide spectrum of techniques for projecting computer-generated materials, such as text, images, and video, onto users' ordinary unaided perceptions of the real world (Kaufmann et al., 2005). Johnson et al. (2011) observe that AR can be especially used in order to support inquiry-based learning. Despite some educational suggestions, Wu et al. (2013) point out that the use of AR in educational settings still remains in its infancy. AR is commonly used by science educators to help students visualize scientific processes, such as chemical reactions, which cannot be observed easily in the real world (e.g. Wu et al., 2013). Importantly, Cheng and Tsai (2013) conclude that while research on the use of AR in educational settings has examined issues such as development, usability, and initial implementation (Blake et al., 2009; Sayed et al. 2011), students' inquiry skills and the learning processes within AR environments have largely been ignored in the scientific literature.

For mathematics education, Sommerauer and Müller (2014) found that most studies on mathematics education using AR examined the effect of this technology on learning spatial abilities (e.g., Kaufmann et al., 2005); this finding is not surprising, as 3D is one of the key affordances of AR. For example, Kaufmann and Schmalstieg (2003) developed a learning environment that allows students to act on and interact with geometrical objects in space. Similarly, Oronzco and colleagues (Orozco et al., 2006) developed an AR-rich learning environment to assist students in visualizing multivariable function curves and reported that it helped develop spatial reasoning and visualizability of 3D mathematical objects.

Within the research project, we will examine the way how a learning environment based on AR, given its ability to combine real-world experiments, real-time data regarding these experiences, and symbolic systems that model them, may help to create the conditions whereby inquiry-based learning will be set into motion in a natural and productive way. This paper reports on the first design-cycle in developing an AR-tool and a learning environment. On the basis of the discussion of the technological aspects of using augmented reality and with reference to the design-principles P1-P3 we formulate the following technology-specific design principles:

- P1\*: The digital tool is used within a real experiment conducted by the students.
- P2\*: The digital tool augments the students experiments with different representations.
- P3\*: The provided data allows students to formulate conjectures and make qualitative experiences with pre-calculus concepts.

### **The Method of Variation inquiry**

This study is guided by the method of the Varied Inquiry (MVI) approach, which is a combination of two theoretical perspectives: the logic of inquiry (Hintikka, 1999) and the variation theory (Marton et al., 2004).

The main idea behind the logic of inquiry involves seeking rational knowledge by questioning (Hintikka, 1999). Hintikka (1999) conceived the process of seeking new knowledge as an interrogative process between two players. The first player (the inquirer) has the role of asking questions, and the second player has the role of answering and is called the verifier (or oracle). The former is the seeker of knowledge who tries to prove a conclusion to be reached from prior experiences or even from theoretical premises. The latter is considered the source of knowledge.

In order to design educational situations that may promote inquiry processes, Arzarello (2016) refers to the variation theory. The variation theory (Marton et al., 2004) defines learning as a change in the way something is discerned, i.e., seen, experienced or understood. According to this theory, meaning emerges as the learner focuses his awareness on the object of learning. In this case, some aspects of the object appear at the forefront of his attention. Yet, not all aspects are discerned at the same time or in the same way. In order to understand an object of learning in a certain way, various specific critical aspects must be discerned by the learner. To facilitate the discerned object of learning, Marton et al. (2004) proposed four interrelated functions (or patterns) of variation to be taken into account when designing educational tasks: (a) contrast: "...in order to experience something, a person must experience something else to compare it with"; (b) generalization: "...in order to fully understand what 'three' is, we must also experience varying appearances of 'three'..."; (c) separation: "In order to experience a certain aspect of something, and in order to separate this aspect from other aspects, it must vary while other aspects remain invariant"; and (d) fusion: "If there are several critical aspects that the learner has to take into consideration at the same time, they must all be experienced simultaneously" (Marton et al. 2004, p. 16).

In the MVI approach, Arzarello (2016) proposes that drawing students' attention to critical aspects, asking to vary them, and observing their effects on the phenomena may foster students' inquiry processes. The main idea of the MVI is creating challenging situations by varying some aspects of

the phenomena (real-world or mathematical) while keeping the others invariant. Exploring various aspects of the same phenomena may lead the students to grasp the intended object of learning.

## Design and Tasks

This paper reports on a first design-cycle of a learning environment and a digital tool using augmented reality in order to develop a learning design for pre-calculus concepts. Such iterative design-cycles are essential within design-based-research approaches (van den Akker et al., 2006; Gravemeijer, 1994; Gravemeijer & Cobb, 2006; Prediger et al., 2015). Within this research paradigm, research and development are strongly connected (Bakker, 2018). A mathematical design consists of learning activities embedded in certain tasks. Hence, the role of the design-principles concerning the tasks is crucial and they should contain “several representations, several kinds of sensory engagement, and several question types” (Watson et al., 2013, p. 10).

<b>Galileo Experiment</b>	
<p><b>Task 1</b> Release the cube from the upper side of the inclined plane. Observe the movement of the cube and answer the following questions. What drew your attention when you observe the cube? Write as many observations as you can. Make a conjecture, which one of the observations will change if the plane inclination changes! Make a conjecture what will change if the time is reduced and give reasons?</p>	<p><b>Task 2</b> Your task is to explore how the change of the plane’s inclination may affect the movement of the cube.</p> <p>A. How will the change of the inclination affect the movement of the cube?</p> <p>B. Change the plane inclination in order to verify or refute the conjectures you raised in (A). Do your conjectures change? If yes, why? If not, prove your conjecture.</p> <p>C. Can you find an equation that describes the movement of the cube movement? Why or why not? Justify your answer.</p>
<p><b>Task 3</b> Your task is to vary the plane inclination in order to explore the motion of the cube. You are requested to explain and justify your answers using the graphic and numeric representations.</p> <p>A. Explain the shape of the curve when the plane inclination changes?</p> <p>B. How do the differences between the y-values of the points on the graphs change when varying the plane inclination?</p> <p>C. Compute the differences of the distance differences. What can you observe? Why? Is your conjecture always true? Can you prove it?</p>	

**Figure 1: Learning environment to explore the Galileo experiment with Augmented Reality**

Based on the formulation of the design-principles above we have developed both elements (task and tool) in a first version. The design principles of the tasks were motivated by the MVI approach. Each task contains a real-world phenomenon and mathematical representations of the dynamic aspects of the phenomenon. The main idea of the design is to create different situations by varying some aspects of the phenomenon while keeping the others invariant (see table 1). For example, the first task may request students to explore the mathematical model of a ball rolling on inclined plane (as in the well-known Galileo experiment). In this situation, the time elapsed and the spaces traversed of the ball vary, and the inclination plane angle is invariant. Hence, the crucial aspect of this task is the time-distance relationship. The second task requests students to explore the same situation as in task 1, but this time the angle of the plane is varied, and the students explore how this new situation

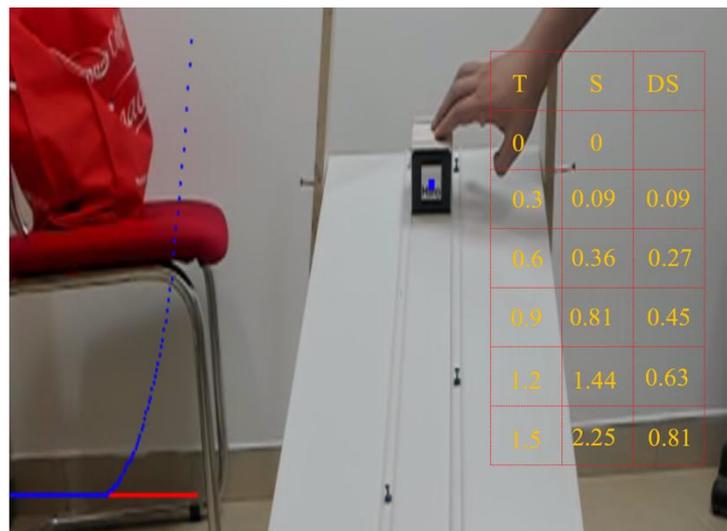
affects the mathematical model. In this case, the students explore quadratic functions based on their graphs and numeric values. Exploring various aspects of the same phenomenon may lead the students to grasp the intended object of learning, namely, quadratic function and its properties.

Task	Mathematics	Crucial aspects
1	Non-propositional relationship	Time-distance relationship
2	Quadratic functions based on their graphs	Parabola, time-distance relationship
3	Quadratic function based on the second differences	Second differences of the space travers, parabola, time-distance relationship

**Table 1: Mathematical ideas, the crucial aspects of the phenomenon**

### First design-cycle: prototype and discussion

This section describes the development of two different prototypes. Based on experiments with the first version of the developed AR-tool we identified various disadvantages of the technology in use. The discussion below highlights some of the features of both versions and discusses the reasons for the technological changes within the developmental process.



**Figure 2: (left side figure, version 1) coded color method for recognizing dynamic object; (right side figure, version 2) Cube movement augmented with graph and table of values**

*First Prototype:* We used MATLAB software in our first attempt to design the AR prototype. MATLAB has the ability to identify the dynamicity of an object and to present it graphically or numerically. To activate this option, the use of a video camera with high resolution is required and the dynamic objects should be color coded. In this way, the specific color given to the dynamic objects is traced and the dynamicity of the physical object is recognized. For example, a mass hanging on a spring was wrapped by yellow coloured paper as well as the beginning and the end of the spring are coded by another colour (figure 2, left side). In this case, a camera followed the coded-colour papers. To recognize the coded colour paper, the users must calibrate and define the color to be traced in each new experiment. For recognizing the color properly—as a result to collect the data of the dynamic object correctly—a uniform distribution of light in the room is required. In

fact, small changes in the light conditions changed the results significantly so that the values and graphs in the different augmented representations showed different results depending on the light conditions of the specific point of view.

*Second Prototype:* The several constraints that are required for the AR prototype to function properly, which are very difficult to ensure, obliged us to look for a new tool to design the AR prototype. We especially looked for tools that are insensitive to light and friendly for users. Therefore, in the second round of the designing, we decided not to use MATLAB in favour of using ARtoolKit library. ARtoolKit is an open-source computer tracking library for creating augmented reality applications that overlay virtual imagery on the real world. The developer's mission was to write an AR code to detect a moving cubic over an inclined plane. The cubic detection is done by using fiducial values or markers. The ARtoolKit provides several methods for tracking and detecting objects in the real world. The most primitive one is the marker-based tracking and detection. We used a specific marker from the markers-collection called "Hiro". The marker is treated as a pre-defined input. The program extracts features from the input marker. Additionally, the program detects these features of the marker in each frame. The markers are attached to the cube in our experiment (figure 2, right side version), so that it allows the detection of the cube movement in real-time (presenting the distance vs. time function) while using a camera. The data is then displayed on the eyeglasses (see figure 2, right side). The type of eyeglass used is HTC VIVE Pro (<https://www.vive.com/eu/product/vive-pro/>). The data of the dynamic object is displayed graphically and numerically. Hence, one sees the real experiment augmented with the data. The data itself is displayed in different representations. Within the evolving graph, one can see a distance-time function graph. The graph itself can be displayed a set of discrete points, or as a continuous function graph. The table shows three columns: time, distance and the difference between two consecutive distances. Both the graph and the table of values are displayed, so that the students can relate and reflect on the relation of the different representation modes.

## **Conclusion and Outlook**

Based on both task- and technological design-principles we discussed a first cycle of the development of a learning design addressing pre-calculus concepts using AR technology. We have reported on the features of an AR-tool with disadvantages in the experimental situations and a second version, which is much more stable in the experimental situations. The outcome of the described iteration is an AR prototype that can be used in order to explore a dynamic real-life situation with mathematical concepts in the field of pre-calculus. The user wears eyeglasses and the data is displayed in different representation modes simultaneously.

The next step will be to conduct empirical experiments with students in order to study their conceptual developments when using the technology and to empirically evaluate the chosen design-principles.

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