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Digital media support functional thinking: How a digital self-assessment tool can help learners to grasp the concept of function

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Developing functional thinking is a central objective in mathematics education. Students especially need to internalize three aspects of functions: mapping, covariation, and object besides learning to translate flexibly between various representations. However, many learners struggle to recognize and interpret functional relationships. To help these students, a formative self-assessment approach with digital media offers new possibilities. Technology can support understanding not only due to the use of multiple, dynamic, interactive, or linked representations, but offer a constraint-support structure to guide students' actions. We present two case studies in form of task-based interviews from the third cycle of a design-based research study that aims at the development and evaluation of the "self-assessment for functional thinking electronic" (SAFE) tool. The cases show how certain technological features of the SAFE tool support learners' self-assessment and functional thinking.

Keywords: Digital media, technology, functions, formative self-assessment, metacognition.

Why support functional thinking?

Many real-world phenomena, such as measuring the temperature at a weather station over a period of time, can be comprehended functionally. Because of this variety of applications and its importance to grasp further mathematical concepts, the discussion of functional relationships is central in many mathematics curricula. For example, it is one of five key content domains in lower as well as upper secondary mathematics education in Germany. Although the function concept is fundamental in the learning of mathematics, many students have difficulties in its comprehension. Thus, they need support to acquire 'functional thinking'. For this reason, we use a formative student self-assessment approach as it has proven to enhance student learning (Stacey & Wiliam, 2013).

Theoretical Background

Formative self-assessment and metacognitive activities

According to Wiliam and Thompson (2008), formative assessment (FA) can be conceptualized in five key strategies. These regard mainly the teacher to be responsible for FA. Especially in computer-based assessment, features such as automatic feedback, a limitation of students' responses by e.g. multiple choice formats, or a focus on measuring achievements (Stacey & Wiliam, 2013), leave little room for students to recognize, reflect upon and react to their work. This is why, the FA framework by Wiliam and Thompson was refined in the EU-project FaSMEd to allow formative self-assessment to be understood as a process performed, observed and reflected on by students. It is conceptualized in these five key strategies (KS): 1) understanding learning intentions and criteria for success, 2) eliciting evidence of understanding, 3) providing self-feedback, 4) activating peers as instructional resources for one's learning, and 5) regulating one's learning process. However, strategy four is only relevant when observing formative self-assessment in classroom situations

(Ruchniewicz, 2017). For all these strategies, but especially for the fifth, learners need to use metacognitive activities. They describe the procedural component of one's metacognition. This means all actions of regulation of one's cognitive activities in learning processes. They include: 1) *planning* problem-solving steps with appropriate mathematical tools, 2) *monitoring* in form of controlling tool-use and comparing what is achieved to set goals, and 3) *reflection* on given problems or understanding of mathematical concepts (Cohors-Fresenborg, Kramer, Pundsack, Sjuts, & Sommer, 2010). To enable students to self-assess their functional thinking abilities, this mathematical content needs careful consideration.

Functional thinking

Functional thinking is a didactical concept, that describes all mental images learners need to build and use when dealing with functions, their representations, and their applications in modelling and problem solving to gain a comprehensive understanding of the function concept. "Functional thinking is a way of thinking, that is typical for working with functions" (translated from Vollrath, 1989, p. 6). Three aspects, noticed in German didactics today as Grundvorstellungen (GVs), characterize this "typical" and allow for various views on functions (Vollrath, 1989): 1) *Mapping*: a function assigns exactly one value of a dependent quantity to the value of an independent one. As such a unique mapping, it is viewed in a local and static way. 2) *Covariation*: in a dynamic view, a function describes how two quantities change in relation to one another. While the independent quantity runs through a set of values from a domain, it causes the values of the dependent quantity to change accordingly. 3) *Object*: a function viewed globally is a mathematical object, that has its own specific properties (e.g. characteristic graph, symmetry) and can be operated upon.

Although the three GVs of functions apply for all forms of semiotic representation, they can appear variously according to a function's visualization. Usually, students interact with functions represented verbally, numerically, symbolically, or graphically. The mapping aspect, for instance, is highlighted when calculating a y-value by inserting the x-value into an equation. By contrast, the covariation aspect is of interest when looking at the change of a graph's values for an interval of x-values. Each representation emphasizes different properties of the function (Duval, 2006). Thus, it is central for functional thinking that students learn to work with and translate between various representations. This leads them to a comprehensive understanding of the function concept (Duval, 2006).

Due to the concept's complexity, such a comprehension and, thus, the acquiring of functional thinking is challenging. Numerous misconceptions are described in regards to learners struggling to conceptualize functions. For example, Clement (1985) states that many students falsely treat graphs as literal pictures of the underlying situations (graph-as-a-picture mistake). Others overgeneralize function types or properties, such as using linear functions in inappropriate situations (illusion of linearity). Further, students might swap the x- and y-coordinates or disregard the uniqueness of a function (e.g. Hadjidemetriou & Williams, 2002; Leinhardt, Zaslavsky, & Stein, 1990). To react to such misconceptions and support students' self-assessment, digital media offer new opportunities.

Digital media

Digital media have the potential to enhance formative self-assessment by altering the assessment process due to changes in, e.g. the nature of tasks, types of feedback, or even assessed skills. What is more, they present new chances for learning by e.g. providing dynamic or interactive representations of mathematical objects (Drijvers et al., 2016). Various studies examine if the use of technology improves achievements. In an ICME survey concerning lower secondary learners, Drijvers et al. (2016, p. 5) summarize: “[t]he overall image is that the use of technology in mathematics education can have a significant positive effect, but with small effect size.” The authors conclude that existing studies give hints for whether achievements can improve, but do not explain why (Drijvers et al., 2016). To answer why digital media can support functional thinking, it is necessary to consider their potential and appraise it against possible risks. Following, we list some of the most important arguments for teaching, learning and assessing functional thinking with digital media:

1) *Fast availability of representations*: Quickly available visualizations of functions leave time for examining functional relationships, generating examples, or checking one’s hypotheses. Yet, the large amount of representations and speed of their availability might lead to a complexity that hinders students to reflect on their actions (Barzel, Hußmann, & Leuders, 2005). Cavanagh and Mitchelmore (2000, p. 118), for instance, identified the tendency “to accept whatever was displayed in the initial window without question” as one of three typical mistakes of 10th and 11th graders asked to interpret linear and quadratic graphs on graphic calculator screens. Students did not reflect upon the visual image on screen or relate it to an inserted algebraic equation (Cavanagh & Mitchelmore, 2000).

2) *Multiple representations*: The aspect of fast availability plays a key role when it comes to screening various representations of the same function at once. Each one stresses different aspects of a function (Duval, 2006). Simultaneous visualizations can support the construction of mappings between those aspects and the translation between representations, which helps learners to solve problems quicker and conceptualize functions (van Someren, Boshuizen, de Jong, & Reinmann, 1998).

3) *Dynamic representations*: In static media, variations of a mathematical object need to be observed, interpreted, and projected upon its representation by the user. Dynamic representations let students experience these changes directly (Kaput, 1992). Especially the covariation GV of functional thinking can, thus, be supported by digital media as the investigation of changes in function values is easier.

4) *Interactivity and linked representations*: Digital media permit „not simply to display representations but especially to allow for actions on those representations“ (Ferrara, Pratt, & Robutti, 2006, p. 242). Moreover, it is possible to link representations so that the variation of one is automatically reflected in another. Such links enable learners to investigate functional relationships as they provide immediate feedback and encourage them to change between representations. Although interactive and linked representations offer great potential to support functional thinking,

the technological speed entails the risk of students being overwhelmed by quick changes in visualizations and driven to act upon them without reflection (Zbiek, Heid, Blume, & Dick, 2007).

5) *Effecting student actions*: Student actions are effected by a digital tool’s design. If learners are asked to draw a function graph, the technology might supply them with a suitable coordinate system. Kaput (1992, p. 526) refers to a tool’s “constraint-support structure” stating that “whether a feature is regarded as one or the other does not depend inherently on the material itself, but on the relation between the user’s intentions and those of the designer of the material and the contexts for its use.”

In our study, these arguments for using digital media to support functional thinking guide the design of a digital self-assessment tool and offer hypotheses for the analysis of students’ learning processes.

Methodology

We use design-based research that aims at developing and evaluating the SAFE tool (Ruchniewicz, 2017; Ruchniewicz & Barzel, in print). Several versions of the tool are designed, investigated, and re-designed. This cyclic process is guided by the following research question: *How do certain technological features of the SAFE tool support students’ self-assessment and functional thinking?*

Design of the SAFE tool

The SAFE tool’s aim is for students to be self-assessors and, thus, no direct feedback is generated. For all tasks, users compare their answer to a sample solution and evaluate on their own whether it is correct or which steps to take next in their learning. The design means to create a balance of providing enough information and autonomy for learners. Therefore, the SAFE tool intends to assess and repeat basic competencies after they have been taught. The learning conjecture is the ability to sketch graphs based on given situations as this reflects a key aspect of functional thinking. The SAFE tool runs as an iPad application and consists of five parts: *Test*, *Check*, *Info*, *Practice*, and *Expand*. These are joined in a hyperlink structure and labelled with various symbols for easy user orientation (fig. 1).

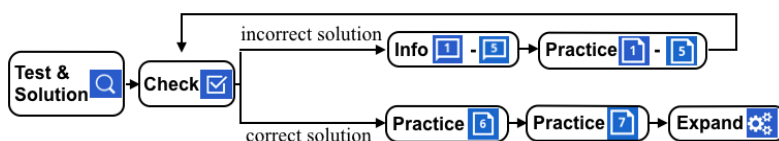


Figure 1: Hyperlink structure of the SAFE tool

Students start by solving the *Test* task, which presents the story of a bike ride and asks them to draw a time-speed graph. They can label the axes using drop-down menus and sketch the graph directly on the screen using their fingers. Afterwards, they move to the task’s sample solution. It consists of a simulation of the situation that is linked to one possible solution graph. The simulation can be started and stopped by the user and a qualitative speedometer can be viewed as well (fig. 3a). The learner moves to the *Check*, that provides five statements regarding important aspects about the graph in question alongside common mistakes that could arise. For each statement, the student decides whether it is true or false for his/her own solution. For this diagnostic step, the *Check*

includes a pictorial visualization of the situation together with a static representation of the sample graph and the learner’s own solution in the same coordinate system (fig. 3b). If an error is identified, the student can choose to read an *Info*, which entails a general explanation to repeat basic ideas about the function concept and specifies them in the time-speed context of the *Test* task, or work on a specified *Practice* task. If the initial graph is correct or a student checks all statements, he/she is presented with two more *Practice* tasks as well as an *Expand* task with a more complex context.

Data collection and analysis

We use case studies in form of task-based interviews with individual students. The interviews are videotaped, transcribed, and analyzed qualitatively. Three subject groups are included: grade 7–9 students (aged 12–15) as they are novices regarding the function concept; grade 10 students (aged 15–16) as they have to repeat curricula contents of years 5–10 for a state-wide assessment; and 2nd semester university students in their bachelor in mathematics education (aged 19–22) as they are experienced, but often need to repeat basic competencies in the transition from school to university.

This paper presents two cases from the third design cycle, which give a first glance of the latest SAFE tool’s potential as data collection is not completed. So far 3 interviews with year 9 and 5 with university students are recorded. One university student solves the *Test* and self-assessment without difficulties. One younger student shows poor knowledge of the content and does not recognize graphs as representations of functions. She is unable to identify her mistake on her own. Six learners show similar self-assessment processes. As the diagnostic step of assessing their solution to the *Test* occurs when viewing either the *Test solution* or *Check*, two of the (university) students were chosen as cases to represent both of these ways of self-assessment. As our focus is assessment, the learner’s work with other tool parts (*Info*, *Practice*, *Expand*) is not included, as it portrays their next steps of learning.

For data analysis, we use qualitative content analysis with deductive categories. Additional inductive categories will be developed once data collection is completed (Mayring, 2000). In relation to our theoretical basis, three main categories with different subcategories were identified to guide analysis:

Student self-assessment	Functional thinking	Technological features (TF)
Key strategies (KS)	GVs	Linked simulation & graph (TF1)
Metacognitive activities (MA)	Change of representations (CR)	Multiple representations (TF2)
	Misconceptions (M)	Constraint-support structure (TF3)

Table1: Categories guiding the case studies’ qualitative content analysis

Results

The *Test* task, that students use for the diagnostic step of their self-assessment, asks them to draw a time-speed graph for this situation: “Niklas gets on his bike and starts a ride from his home. Then he rides along the street with constant speed before it carves up a hill. On top of the hill, he pauses for a few minutes to enjoy the view. After that he rides down and stops at the bottom of the hill.” Paul and Ayse solve the task and describe their reasoning in the interviews (fig. 2). Thus, they elicit

evidence of their own understanding (KS2). Afterwards, they both move to the sample solution (fig. 3a).

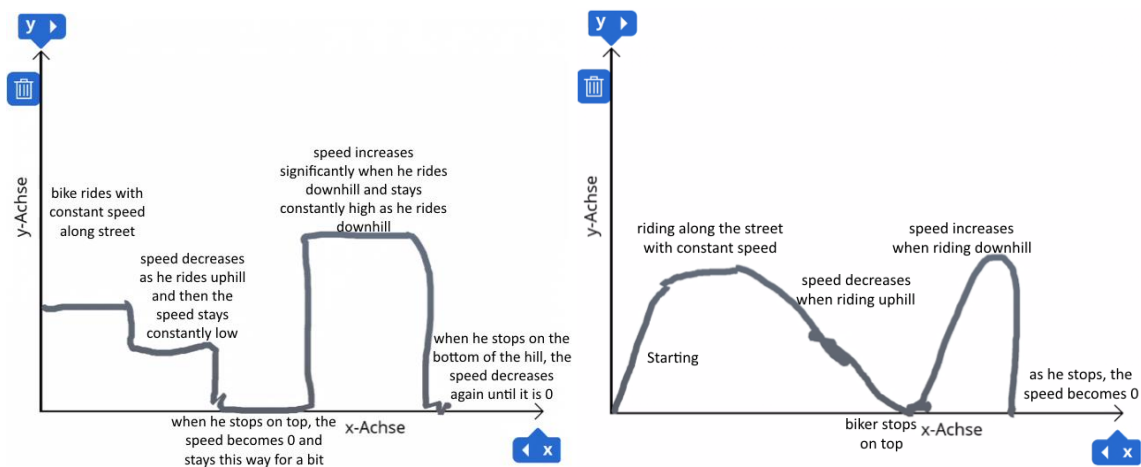


Figure 2: a) Paul's and b) Ayse's Test solution and reasoning

Paul starts the simulation, stops it when the bike starts to ride uphill and says: *“The first thing I saw was that it is assumed that not - (points at the origin in the sample graph) so that he does not start directly with constant speed (points along the first increasing graph segment) but that starting from home is also its own time period. That means that you also have to consider that he has to start first and that he is not at any speed at first.”* Thus, the linked simulation encourages him to reflect his own solution. He identifies a mistake in his answer by realizing that he previously did not model the first part of the situation, namely starting the bike ride from home. Afterwards, he resumes the simulation and stops it again when the bike stops on top of the hill: *“Okay, and then he becomes again, when he rides up the hill, that he does not reach the speed with which he rides uphill at once, but that the speed decreases in shifts.”* Here, Paul reflects part of his answer (riding uphill) by comparing it to the sample graph. He recognizes that the speed and, thus, the graph cannot decrease suddenly, but that it slows down with time. Even if Paul does not realize that his graph disregards the uniqueness of the underlying functional relationship (vertical lines in his solution, fig. 2a), the simulation does inspire him to observe the covariation of the quantities time and speed more closely.

Ayse watches the entire simulation at once and directly moves on to the *Check*. Her screen shows the checklist and a multiple visualization of the bike's path together with her own and the sample solution's graph in one coordinate system (fig. 3b). Ayse reads checkpoint 2: *“I realized correctly when the graph is increasing, decreasing, or remaining constant.”* Relating to this, she assesses her solution: *“Basically yes, but the duration! So with the variable with the time, so I should have much longer (points to the part of her graph that models the bike stopping on top of the hill) along zero – should have gone along the x-axis when he stops for a few minutes.”* Thus, she denies the statement of the checkpoint for her answer and crosses it off (fig. 3b). The checkpoint together with the multiple representation of the functional relationship prompt Ayse to reflect on her solution and discover an error regarding her graph's slope: while she depicted the bike stopping on top of the hill only as one point of her graph reaching the x-axis, the tool reveals that it should be a constant graph segment with the value of zero. Her reflection shows that Ayse did regard the variation and value of

the dependent quantity (speed) while drawing her graph, but she missed to consider the change of the independent quantity (time) as well. The SAFE tool helps her to focus more on the covariation of both quantities.

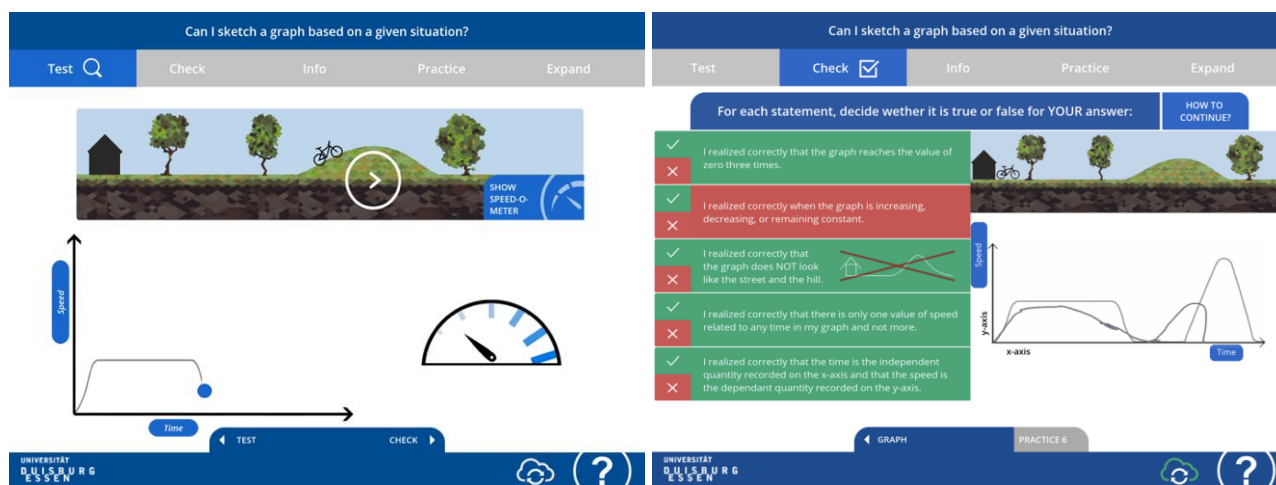


Figure 3: a) Linked simulation and graph as the *Test's* sample solution, b) Ayse's *Check* screen

Conclusion

The two cases of Paul and Ayse show that the SAFE tool can support students' self-assessment and functional thinking. This becomes apparent when considering how the analysis' categories of self-assessment, functional thinking, and technological features interact with each other in the interviews.

Paul's case reveals that an interactive representation of a simulation linked to a graph (TF1) stimulates him to use the metacognitive activity of reflection (MA3). Further, he starts and stops the simulation several times in order to evaluate different aspects of the graph, which shows that his reflection is supported by this tool functionality (TF3). He evaluates his own graph by comparing it to the sample solution and identifies his own mistakes while regulating this process (KS5). Paul realizes that he missed to model part of the situation (start of bike ride) and translate it into its graphical representation (CR). For another part of the situation (riding uphill), he focusses on the covariation of both quantities. While he initially assumes a more prototypical graph with many constant segments (fig. 2a, M), he addresses a non-linear decrease of the graph's slope to model the bike slowing down when looking at the simulation (GV2). In his reflection, he shows an understanding of criteria for success, namely translating all parts of the situation and correctly sketching the graph's slope (KS1). Additionally, he describes how to change his initial work to correct it, thus, giving himself feedback (KS3).

Likewise, Ayse's case shows that the SAFE tool encourages a reflection of her solution (MA3) and points her attention towards the covariation of both variables (GV2). Her reflection is initiated by the *Check's* multiple representation of the situation together with her and a sample graph (TF2) as well as a provided checkpoint (TF3). Ayse grasps that change in both variables needs consideration when sketching graphs (KS1). Finally, she formulates a self-feedback (KS3) and regulates her

assessment process by reflecting her work, identifying a mistake and crossing off the checkpoint (KS5).

Both students use four key strategies of formative self-assessment and the metacognitive activity of reflection when working on technological features, such as multiple, interactive, and linked visualizations. This enhances their functional thinking by shifting the learners' focus towards the Grundvorstellung of covariation, paying attention to the graph's slope, and translating the entire situation into a graphical representation of the underlying functional relationship. Future investigations of students' work with the SAFE tool will reveal whether these findings can be generalized to inform the design of other digital media to support learners' conceptions of functions.

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