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# **A virtual environment dedicated to spatial geometry to help students to see better in space**

Xavier Nicolas and Jana Trgalova

University of Lyon, Claude Bernard University Lyon 1, S2HEP (EA4148), France;

[xavier.nicolas@uca.fr](mailto:xavier.nicolas@uca.fr); [jana.trgalova@univ-lyon1.fr](mailto:jana.trgalova@univ-lyon1.fr)

*We report about an experiment carried out in a school context of teaching spatial geometry with a dynamic geometry environment, based on gesture-based interface for use with immersive, room-scale virtual reality. The work reported in this paper is the first iteration of a research cycle involving teachers, researchers and developers to help high school students better “see” in space and propose principles for the design and use of such environments in a school context. Our study focuses on the impact of “dimensional construction” techniques on the way students perceive a geometric figure represented in a cavalier perspective. Our first results show a significant improvement of students’ performance, demonstrating the appropriation and transfer of these dimensional construction techniques to 2D supports as well as the transition from an iconic to a non-iconic vision of figures, which is an essential condition for geometric activity.*

*Keywords: Mathematics, learning, geometry, visualization, technology*

## **Introduction**

Teaching geometry in space is considered difficult because it is necessary to “see in space,” objects or situations most often represented by drawings in cavalier perspective. In geometry, dynamic geometry environments (DGE) have revolutionized not only the way geometric objects are created and manipulated, but also allowed new didactic situations and modified our very relationship to geometric objects (Laborde, 2003). However, DGE are constrained by a 2D representation mode (screen), which poses perception and usability problems (Dimmel & Bock, 2017). In the domain of technology in education, unlike the professional world, virtual reality (VR) has made a timid entry. A DGE in VR could be relevant to better see in space or at least avoid the problem, but what will happen once the HMD (Head Mounted Display) is removed and the students are back on their tables? An interesting aspect of virtual worlds that can be exploited is their modes of interaction and assistance that are impossible in the physical world as a way of approaching learning differently (Burkhardt, Lourdeaux, & Mellet-d’Huart, 2006). VR can reveal and make tangible concepts or information normally outside the field of human perception (Winn, 2003). This leads us to question whether concrete and intelligible manipulation of geometric objects performed by students in a VR environment can change the way they later see them in cavalier perspective drawings? In this paper, we present a research aiming to evaluate the potential of VR environment in geometry in space, but also to set some benchmarks for academic learning.

## **Theoretical framework**

### **Seeing in space in geometry**

In the context of geometry, the act of seeing involves complex perceptive and cognitive processes. For Duval (2005), a major cognitive problem is how to move from discriminative recognition of

shapes to the identification of objects to be seen. This transition can take two paths: an iconic and a non-iconic path. The iconic path is based on the similarity of the shapes with the object and is not specific to geometry. Seeing in space in an iconic way can also be considered from the perspective of spatial skills as "the ability to generate, preserve, recover and transform well-structured visual images" (Lohman, 1996, p. 98). The non-iconic way relies on considering a representation of figural units linked by relations as a whole. Most of spatial geometry tasks imply this flexibility of vision. The dimensional deconstruction (Duval, 2005), i.e., considering the figural units of smaller dimensions in a geometric object, is the central point for the geometric processing of representations. Following Duval, we start from the premise that getting students to mobilize dimensional deconstruction on a 3D object represented in 2D can help them better "see in space".

According to Chaachoua (1997), to be efficient in geometry, the representations must fulfill three functions: illustration, supporting assumptions and allowing the heuristic change of point of view like the dimensional deconstruction. In spatial geometry, the drawings in cavalier perspective are the most commonly used representations. Mithalal (2014) notes that drawings in perspective struggle to fulfill these functions, and that dynamic geometry environments (DGE) allow them to be partially restored.

In order to tackle our initial question, we need to identify different ways of seeing in space students mobilize in a geometric activity. This involves modelling the ability to "see in space" in an ecological approach, which leads us to choose the anthropological theory of didactics.

### **The anthropological theory of didactics**

The anthropological theory of didactics (ATD, Chevallard, 2012) provides an epistemological framework aiming at the understanding of the ecology of mathematical knowledge. In ATD, any human activity is modelled through the notion of *praxeology* represented by so called "4T-models (T,  $\tau$ ,  $\theta$ ,  $\Theta$ )". A *type of task* (T) and the relevant *techniques* ( $\tau$ ) for solving tasks t of the type T constitute a practical block or *praxis* (know-how). The technologico-theoretical block or *logos* (know why) covers a *technology* ( $\theta$ ) explaining and justifying the technique and a *theory* ( $\Theta$ ) justifying the underlying technology. In the case where the type of tasks T relates to the field of mathematics, we use the term of *mathematical organization* (MO) rather than praxeology. For Duval, "the way of seeing a figure depends on the activity in which it is mobilized" (Duval, 2005, p. 8). From the ATD perspective, we consider the different ways of seeing as intrinsic *elementary praxis* (EP) because, on the one hand, they only exist through their mobilization within techniques of prescriptible types of tasks and, on the other hand, they are present at a level of granularity that the didactics of mathematics can no longer exceed. We propose the following elementary praxis to describe the possible transitions from a figure to a geometric object. EP1 and EP2 follow the iconic way (Chaachoua, 1997), EP3 the non-iconic way (Duval, 2005).

- EP1: Associate the recognized shapes with the appearance of a physical object seen from a certain point of view (identification by natural observation and resemblance).
- EP2: Associate the recognized shapes with a prototypical image of the object available in a mental catalogue (identification by prototypical shapes).

- EP3: Identify figural units and their relationships (identification by processes supported by dimensional deconstruction).

### *The relationship to knowledge*

In ATD, the notion of *relationship to knowledge* (Chevallard, 1992) leads to considering the didactics from the anthropological perspective. An individual's relationship to particular (piece of) knowledge may be different from the relationship that the educational institution has with that knowledge. In the case of the ability to see in space, the student develops his/her personal relationship within and outside the school walls. The analysis of student productions, and in particular of their mistakes with regard to the different EPs mobilized in the techniques used to solve a geometric task, can provide us with information on the student's personal relationship with the notion of seeing in space. At the secondary level, in geometry, as well as more broadly in mathematics, the curriculum clearly aims to develop a deductive approach. From the point of view of the institutional relationship to "seeing in space", EP1 and EP2 are non-compliant and error-prone relationships. It is the EP3 that is targeted by the teaching of geometry in space.

### *The concept of ostensive/non-ostensive*

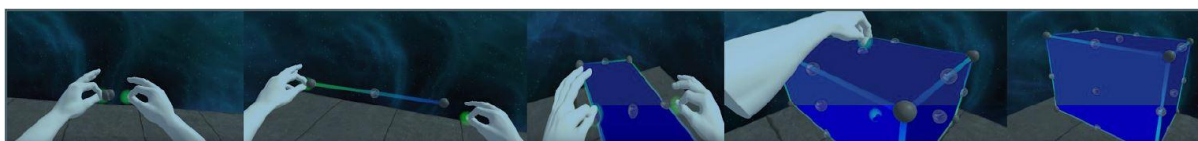
The implementation of a technique involves manipulation of *ostensives*, i.e., tangible and manipulable objects, regulated by *non-ostensives*, i.e., theoretical objects as concepts, notions, ideas (Bosch & Chevallard, 1999). Ostensives can belong to different registers: graphic, gestural, discursive and scriptural. The performance of an ostensive can be evaluated according to two criteria: its *instrumental valence*, which is its ability to act, to work and its *semiotic valence*, which is its ability to see, to appreciate in a sensitive way the work done or to plan the work to be done.

According to Chevallard (1994, p 8), "one of the factors of progress in mathematics is the creation of ostensives that are effective from both an instrumental and a semiotic point of view." (our translation).

We have seen in the introduction that drawings in perspective are ostensives whose instrumental and semiotic valences are not sufficient to develop a non-iconic way of seeing in space.

### **Analysis of *Handwaver*, a DGE in virtual reality**

The *Handwaver* application is developed by a research team at the University of Maine to exploit the modes of representation and interaction available in virtual environments to create experiences where learners use pseudo natural gestures to observe, create and manipulate mathematical objects (Dimmel & Bock, 2017). In *Handwaver*, the "stretch" and the "extrude" operators allow to generate geometric objects of  $n$  dimensions from an object of  $n-1$  dimension (Figure 1).



**Figure 1. Different cases of the “stretch” operator: a point is stretched into a line segment, the segment is stretched into a polygon, and the polygon is stretched into a prism (Dimmel & Bock, 2017)**

As we focus on 3D objects, we consider the generation of a polyhedron from a polygon as a type of task T, made up of several punctual mathematical organizations (PMO), i.e., relating to a single subtask of T. We describe these praxeologies and the involved ostensives in Table 1.

| Type of task T   | Construct a polyhedron from a polygon   |   |
|--|---|---|
| <b>Punctual Mathematic Organization</b>  | Dimensional construction of prisms<br>PMOdim_prism  | Dimensional construction of pyramids<br>PMOdim_pyramid  |
| <b>Subtype of task T</b>   | Construct a prism from a polygon  | Construct a pyramid from a polygon  |
| <b>Technique</b>   | Make a translation of a polygon and join corresponding vertices of the two polygons by segments generating faces of the prism | Extract an apex from the polygon, pull it out of the plane and join it with all vertices of the polygon generating faces of the pyramid |
| <b>Technology</b>  | Definition of a prism: a polyhedron with two polygonal faces lying in parallel planes and with the other faces parallelograms | Definition of a pyramid: a polyhedron with a polygonal base and three or more triangular faces that meet at a point, called apex        |
| <b>Theory</b>  | Euclidean geometry  | Euclidean geometry  |
| <b>Ostensives</b>  | The stretch operator  | The extrude operator  |
| <b>Graphical register</b>  | Virtual model of the prism reacting in real time to the manipulation of polygonal bases                                       | Virtual model of the pyramid reacting in real time to the manipulation of the apex and the polygonal base                               |
| <b>Gestural register</b>   | Grasp and stretch (with hands apart) the polygon  | Touch the polygon with the extrude tool and move away the apex created (with hands apart)   |
| <b>Discursive register</b><br>(later introduced by the teachers of our experiment) | “I stretch the polygon”   | “I pinch the polygon”   |

**Table 1: Two examples of Punctual Mathematical Organizations related to the type of task T**

The a priori analysis of the stretch and extrude ostensives leads us to make the following observations:

- From the point of view of their instrumental valence, the mobilization of pseudo natural gestures makes the techniques easy to apply and retain.
- From the point of view of their semiotic valence, the environment provides immediate feedback on the manipulation of objects in the form of visualization of the polyhedron being generated. This enables the student to control the process of generating polyhedrons, and thus makes the techniques, and more generally the continuity of the process leading from the polygon to the polyhedron, readable and intelligible.

With regard to these two considerations, we hypothesize that these techniques have a strong reversibility potential, i.e., they allow a dimensional deconstruction of the polyhedrons enabling to identify the starting polygon. This leads us to reformulate our initial questioning:

To what extent the use of Handwaver to solve tasks involving dimensional construction techniques help students to move from an iconic (EP1 & EP2) to a non-iconic (EP3) way of seeing drawings in a cavalier perspective?

## Methodology

Our methodology relies on a teaching experiment involving two grade 6 classes and their mathematics teachers. Our aim was to design and test activities with *HandWaver* involving the above mentioned techniques. The methodology comprises several phases outlined below.

### Phase 1: Identification of errors related with seeing in space

We interviewed six secondary school mathematics teachers through two focus groups in two schools to determine types of tasks where seeing in space is problematic for students. We present here two errors that are widespread according to the teachers at the beginning of secondary school and that persist throughout compulsory schooling:

- Error 1 (E1): The student does not consider cubes or rectangular cuboids as prisms.
- Error 2 (E2): The student does not identify properly the base of a prism or a pyramid in a non-prototypical position.

From the point of view of our theoretical framework, E1 and E2 are characteristic of a relationship of knowledge that does not conform to the one expected by the institution. EP1 leads for example to difficulties with perceiving geometric properties shared by different solids as this requires identifying components or figurative units and their relationships, which EP1 does not allow. EP2 comes from observation of real objects and leads for example to always identify the base of a solid as a face on which the solid sits on. E1 and E2 errors prove the non-mobilization of EP3.

### Phase 2: Diagnostic assessment

To investigate these statements, we analyzed the outcomes of a diagnostic assessment administered to 40 grade 6 students (11-12 years old, two classes) who have not had yet teaching of geometry in space. The test consisted of two exercises: classify given solids by family and identify a base for each of the given solids. The solids are represented by drawings in cavalier perspective. E1 and E2 errors were present in all students, confirming the statements collected from the focus group teachers. We note that only EP1 and EP2 are present in students' personal praxeology for these types of tasks. We assume that we can evaluate changes in ways of seeing in space with respect to the presence of these errors in students' productions. We can therefore test the following hypothesis:

The dimensional construction technique makes it possible to develop the EP3 praxis in the students' personal praxeology.

### Phase 3: Design and implementation of tasks in Handwaver environment

With the two teachers, we have designed a scenario to allow students to implement the dimensional construction PMOdim\_prism and PMOdim\_pyramid (Table 1). The scenario outlined below was implemented by each of the two teachers in their 6th grade class of 20 students who passed the diagnostic assessment and whose results are reported above. It should be noted that this was the first session where the notions of prisms and pyramids were introduced at this level of class.



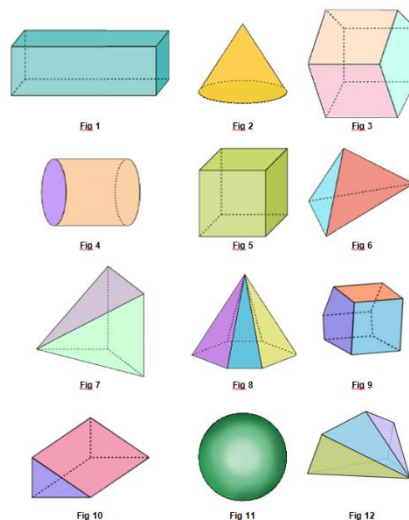
**Figure 2. The classroom setting and the generation of a prism using the stretch operator**

The implementation went through the following phases in the classroom setting shown in Figure 2 (left):

- Introduction of the activity and demonstration of the environment by the teacher
- Generation of solids in the environment by students with the stretch or the extrude operator - one student manipulated at a time, the others observed the screen (Figure 2, right)
- Classification, by each student, of the solids generated on the basis of the techniques used by the group (mental re-mobilization and reversibility of techniques)
- Discussion, argumentation, validation of the students' classifications (development of a discourse on techniques) - whole class
- Institutionalization by the teacher of prisms and pyramids in relation to the techniques used
- Mental reinvestment of the techniques considering a disc (to generate either a cylinder or a cone) - whole-class discussion

#### **Phase 4: Post-test**

A post-test was administered to the students 10 days after the session. As the students were on a school trip during these 10 days, they did not have any mathematics lessons during this period.



**Figure 3. The set of solids from the post-test**

The post-test includes the following two exercises:

- Classify the solids in the families of prisms, pyramids, or other.

- Color a base of the solid on each figure (if possible).

In terms of MO, the type of task underpinning the first exercise is “Identify prisms and pyramids in a set of solids in cavalier perspective“(T1) and the type of task underpinning the second one is “Identify the base of solids in cavalier perspective” (T2). The test includes 12 solids represented in cavalier perspective whose visible faces are colored to avoid perceptual aberrations. Among the solids are 5 prisms including a cube and a rectangular cuboid, 4 pyramids, 1 cone, 1 cylinder of revolution, and 1 sphere (Figure 3). A particular attention was paid to draw some of the solids in a non-prototypical position.

## Findings and discussion

In the analysis of the post-test results, we focus on those that are relevant to the types of tasks T1 and T2. We obtain the following success rates:

- 79% of students correctly solved T1 (68% for prism and 93% for pyramids).
- 75% of students correctly solved T2

If we consider only the object in the post test who can generate the errors E1 and E2:

- Error 1 (E1) 70% of students correctly solved T1 (Fig:1, 2)
- Error 2 (E2): 59% of student correctly solved T2 (Fig: 3, 4, 7, 9, 10, 12).

Even if we have not submitted a real pre-test, only a diagnostic assessment, the very significant decrease in the presence of E1 and E2 in student production seems indicate the non-mobilization of EP1 or EP2. The new technique used by student is more relevant, that could indicate the mobilization of EP3 and could be link to the PMOs used in the VR environment. This seems to confirm our hypothesis and the a priori analysis of stretch and extrude ostensives in terms of their instrumental and semiotic valence. However the solids generation by extrusion is not new, it is often used in an imaginative way by teachers or in CAD software used by students in technology teaching. Yet mistakes persists. The novelty in VR lies in the use of a behavioral interface that affects the graphical and gestural registers of the ostensive. We must consider how cognition operates within constraints imposed by our physiology. First about the graphical register, artificial environments can use computer technology to create metaphorical representations in order to bring to students concepts and principles that normally lie outside the reach of direct experience. (Winn, 2003). Secondly about the gestural register, in the embodied cognition approach, the mathematical concept of dimensional construction that we mobilized in our experiment, could be considered as a conceptual metaphor (Nunez, 2009), grounded in bodily-based mechanisms : the stretch gesture like stretching gum from a ball to a thread (from a 0D to 1D object), or pulling a blind down (from a 1D to 2D object). So we can suppose in our case that students can easily accept that stretching a polygon makes a 3D object so they can mobilize spontaneously this technique later.

## Conclusion

The VR activity implemented seems to have helped students develop a heuristic understanding of drawings in cavalier perspective, based on the identification of figurative units of smaller dimensions, and thus helping to better see in space. In a VR environment we call presence the belief that you are “in” the artificial environment, not in the laboratory or classroom interacting with a



computer. The coupling effect leads to a permeability at the cognitive level between the real world and the artificial world. An environment considered as real by users but where we program the interactions and rules, open a field of great opportunities for learning.

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