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Investigating students' use of dynamic materials addressing conceptions related to functional thinking

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This paper reports on a qualitative case study about the use of dynamic worksheets addressing typical difficulties and misconceptions concerning functional thinking. The dynamic worksheets designed in this project visualize the transfer between iconic situational and graphical representations and were utilized during an intervention with students in a 7th grade Austrian middle school class. In addition to observational data, diagnostic tests and interviews regarding students' conceptions in the field of functional thinking were conducted. The case study particularly pays attention to the intuitive conceptions of students, the influence of the dynamic worksheets on these conceptions, and whether or not these materials are able to support students in developing appropriate mathematical conceptions. In this paper, an overview of the study and the part of the results related to students' use of dynamic worksheets are presented.

Keywords: Functional thinking, technology, representational transfer, lower secondary school.

Introduction

Functional thinking is an important concept in mathematics education and various students' problems and (mis-)conceptions have been widely researched in this context. The development of technology and technology-based resources now offers new opportunities with regard to multiple, dynamically linked representations. The question arose, whether such materials are able to support students in an early stage of learning functions as well as whether and in what ways they may influence students' conceptions. Based on these considerations, I developed several dynamic worksheets that visualize the representational transfer between iconic situational and graphical representations and integrated them into a qualitative case study to investigate this question.

Theoretical background

Literature review reveals various problems and misconceptions in the field of functional thinking; for example, illusion of linearity (De Bock, Van Dooren, Janssens, & Verschaffel, 2002), problems related to representations of functions such as graph-as-picture errors and slope-height confusions (e.g., Clement, 1989), and difficulties related to aspects of functional thinking whereby particularly the dynamic co-variational aspect is difficult for students to comprehend (Goldenberg, Lewis, & O'Keefe, 1992). These problems can cause misinterpretations of functions and particularly of function graphs. In this study, I follow Vollrath's (1989) description of functional thinking mainly including relational and co-variational aspects, because it best fits for the purpose of my study.

Dynamic Mathematics Software (DMS) allows to examine multiple, dynamically linked representations because changing one representation immediately affects the other(s). Therefore, it may support students' development of functional thinking because it is suitable to emphasize relational as well as co-variational functional aspects, the latter providing a dynamic perspective on functional dependencies (Falcade, Laborde, & Mariotti, 2007). So far, quantitative studies in the

field of technology reveal at best moderate effects on students' achievement, but research related to dynamic representations appears to be more promising (Drijvers et al., 2016; Hoyles, Noss, Vahey, & Roschelle, 2013).

At present, Cartesian coordinates seem to be dominant for representing function graphs in mathematics learning and teaching. Another way to represent functional dependencies is based on a software called DynaGraph providing two parallel lines instead of perpendicular axes (Goldenberg et al., 1992). According to Goldenberg et al. (1992), dynagraphs are powerful tools to examine characteristic properties of functions, to analyze functions qualitatively and therefore to support the understanding of functions. For instance, it is not possible to perceive graphical representations as static "images", which could lead to graph-as-picture misinterpretations. Dynagraphs are able to complement static representations by adding dynamic perspectives, thus enriching the conceptual development of students. However, we need to examine in more detail the influence of dynamic materials (utilizing Cartesian as well as parallel coordinates) on students' conceptions.

Several researchers (e.g., Vosniadou & Vamvakoussi, 2006) suggest introducing mathematical concepts at an earlier stage in mathematics education in order to help students building a diverse concept image and to avoid that intuitive conceptions develop to misconceptions. This led to my research interest in students in an early phase of learning functions. In Austria, this group is represented by students in grade 7 and in the beginning grade 8 (age 12 to 13), because they have already learned how to interpret Cartesian coordinates. Therefore, they are able to study graphical representations, but they are not accustomed to the concept of the function and have only little experience with different functional relationships.

Dynamic materials

For creating dynamic worksheets, I decided to use the DMS GeoGebra, because it provides the possibility of dynamically linked, multiple representations, it is widely used in Austrian schools, and it is an open-source and free software. Moreover, I focused on the representational transfer between iconic situational and graphical representation due to the age and prior knowledge of the participating students, and additionally because it is particularly problematic (Bossé, Adu-Gyamfi, & Cheetham, 2011). An English version of the worksheets can be retrieved here: <https://ggbm.at/ftqpETqJ>.

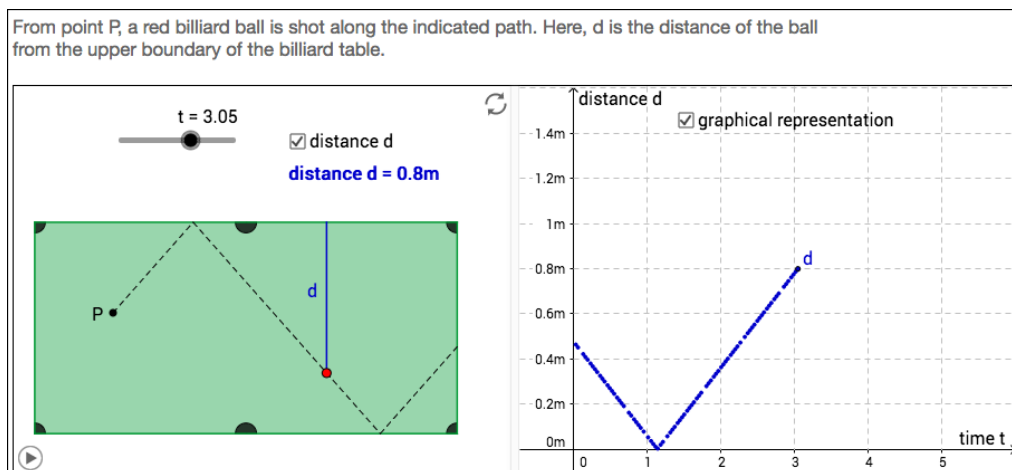


Figure 1: Dynamic worksheet “Billiard”, <https://ggbm.at/Dvf9Pwxj>

Figure 1 is based on a task by Schlöglhofer (2000) and illustrates a typical example of the dynamic materials designed for this project. Each dynamic worksheet concentrates on problems concerning functional thinking described in literature and consists of an interactive GeoGebra applet, which presents an iconic situational model and a graphical representation of a specific example (Cartesian coordinates or dynagraphs). Dynamically linked and interactive representations should address aspects of functional thinking.

Research design

The literature review drew my attention to several research questions (Lindenbauer & Lavicza, 2017). In this paper, I discuss a part of the results related to the following aspect: What kinds of influences of dynamic materials exist on lower secondary students' conceptions concerning functional thinking?

Two 7th grade classes of a middle school in Austria including 28 students with a broad range of different achievement levels participated in the study. I chose a qualitative inductive approach, more exactly Eisenhardt's (1989) approach that integrates features of Grounded Theory in a case study design in which the students represent different cases. Several types of data were collected in this study by diagnostic tests, diagnostic interviews, students' worksheets on paper, and observations during the intervention. These methods were arranged to a research design including five data collection stages: (1) diagnostic test 1, (2) diagnostic interviews, (3) intervention, (4) diagnostic test 2, and (5) diagnostic interviews (Lindenbauer & Lavicza, 2017). During the three-lesson-intervention, students worked in pairs with the designed dynamic worksheets without teacher guidance, because first I wanted to examine the influence of these interactive materials. Ten students were audio- and videotaped when working and the screens of their laptops were recorded, and I supervised these students in my role as researcher. The other students were working in another school's computer lab supervised by their mathematics teachers. Both teachers were instructed before the intervention process in order to guarantee that the participants were not influenced in their learning processes by the teachers' help.

Results concerning dynamic materials

For analyzing data, I followed coding procedures related to Grounded Theory (initial, focused, and theoretical coding). Various categories emerged from the common analysis of data from the second diagnostic tests and interviews as well as observation data. For this paper, I chose to outline results concerning students' use of dynamic materials and representations. Further results will be presented in upcoming papers.

As can be seen in Figure 2, when students worked with dynamic materials, they observed what happened within the dynamically linked representations, either more actively when *manipulating* dynamic representations (e.g., by moving sliders) or more passively by *observing* an animation. The variety and number of codes assigned to data imply an influence of students' *achievement level* on how different students utilized dynamic worksheets; in essence, higher achieving learners appear to

use dynamic materials in more diverse ways. For instance, only Harald¹ and Wolfgang, the highest achieving students who I grouped together during the intervention, utilized the trace mode to mark extrema of the functional dependency within a dynagraph representation (“Billiard dynagraph”, see <https://ggbm.at/hSP3H9wv>) and to compare with other function values. Lower achieving students Konstantin and Mario, however, mainly worked with the same dynamic worksheet passively by watching the animation. Possibly, such students’ behavior could be related to the use of visual representations in problem-solving processes. Stylianou and Silver (2004) compared experts and novices and found out that the former utilize visual representations in more diverse ways.

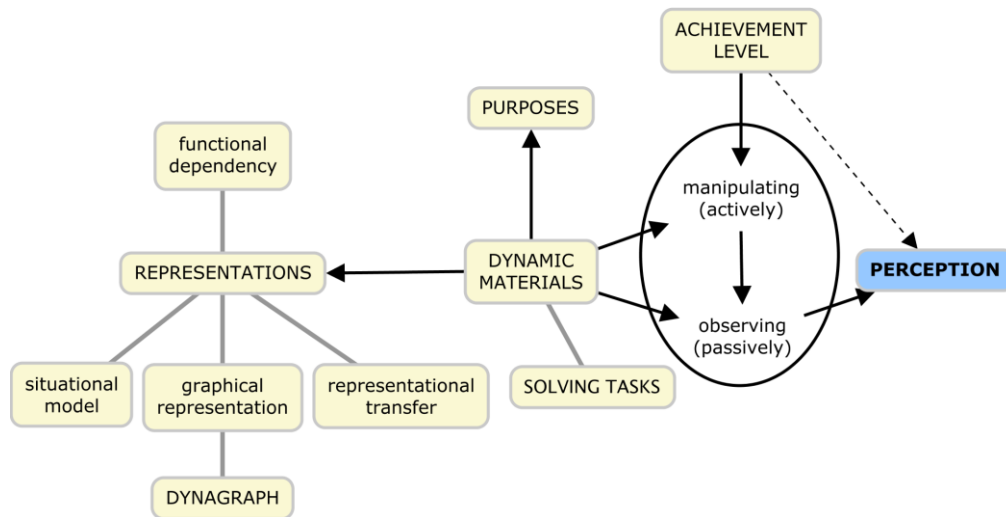


Figure 2: Dynamic materials and related issues

The way of utilizing dynamic worksheets influenced what students observed and consequently what they perceived. *Perception*, which in this context means that students consciously observe a feature and further consider or interpret it, evolved as main category when analyzing data related to the research question (see Figure 2). In brief, data analysis indicates an intention-reality discrepancy between the mathematical content the dynamic materials are intended to visualize and what students really perceive, especially when students focus on visual features of dynamic worksheets (Lindenbauer, 2018). Due to space limitations this issue will not be outlined further in this paper.

As the dynamic materials designed for this project address the transfer between iconic situational and graphical *representations* and visualize functional dependencies, these topics also emerged during the data analysis (see Figure 2). They mainly contain several students’ activities such as manipulating and observing dynamic representations, describing variables or functional dependencies, relating variables and representations, and interpreting graphical representations. Results suggest that for solving tasks, students tend to rely on the more familiar representation – the situational model – if possible. Particularly higher achieving students utilized representations more flexibly while lower achieving learners seemed to focus on one representation. Research results about connections between a flexible use of representations and students’ achievement levels and

¹ All student names are pseudonyms to preserve anonymity.

experience support these observations (Acevedo Nistal, Van Dooren, & Verschaffel, 2014; Stylianou & Silver, 2004).

Potentials and difficulties of working with dynagraphs

During the intervention, students discussed two dynamic worksheets that first utilized Cartesian coordinates and within a second version dynagraph representations instead (“Billiard dynagraph”, see <https://ggbm.at/hSP3H9wv>, and “Triangle dynagraph”, see Figure 3). At this time, they were not used to dynagraphs, and included paper worksheets only provided a brief explanation.

Intervention data reveals that *dynagraphs* are intuitive for students to work with when solving tasks addressing relational as well as co-variational aspects of presented functional dependencies (e.g., to describe co-variational behavior of functional dependencies, to read off function values) and, thus, provide easy access to graphical representations. For example, during an interview Hannah applied the dynagraph representation within worksheet “Triangle dynagraph” (see Figure 3) correctly for answering questions addressing relational aspects (e.g., minimum of the function) while she had difficulties in answering similar questions by interpreting Cartesian coordinates (Lindenbauer, 2018).

Furthermore, dynagraph representations appear to emphasize co-variational aspects of the presented functional dependency probably because the function value y “moves” along the upper axis when the x value changes. Participants repeatedly described the dependency of function values from arguments in terms of speed and similarly observed the speed of the points representing x and y values relative to each other. Additionally, data implies that this representation also stresses rates of change, which can be experienced directly from changing speed of these moving points. One example provided by Pia, a student between average and low achieving level with lack of understanding of Cartesian coordinates, who rather described the functional dependency in dynamic worksheet “Billiard dynagraph” focusing on co-variational aspects than she did when interpreting Cartesian coordinates. However, students’ focus on speed could also lead to a confusion of variables, especially the x variable with time. In sum, dynagraph representations actually appear to provide a more dynamic perspective on functional dependencies than Cartesian coordinates and enable a more profound examination of the co-variational aspect of functional dependences as already outlined by Goldenberg et al. (1992). In addition, dynagraphs emphasize specific properties of functions including inflection or fixed points. Several students implicitly recognized such points without being asked. When asked to describe how the function value changes when x is increased within

The dashed line can be moved left and right. F indicates the area of the blue shaded figure. The graphical representation uses two parallel coordinate axes - a so called dynagraph representation - in the lower graphics window.

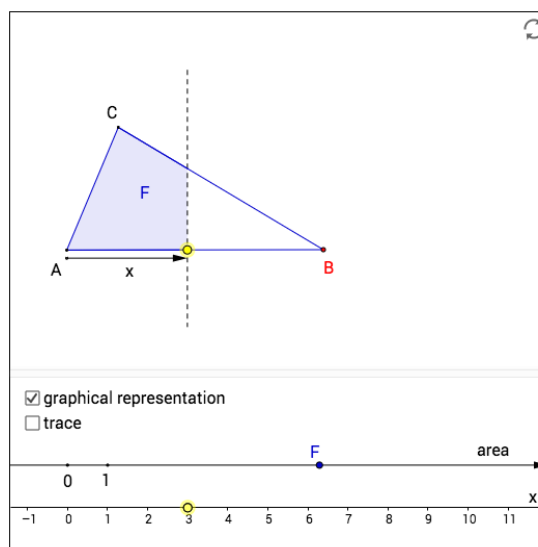


Figure 3: Dynamic worksheet “Triangle dynagraph”, <https://ggbm.at/wrVPlwHE>

dynamic worksheet “Triangle dynagraph” (see Figure 3), Franziska, for instance, implicitly observed the inflection point of the presented functional dependency. She replied:

56 Franziska: Here it [area] is increasing faster (she moves the x value from $x = 0$ to $x = 1$) . . . and then slower again.

57 Interviewer: Do you have any idea from where it slows down?

58 Franziska: So, from here on (She moves dashed line to vertex C).

As the dialogue reveals, Franziska identified the inflection point from which on the change of rate decreases. Apparently, the “speed” of the point representing the function value relatively to those representing the argument x can be easily perceived. For these reasons, characteristic features of dynagraphs could be utilized in mathematics teaching to support students in accessing graphical representations, possibly in scaffolding the representational transfer to Cartesian coordinates.

Although students had no difficulties in recognizing dynamic properties of dynagraphs, they did not always interpret them correctly. Konstantin, a lower achieving student, recognized the inflection point as position where the increase of speed changes similar to Franziska (see previous paragraph). However, he related this change of speed to the decreasing non-colored area within the triangle (see Figure 3) when dragging the dashed line from vertex C to the right. In addition, students tried to interpret visual features, such as the denseness of points created by trace mode on the y-axis, similar as they did when working with Cartesian coordinates.

A further student problem, which I call a “projecting error”, appeared when reading off function values. Following solution from Tina when working with dynamic worksheet “Billiard dynagraph” exemplifies this error. Tina described when trying to answer the question about the distance of the ball at $t = 1$ second:

Tina: At one second it [the distance] is about 0.2.

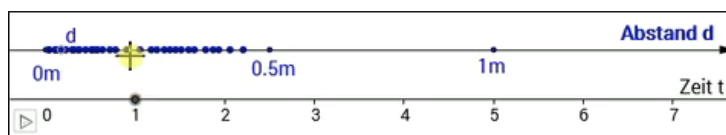


Figure 4: Screenshot of students’ work with dynamic worksheet “Billiard dynagraph”

Figure 4 illustrates that Tina correctly adjusted the value for time on the lower axis to $t = 1$. However, she did not identify the correct function value of about less than 0.1 m (possibly due to the distracting influence of the trace). Instead, as the yellow shaded cursor indicates, she replied the function value exactly above $t = 1$, which would actually be about 0.2 meters.

Dynamic materials

Data related to dynamic worksheets was additionally analyzed separately from other intervention data to provide further insight. On the one hand, in essence the structure of categories manipulating, observing, and interpreting perceived features also evolved from this data analysis and thus triangulates the results. On the other hand, data revealed further typical students’ activities when solving tasks and purposes of dynamic worksheets (see Figure 2). As these results are not specific

for this research project but typical activities when working with computer-based materials or technology in general (e.g., Artigue, 2002), I will not provide further details.

Potentials and problems of dynamic materials

This issue summarizes further potentials and problems of working with these dynamic materials. First, they can be supportive within the first steps students have to conduct when translating from iconic situational to graphical representations, namely for understanding and visualizing a situation as well as for identifying and describing the dependent variable. Second, results related to another research question in this project revealed a comprehension gap as obstacle for students to overcome during this transfer. Data analyses indicate that if students are basically not able to understand and interpret graphical representations in Cartesian coordinates, without teacher guidance students could rather not manage to overcome this comprehension gap by working with the dynamic worksheets alone. Apparently, they would profit from teachers' assistance to help them reflect and reconsider their perceptions and interpretations.

Third, for students who are basically able to make representational transfers, dynamic materials seem to be helpful, because they appear to induce adaptations of students' conceptions. When first confronted with the dynamic worksheet "Triangle" (for situational model see Figure 3), Wolfgang and his colleague represented the functional dependency by a piecewise linear function (see Figure 5). After the second test, in which both students solved a corresponding task correctly, Wolfgang resumed:

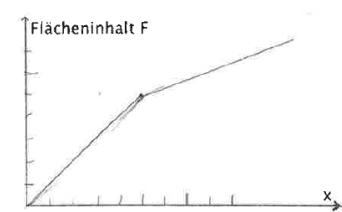


Figure 5: Students' solution

76 Wolfgang: . . . because at first, I thought that it [the area] regularly becomes larger, but then I found out that it grows increasingly.

Apparently, this dynamic worksheet supported Wolfgang in adjusting his conception from regular to irregular increase of the function value and, consequently, from a linear to a curved graph.

Conclusions

The data analyses reveal *perception* as main category of working with dynamic worksheets. In particular, students appear to focus on visual or structural features of iconic situational and graphical representation. Perception seems to depend, among others, on students' achievement levels and their prior knowledge. Therefore, the problem remains how to draw students' attention to those features that are relevant for mathematical learning.

Dynagraphs provide an intuitive access to graphical representations and seem to be easy for students to interpret. Minor difficulties have to be considered when introducing dynagraphs in school, mainly concerning reading off function values. Finally, these results could be influenced by the order of dynamic worksheets utilized during the intervention. Possibly letting students first work with dynagraph representation and then with Cartesian coordinates would provide further insight.

Depending on students' prior conceptions and their understanding of Cartesian coordinates, the dynamic materials utilized in this project have an adaptational influence on students' conceptions.

In other words, they appear to support students in improving descriptions of functional dependencies (e.g., leading to qualitative correct ideas about changing slopes of function graphs). However, these materials could also generate new misconceptions, especially when only superficially perceived. Lower achieving students in particular would profit from teachers' assistance to reflect their perceptions and interpretations.

Additional questions arose from this project, for example, how to improve these materials and how to utilize them in regular teaching to support students. Design experiments and instrumental approach respectively could be suitable frameworks to investigate these questions further in other studies.

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