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Topology and field strength in spherical, anelastic dynamo simulations

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ABSTRACT

Dynamo action, i.e. the self-amplification of a magnetic field by the flow of an electrically conducting fluid, is considered to be the main mechanism for the generation of magnetic fields of stars and planets. Intensive and systematic parameter studies by direct numerical simulations using the Boussinesq approximation revealed fundamental properties of these models. However, this approximation considers an incompressible conducting fluid, and is therefore not adequate to describe convection in highly stratified systems like stars or gas giants. A common approach to overcome this difficulty is then to use the anelastic approximation, that allows for a reference density profile while filtering out sound waves for a faster numerical integration. We present the results of a systematic parameter study of spherical anelastic dynamo models, and compare them with previous results obtained in the Boussinesq approximation. We discuss the influence of the stratification on the field geometry and the field strength, and also compare the different scaling laws for the velocity amplitude, the magnetic dissipation time, and the convective heat flux.

1. ANELASTIC MHD EQUATIONS

- The reference state: decomposition of the thermodynamics variables into the sum of a steady variable corresponding to the reference atmosphere and a convective disturbance $f = f_a + f_c$. The reference state must be in mechanical and thermal quasiequilibrium, defined by :

- hydrostatic balance

$$-\nabla P_a + \rho_a \mathbf{g} = 0$$

- “well-mixed” isentropic reference state

$$\nabla S_a = 0$$

Then, $\bar{P} = P_c \zeta^{n+1}$, $\bar{\rho} = \rho_c \zeta^n$, $\bar{T} = T_c \zeta$,

with $\zeta = f(r, N_\varrho, n, \chi)$

- Navier-Stokes equation

$$D_t \mathbf{v} = Pm \left[-\frac{1}{E} \nabla \frac{P'}{\zeta^n} + \frac{Pm}{Pr} Ra \frac{S}{r^2} \hat{\mathbf{r}} - \frac{2}{E} \hat{\mathbf{z}} \times \mathbf{v} + \mathbf{F}^\nu + \frac{1}{E \zeta^n} (\nabla \times \mathbf{B}) \times \mathbf{B} \right]$$

$$F_i^\nu = \zeta^{-n} \partial_j \left[\zeta^n \left((\partial_i v_j + \partial_j v_i) - \frac{2}{3} \delta_{ij} \partial_k v_k \right) \right]$$

- Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla^2 \mathbf{B}$$

- Heat transfert equation

$$D_t S = \zeta^{-n-1} \frac{Pm}{Pr} \nabla \cdot (\zeta^{n+1} \nabla S) + \frac{Di}{\zeta} [E^{-1} \zeta^{-n} (\nabla \times \mathbf{B})^2 + Q^\nu]$$

with $Q^\nu = 2 [e_{ij} e_{ij} - \frac{1}{3} (\nabla \cdot \mathbf{v})^2]$

together with the constraints

$$\begin{aligned} \nabla \cdot (\zeta^n \mathbf{v}) &= 0 \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned}$$

Our numerical solver PARODY reproduces the anelastic dynamo benchmark (Jones et al. 2011).

2. CONTROL PARAMETERS

The system involves 7 dimensionless numbers:

$$\begin{aligned} Ra &= \frac{GMd\Delta S}{\nu \kappa c_p} \in [10^4, 10^7], \quad Pr = \frac{\nu}{\kappa} \in [1, 2], \\ Pm &= \frac{\nu}{\eta} \in [1, 5], \quad E = \frac{\nu}{\Omega d^2} \in [10^{-3}, 10^{-5}], \\ N_\varrho &= \ln \left(\frac{\rho_i}{\rho_o} \right) \in [0.1, 3.5], \\ n &= 2, \quad \chi = \frac{r_i}{r_o} \in [0.35, 0.60]. \end{aligned}$$

3. THEORETICAL ISSUES

- use of a non steady reference state
- consistency of the different variants of the anelastic approximation
- averaging the heat transfert equation

$$\rho T D_t S = \nabla \cdot (\rho T \kappa^t \nabla S) + \nabla \cdot (K \nabla T) + Q^\nu$$

4. OUTPUT PARAMETERS

- Scaling laws

- $Nu^* = (Nu_{\text{bot}} - 1) \frac{E}{Pr}$
- $Ra_Q = (Nu_{\text{bot}} - 1) \frac{Ra E^3}{r_o^2 Pr^2}$
- dynamo efficiency $f_{\text{ohm}} = D/P$, where P is the power released by buoyancy and D is the ohmic dissipation

- Field topology

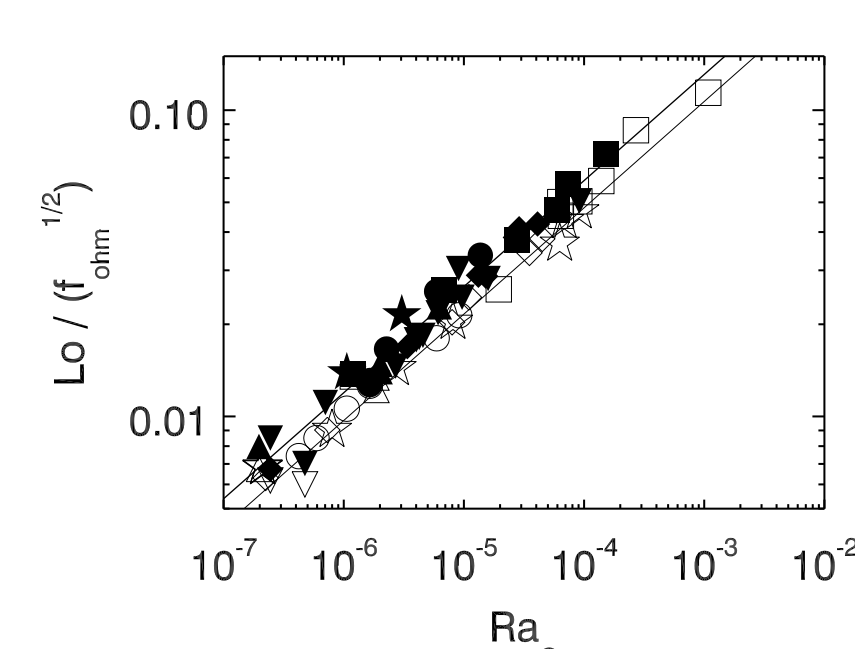
- kinetic energy density E_k
→ $Ro = \sqrt{2E_k} E / Pm$
- magnetic energy density E_m
→ $Lo = \sqrt{2E_m} E / Pm$
- local Rossby number $Ro_\ell = Ro_c \ell_c / \pi$
 ℓ_c stands for the mean harmonic degree of the velocity component \mathbf{v}_c from which the mean zonal flow has been subtracted

$$\ell_c = \sum_\ell \ell \frac{\langle (\mathbf{v}_c)_\ell \cdot (\mathbf{v}_c)_\ell \rangle}{\langle \mathbf{v}_c \cdot \mathbf{v}_c \rangle}$$

- modified tilt angle of the dipole

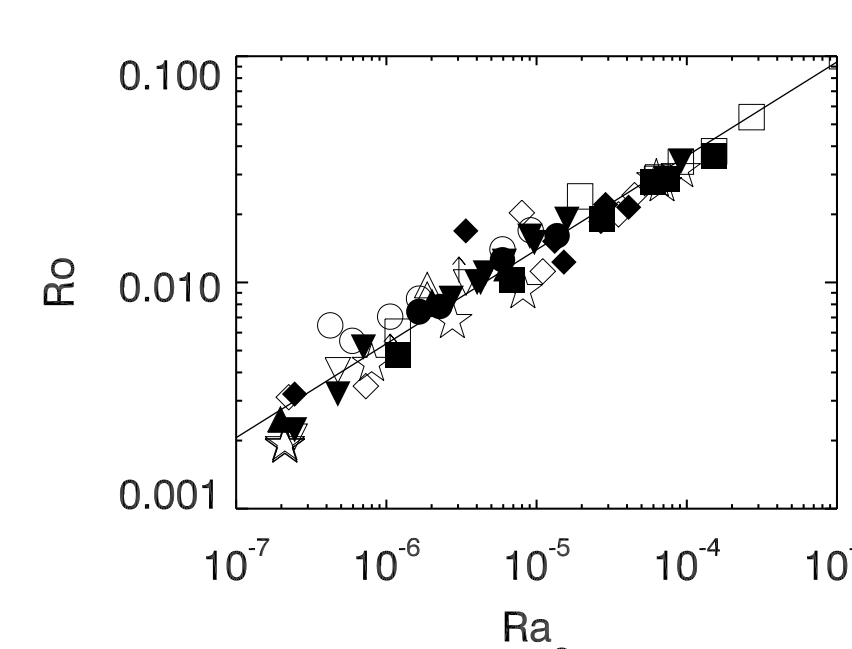
$$\Theta^* = \frac{2}{\pi} \left\langle \sqrt{\left(\Theta(t) - \frac{\pi}{2} \right)^2} \right\rangle_t$$

5. SCALING LAWS



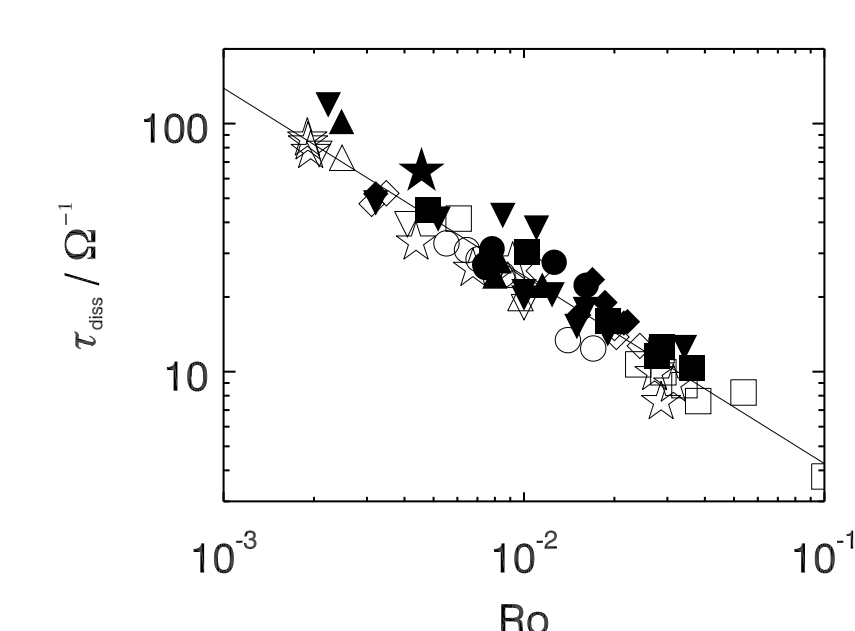
Field strength

$$\frac{Lo}{f_{\text{ohm}}^{1/2}} = \begin{cases} 1.58 Ra_Q^{0.35} \\ 1.19 Ra_Q^{0.34} \end{cases}$$



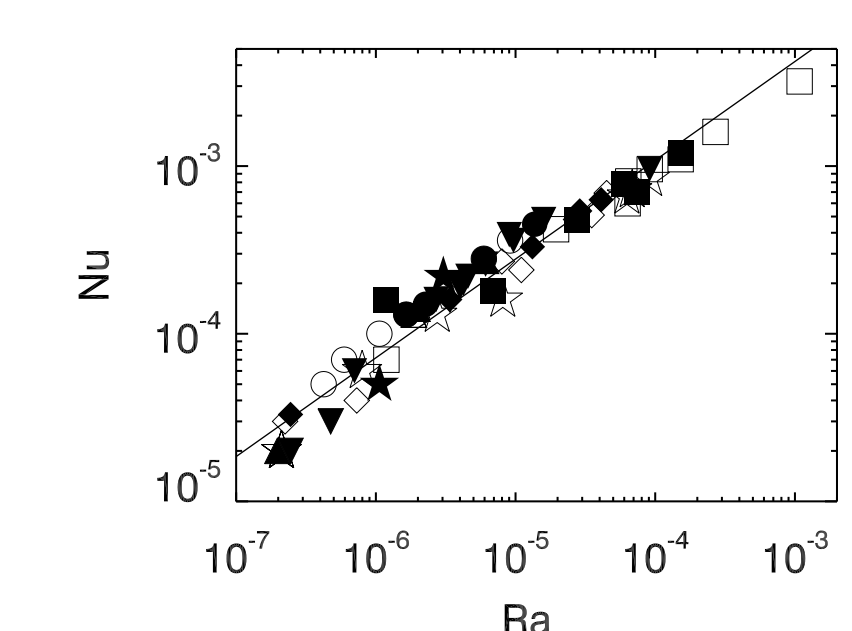
Velocity

$$Ro = 1.66 Ra_Q^{0.42}$$



Ohmic dissipation time

$$\tau_{\text{diss}} = 0.75 Ro^{-0.76}$$



Heat flux

$$Nu^* = 0.25 Ra_Q^{0.59}$$

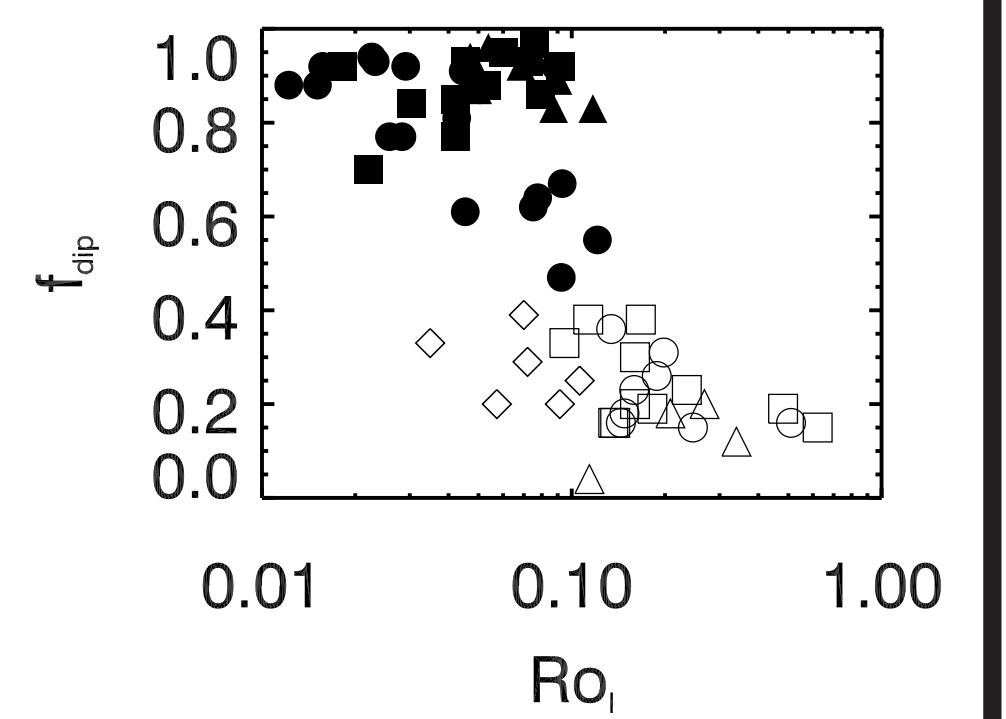
6. FIELD TOPOLOGY

- Dipolarity

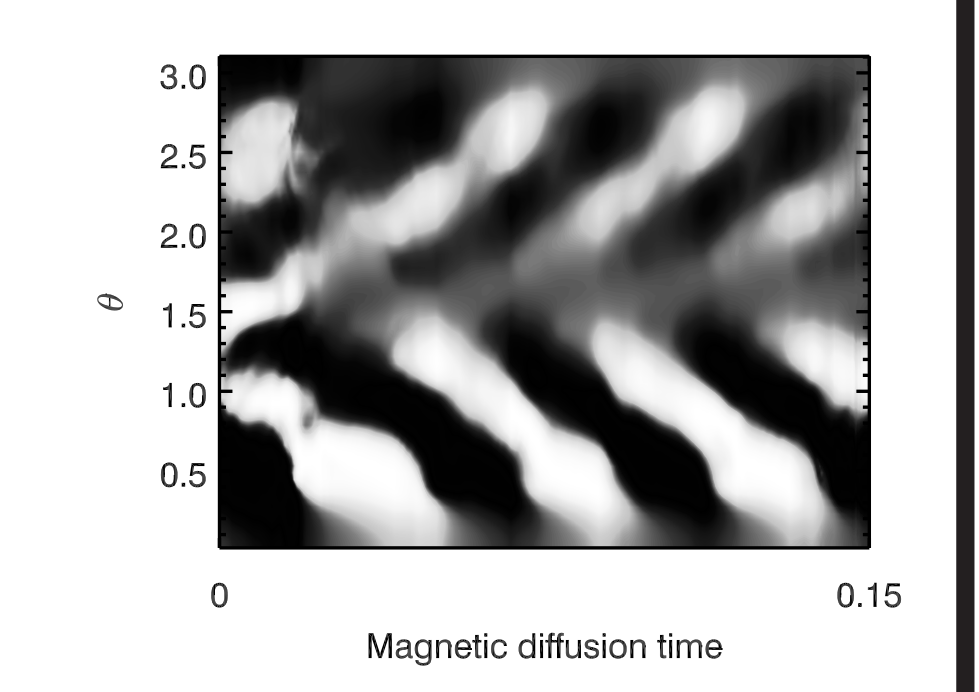
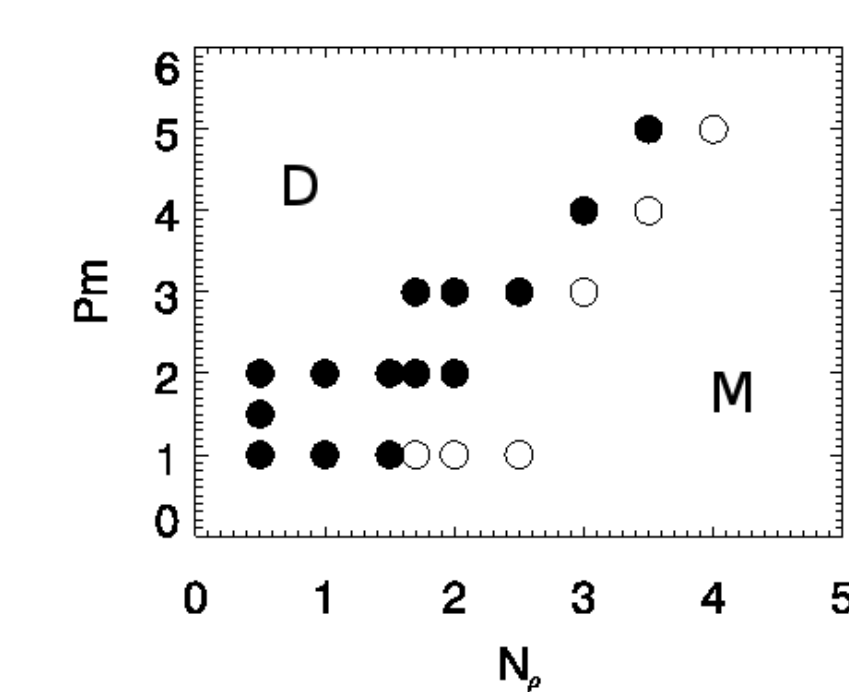
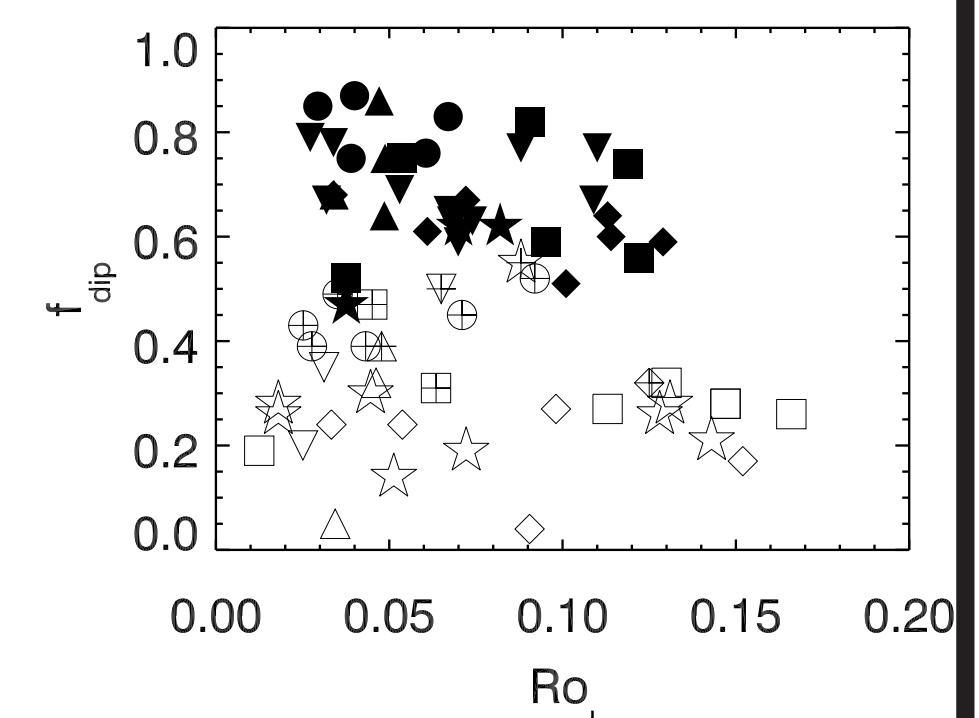
Boussinesq

f_{dip} vs. Ro_ℓ

(Schrunner et al., 2012)



- Anelastic (crosses indicate the presence of a strong equatorial dipole component)

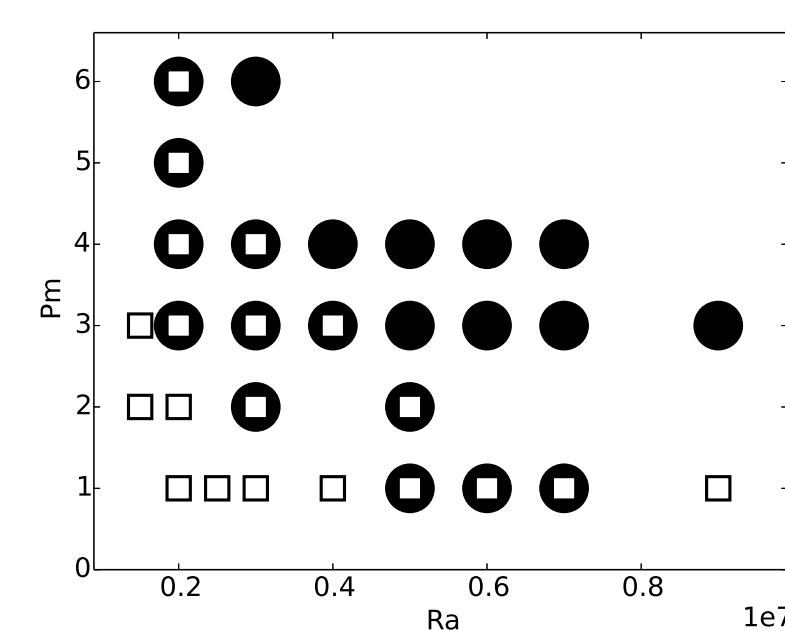


Pm vs N_ϱ
($E = 10^{-4}$, $Pr = 1$, variable Ra)

$\langle B_r \rangle_\phi$
vs. time and colatitude
($E = 10^{-4}$, $Pr = 2$, $Pm = 4$, $Ra = 5 \times 10^6$, $N_\varrho = 3$)

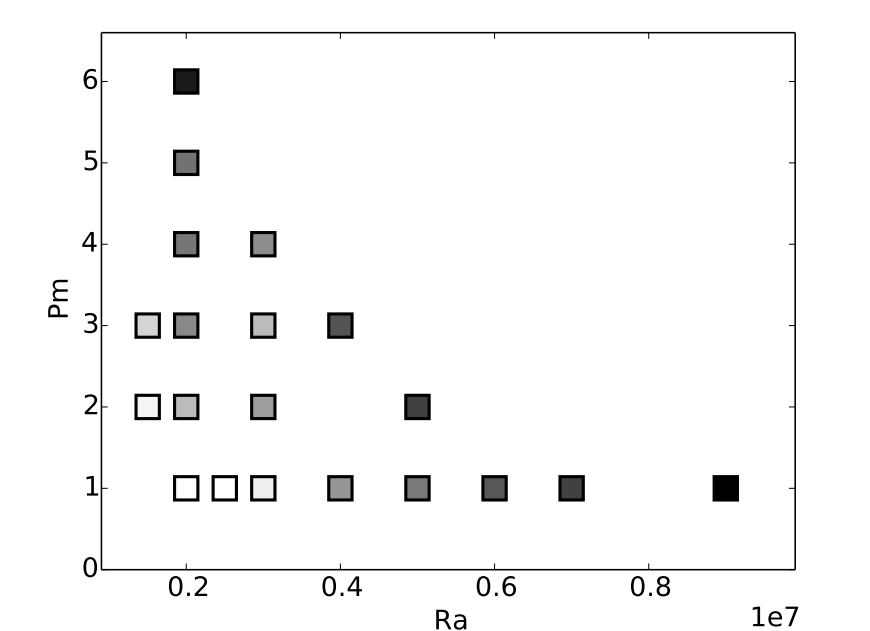
- Equatorial dipole

$E = 10^{-4}$, $Pr = 1$, $N_\varrho = 0.1$



bistability

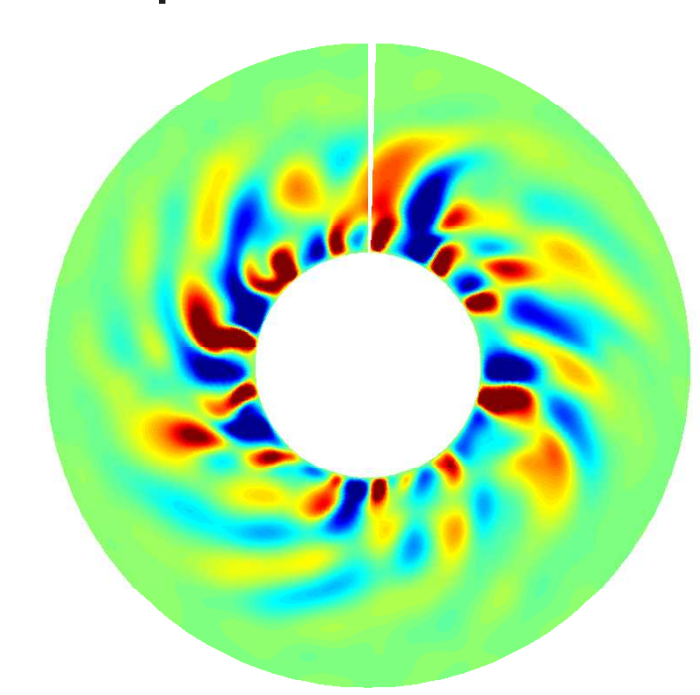
(dip.: circle, mult.:square)



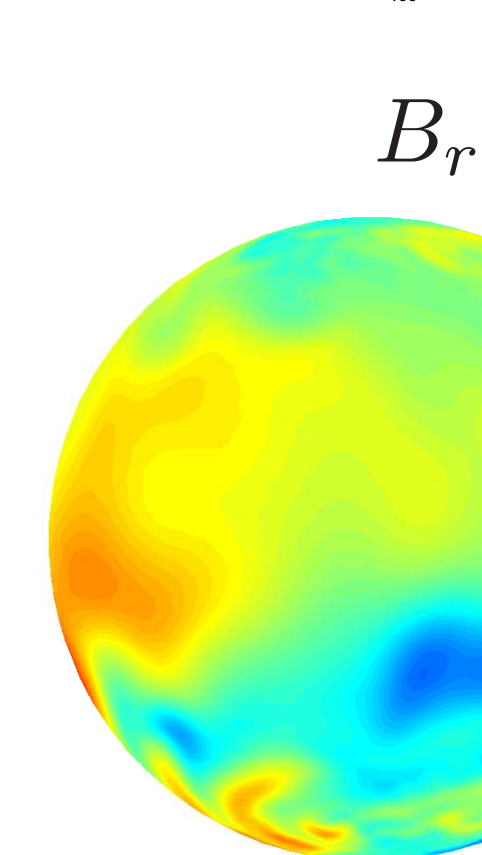
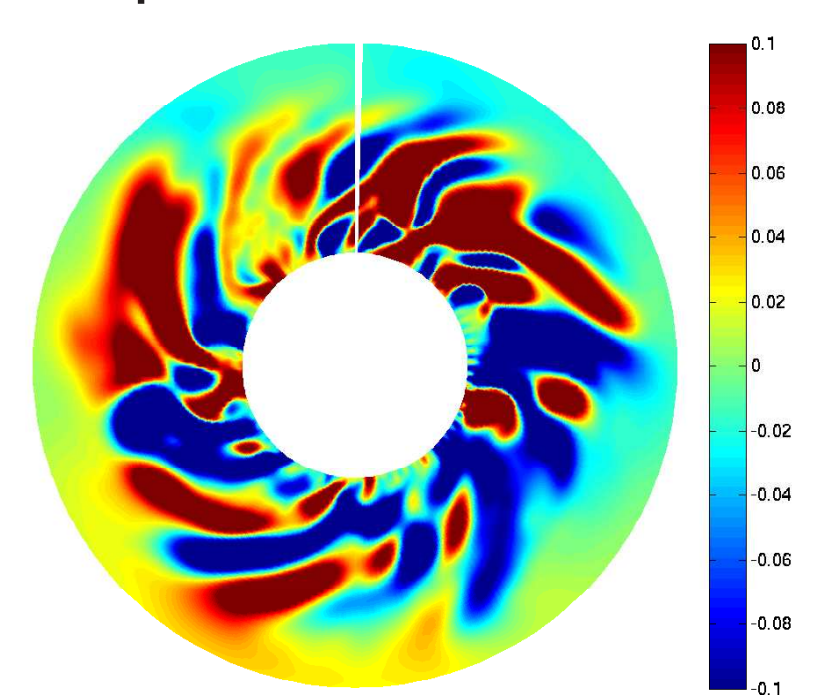
modified tilt angle Θ^*

(white:0, black:1)

Equatorial cut of V_r



Equatorial cut of B_r



$E = 10^{-4}$, $Pr = 1$, $Ra/Ra_c = 9$

CONCLUSIONS

- dichotomy between dipolar and multipolar dynamos extends to anelastic simulations
- equatorial dipole not related to large N_ϱ
- consistency of the scaling laws between Boussinesq and anelastic simulations
- toward observations: M dwarfs (bistability), equatorial dipole

References:

- Braginsky & Roberts, 1995, GAJD, 79, 1
- Christensen, 2010, Space Sci. Rev., 152, 565
- Duarte et al., 2013, PEPI, 222, 22
- Jones et al., 2011, Icarus, 216, 120
- Schrunner et al., 2012, APJ, 752, 121
- Schrunner et al., 2013, A&A, submitted
- Yadav et al., 2013, Icarus, 225, 185