On-off intermittency in spherical Couette flow

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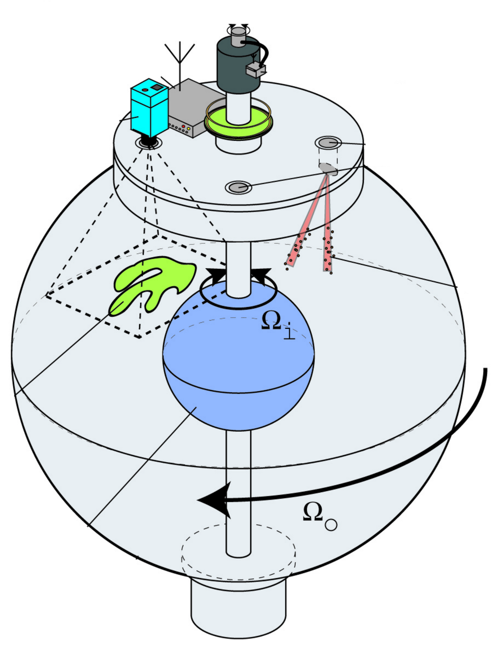
Abstract



* figure from DPL Nonlinear Dynamics Lab

We perform 3D numerical simulations of dynamos in a Couette flow generated by two contra-rotating spheres. We show that the magnetic field may display on-off intermittency at relatively low magnetic Prandtl number, close to the onset of dynamo action. The basic signature of this phenomenon lies in series of short bursts of the magnetic energy ("on" phases) separated by low energy phases ("off" phases). The length of the "off" phases increases as the system gets closer to the threshold. We successfully compare our observations to predictions based on a canonical model and find it provides an accurate estimate of the threshold.

Equations of magnetohydrodynamics

- Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} + \frac{2}{\mathrm{ERe}}\,(\mathbf{e}_z \times \mathbf{u}) = -\frac{1}{\mathrm{Re}}\nabla\Pi + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} + \frac{1}{\mathrm{Re}}\left(\frac{1}{\mathrm{E}} + \frac{\mathrm{Re}}{\chi}\right)(\nabla\times\mathbf{B})\times\mathbf{B}$$

- Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \frac{1}{\mathrm{Rm}}\nabla^2\mathbf{B}$$

- Boundary conditions
  - no-slip conditions for the velocity field
  - insulating or ferromagnetic ($\mathbf{B}\times\mathbf{n}=0$) outer sphere
  - insulating, conducting, or ferromagnetic inner sphere

On-off intermittency: a simple model

$$\boxed{\dot{X} = [\mu + \zeta(t)]\,X - X^3}$$

with

μ distance to the threshold, $\mu \to 0$

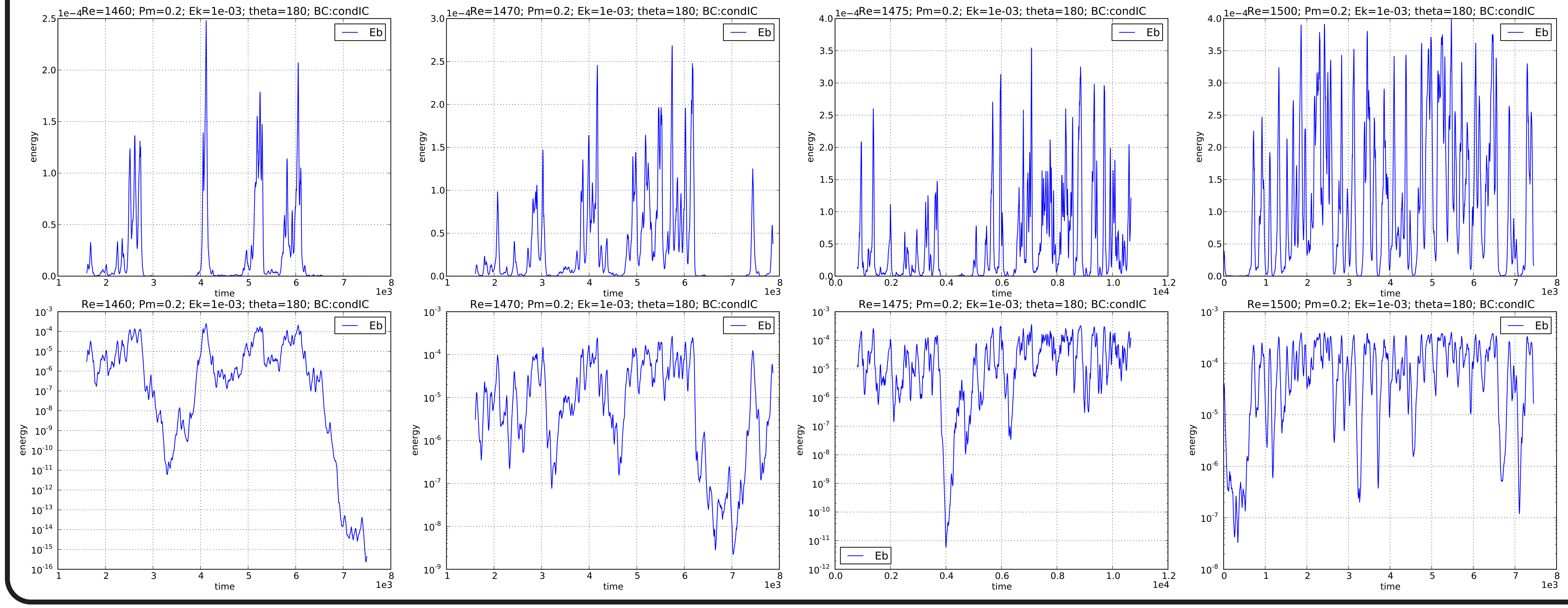
ζ Gaussian white noise, $\begin{cases}\langle\zeta(t)\rangle = 0\\ \langle\zeta(t)\zeta(t')\rangle = 2D\delta(t-t')\end{cases}$

Properties:

- "off" phases $\Longrightarrow$ $\ln(E_b)$ follows a random walk
- PDF $\Longrightarrow$ $P(E_b) \sim E_b^{(2\mu/D)-1}$
- moments $\Longrightarrow$ $\langle E_b{}^n\rangle \propto \mu\,,\forall n$

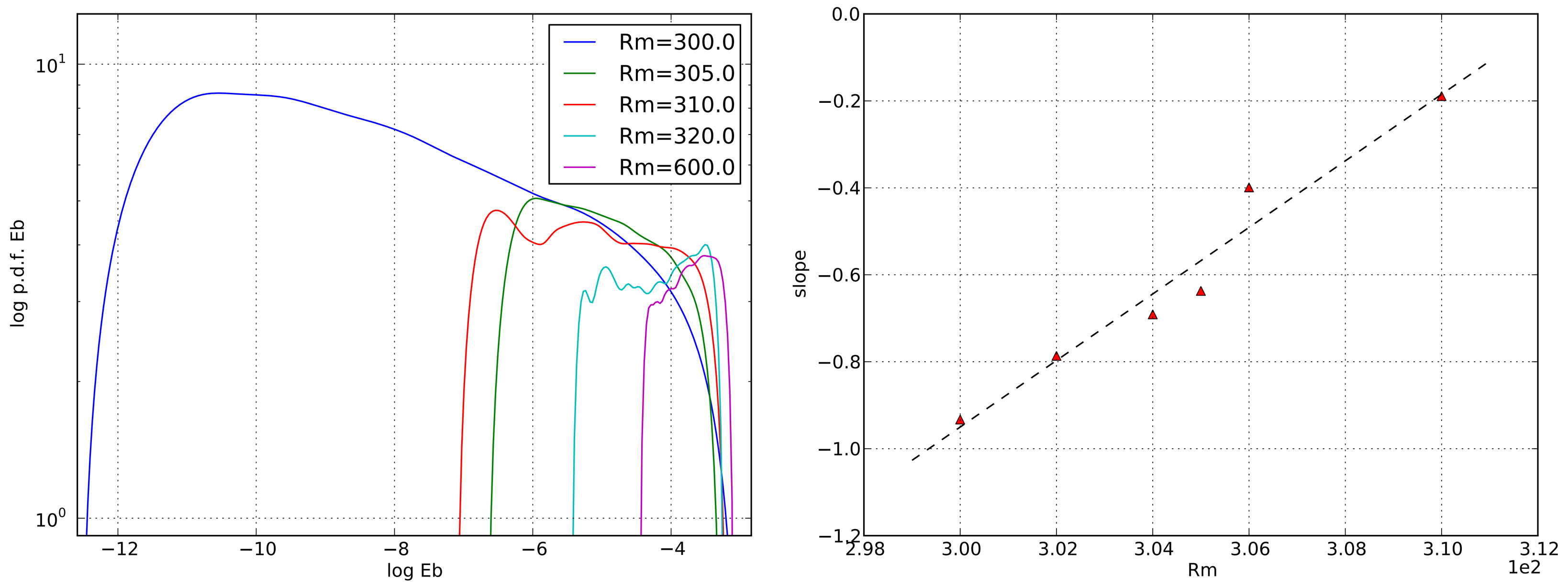
Results

Time series of the magnetic energy in linear & log scale

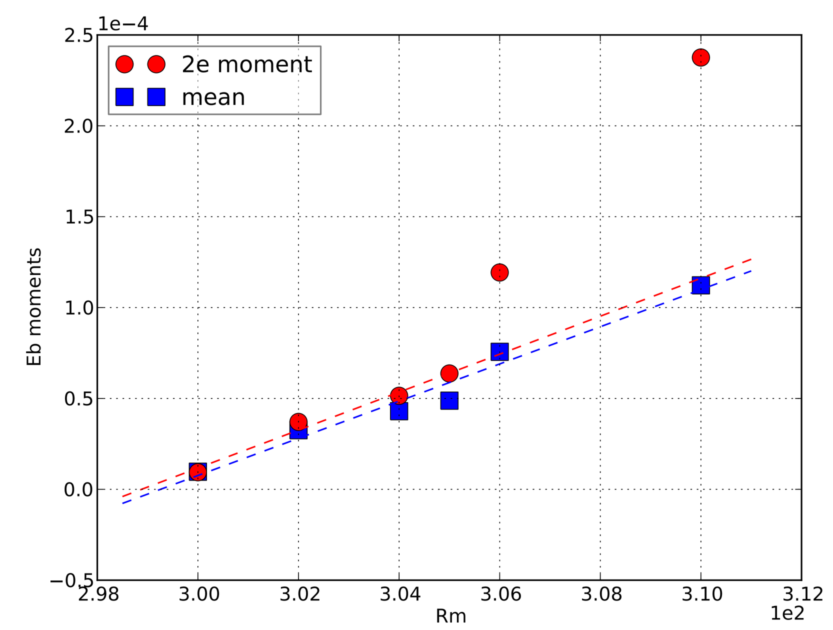


Statistics of the magnetic energy

PDF (with ferromagnetic boundary conditions)

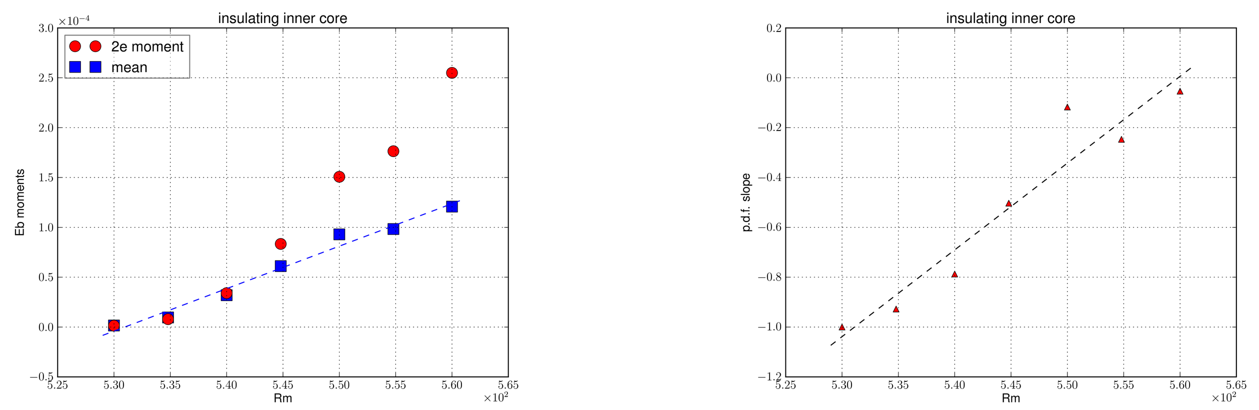


Linear scaling of the moments

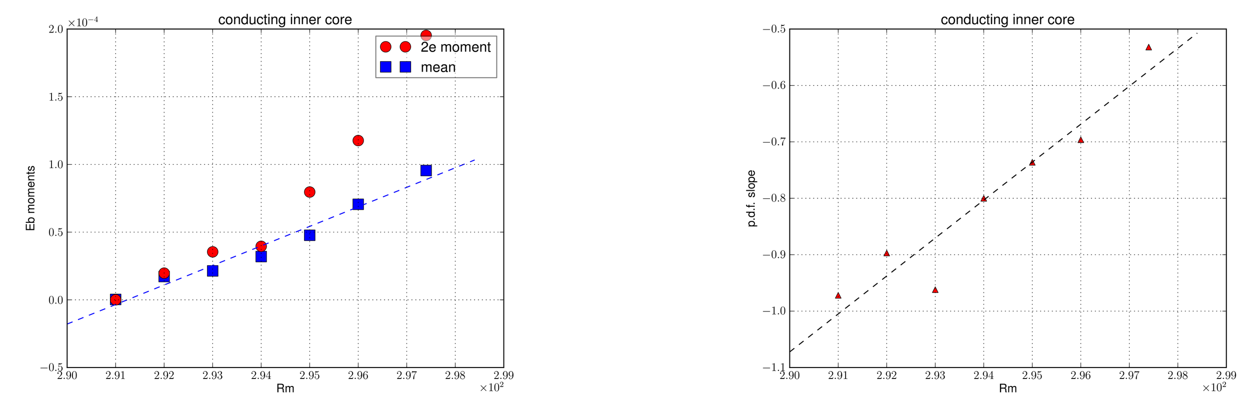


Changing the boundary conditions

With insulating boundary conditions



With a conducting inner core and an insulating outer sphere



Conclusions

On-off intermittency has so far never been observed experimentally in dynamo experiments. This phenomenon has already been reported in a few numerical simulations using an analytic ABC forcing and periodic boundary conditions (Sweet 2001, Alexakis and Ponty 2008). Here we show that on-off intermittency can also be observed with more realistic forcing and boundary conditions.