

# Multiple Partitioning of Multiplex Signed Networks: Application to European Parliament Votes

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## 1 Graph Theory & Operations Research

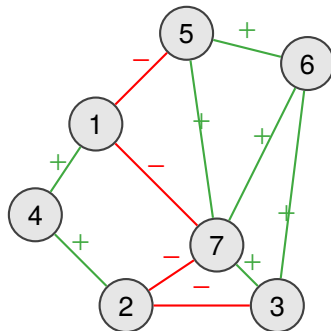
- Structural Balance and Signed Graph Partitioning
- ILP Formulation

## 2 Data Mining & Political Science

- Proposed Approach
- Results

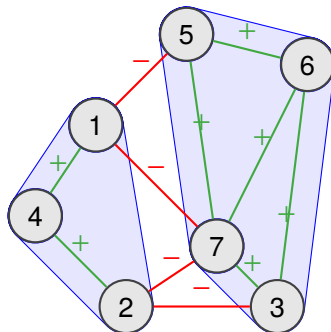
# Structural Balance and Signed Graph Partitioning

- **Signed** graphs



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- **Structural Balance:** Partitioning into two [Heider, 1946] or more [Davis, 1964] mutually hostile subgroups each having internal solidarity

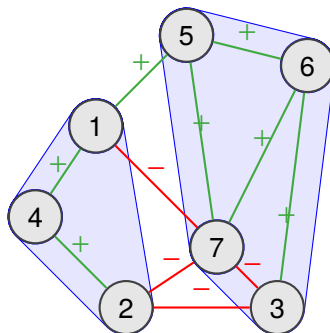


[Heider, 1946] F. Heider, "Attitudes and cognitive organization", Journal of Psychology, 21:107-112, 1946.

[Davis, 1964] J. Davis, "Clustering and structural balance in graphs", Human Relations, 20:181-187, 1967.

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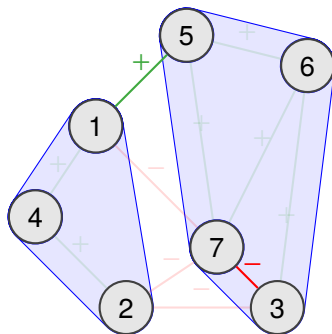


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# Structural Balance and Signed Graph Partitioning

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- **Structural Balance:** Partitioning into two [Heider, 1946] or more [Davis, 1964] mutually hostile subgroups each having internal solidarity
- Most real networks are not structurally balanced → need to measure graph **imbalance**



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- Imbalance of a partition

$P = \{S_1, S_2, \dots, S_l\}$  of  $V$

$$I(P) = \sum_{1 \leq i \leq l} \Omega^-(S_i, S_i) + \sum_{1 \leq i < j \leq l} \Omega^+(S_i, S_j).$$

where  $\Omega^+(S_i, S_j) = \sum_{e \in E^+ \cap E[S_i:S_j]} w_e$

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# Measuring imbalance - Correlation Clustering problem

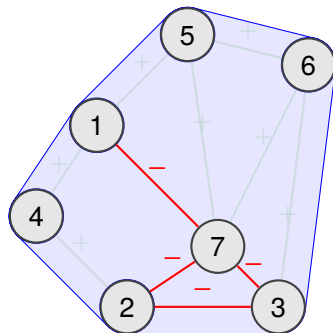
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$$I(P) = 4 + 0 = 4$$



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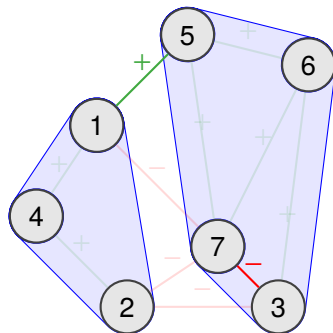
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$$I(P) = 1 + 1 = 2$$

# Measuring imbalance - Correlation Clustering problem

## Definition (Bansal, 2002)

Consider a signed graph  $G = (V, E, s)$  with a nonnegative weight  $w_e$  for each  $e \in E$ , and  $s \in \{-, +\}$ . The **Correlation Clustering (CC) prob.** consists in finding the partition  $P$  of  $V$  such that the imbalance  $I(P)$  is minimized.

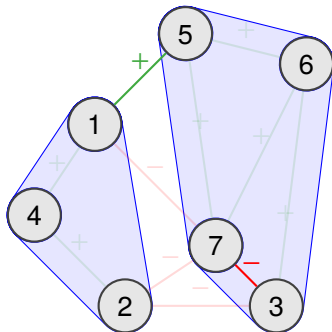
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$$I(P) = 1 + 1 = 2$$

$$x_{pq} = \begin{cases} 1, & \text{if } p \text{ and } q \text{ are in the same cluster,} \\ 0, & \text{otherwise} \end{cases}$$

$$I(P) \left\{ \begin{array}{l} \text{Min} \quad \sum_{p,q \in V: pq \in A^-} w_{pq} x_{pq} + \sum_{p,q \in V: pq \in A^+} w_{pq} (1 - x_{pq}) \end{array} \right. \quad (1)$$

s.t.

$$\text{Triangle constr.} \left\{ \begin{array}{l} +x_{pq} + x_{qr} - x_{pr} \leq 1, \quad \forall 1 \leq r < q \leq p \leq n \\ +x_{pq} - x_{qr} + x_{pr} \leq 1, \quad \forall 1 \leq r < q \leq p \leq n \\ -x_{pq} + x_{qr} + x_{pr} \leq 1, \quad \forall 1 \leq r < q \leq p \leq n \end{array} \right. \quad (2)$$

$$x_{pq} \in \{0, 1\}, \forall 1 \leq q < p \leq n \quad (3)$$

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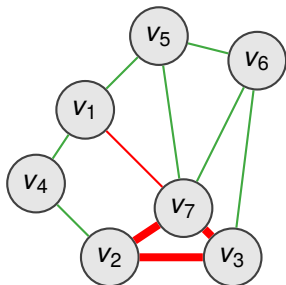
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# Redundant Triangle Inequalities

- Redundant Triangle Inequalities [Miyachi, 2015]



$$\begin{aligned}x_{pq} + x_{qr} - x_{pr} &\leq 1, \forall p, r, q \in V, w_{pq} \leq 0 \wedge w_{qr} \leq 0 \\x_{pq} - x_{qr} + x_{pr} &\leq 1, \forall p, r, q \in V, w_{pq} \leq 0 \wedge w_{pr} \leq 0 \\-x_{pq} + x_{qr} + x_{pr} &\leq 1, \forall p, r, q \in V, w_{qr} \leq 0 \wedge w_{pr} \leq 0\end{aligned}$$

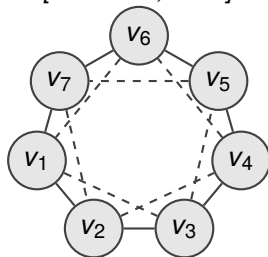
[Miyachi, 2015] A. Miyachi and N. Sukegawa, "Redundant constraints in the standard formulation for the clique partitioning problem", Optimization Letters, 9(1):199–207, 2015.

# Cutting Plane with Generalized Triangle Inequalities

- Cutting plane approach [Ales et al., 2016]

## 2-chorded cycle inequalities

[Grötschel, 1990]



for every cycle  $C \subseteq E$  of length at least 5,  
the set of 2-chords

$$\bar{C} = \{v_i v_{i+2} \mid i = 1, \dots, |C| - 2\} \cup \{v_1 v_{|C|-1}, v_2 v_{|C|}\}$$

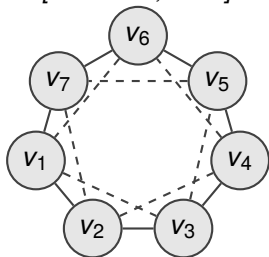
$$x(C) - c(\bar{C}) \leq \left\lfloor \frac{|C|}{2} \right\rfloor$$

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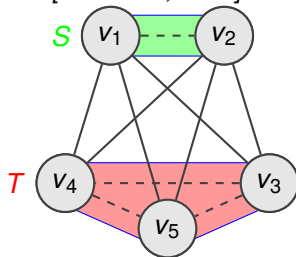
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## 2-partition inequalities

[Grötschel, 1990]



For every  $n \geq 3$  and every two nonempty disjoint  
subsets  $S, T$  of  $V$

$$x([S : T]) - x(E(S)) - x(E(T)) \leq \min\{|S|, |T|\}$$

[Grötschel, 1990] M. Grötschel and Y. Wakabayashi, "Facets of the clique partitioning polytope", Math. Prog. 47:367–387, 1990.

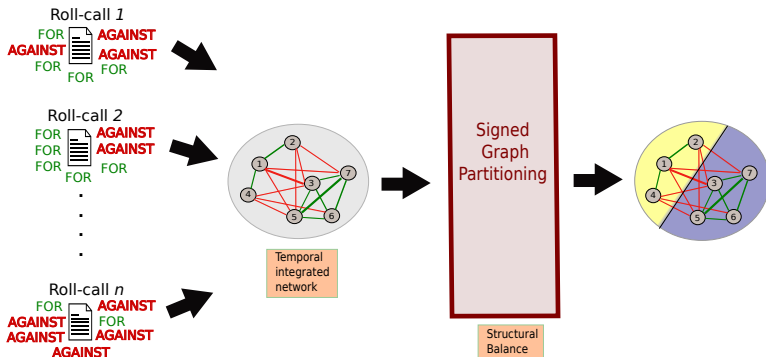
[Ales, 2016] Z. Ales and A. Knippel and A. Pauchet, "Polyhedral combinatorics of the  $K$ -partitioning problem with representative variables", Discrete Applied Mathematics, 211:1–14, 2016.

# Voting Analysis: Network Extraction

- Raw data (from [itsyourparliament.eu](http://itsyourparliament.eu)):
  - Nature: Voting activity at the European Parliament
  - Period: 7<sup>th</sup> term (June 2009–June 2014)
  - Size: 840 Eurodeputies, 851 roll-calls, 21 topics
  
- Legislative proposition networks:
  - Nodes: Eurodeputies
  - Edges:  $\rightarrow -1/+1$  (unweighted), or  $[-1,+1]$  (weighted)
    - Agreement: +1 (FOR vs. FOR, AGAINST vs. AGAINST, ABSTAIN vs. ABSTAIN)
    - Disagreement: -1 (FOR vs. AGAINST)
    - Undetermined: 0 (ABSTAIN/ABSENT vs. \*)
  - Dimensions: country  $\times$  topic  $\times$  time period  $\times$  roll-call
    - For instance, a roll-call voted by French Eurodeputies on Agriculture in 2012-2013

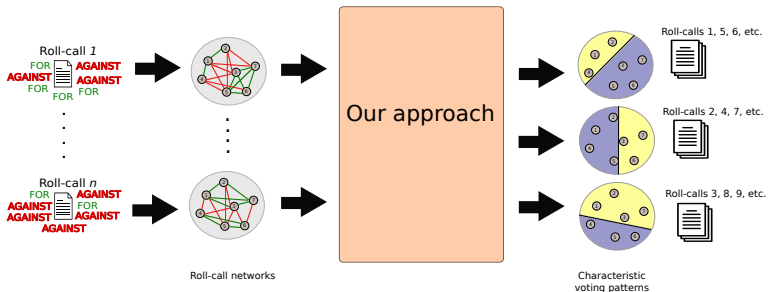


# Traditional Approach: Temporal integration



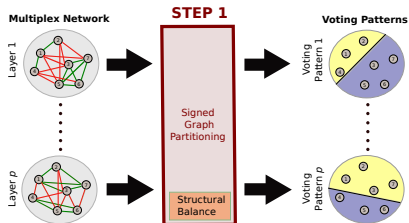
[Arinik et al., 2017] N. Arinik, and R. Figueiredo, and V. Labatut. "Signed Graph Analysis for the Interpretation of Voting Behavior". International Conference on Knowledge Technologies and Data-driven Business - International Workshop on Social Network Analysis and Digital Humanities. 2017.

# Proposed Approach

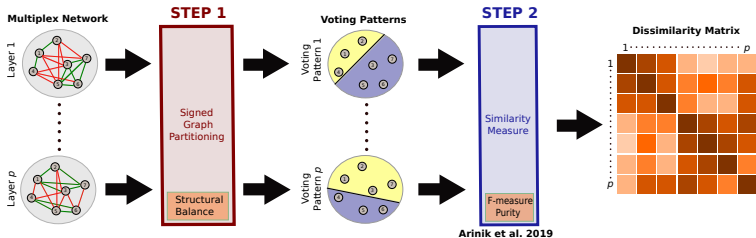


[Arinik et al., 2020] N. Arinik. and R. Figueiredo, and V. Labatut. "Multiple partitioning of multiplex signed networks". Social Networks 60:83–102. 2020.

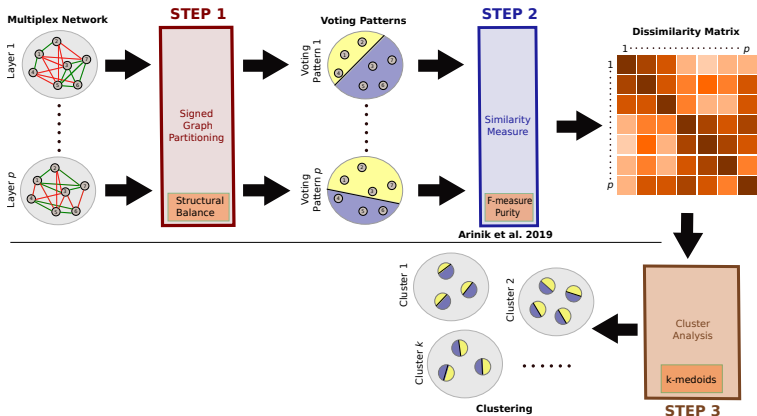
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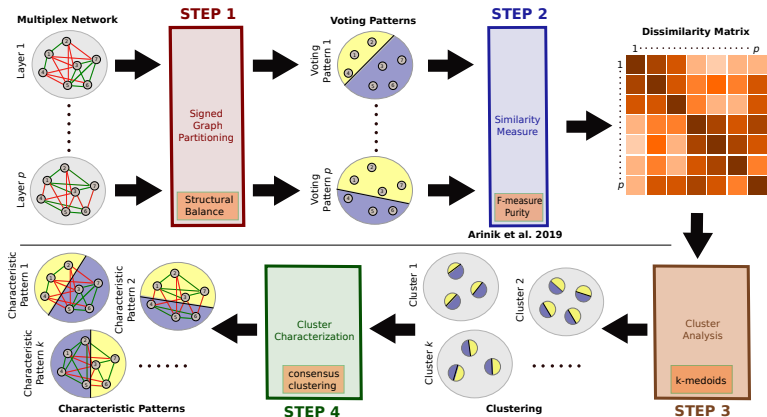
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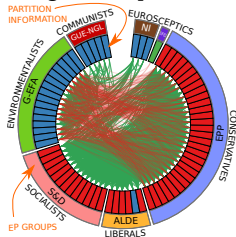
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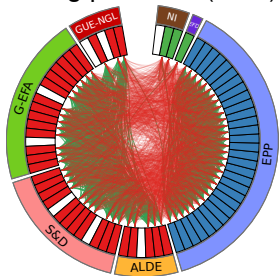
[Arinik, 2019] N. Arinik and R. Figueiredo and V. Labatut, "A new methodology for comparing partitions and evaluating external measures", Working paper, 2019.

# Results: France, AGRI, 2012-13

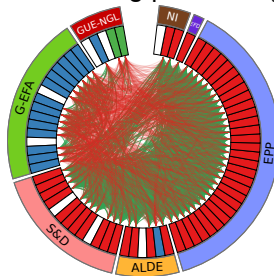
Temporal integration [Arinik et al., 2017]



Voting pattern 1 (%16)

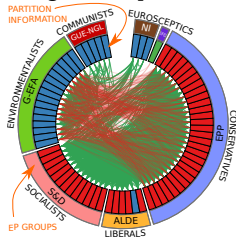


Voting pattern 2 (%40)

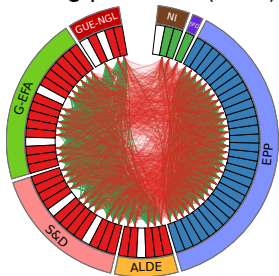


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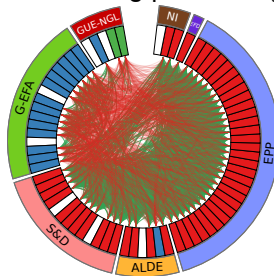


Voting pattern 1 (%16)



Reduction of direct payments,  
etc.

Voting pattern 2 (%40)

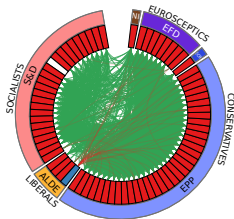


Inclusion of crop diversification,  
etc.

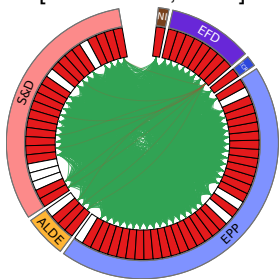


# Results: Italy, AGRI, 2012-13

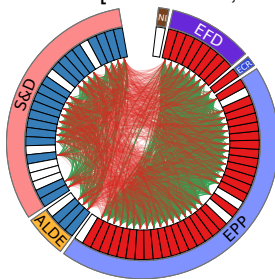
Temporal integration [Arinik et al., 2017]



Voting pattern 1 (%85)  
[Arinik et al., 2020]

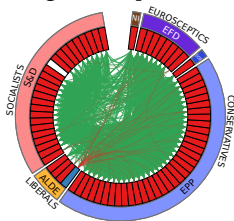


Voting pattern 2 (%15)  
[Arinik et al., 2020]

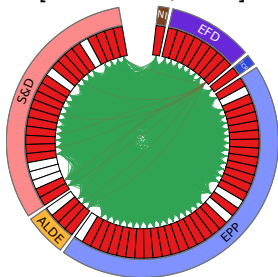


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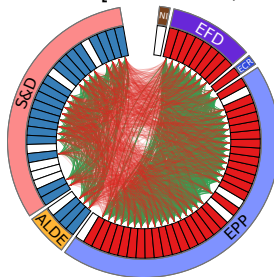
Temporal integration [Arinik et al., 2017]



Voting pattern 1 (%85)  
[Arinik et al., 2020]



Voting pattern 2 (%15)  
[Arinik et al., 2020]



Inclusion of crop diversification,  
etc.

Reduction of direct payments, etc.

## Contributions:

- resources
  - data: <https://doi.org/10.6084/m9.figshare.5785833.v2>
  - source code: <https://github.com/CompNet/MultiNetVotes>
- Highlighted the limitations of the traditional approach
- Proposed a *generic method* to address those issues
  
- Analysis of External Evaluation Measures
- Multiplicity of optimal solutions for Correlation Clustering problem
- 2-Edge Connected Balanced Subgraphs for Correlation Clustering Problem

# Thank you for your attention!

## **Contact Information:**

Nejat ARINIK

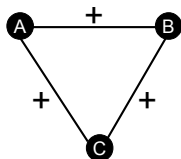
nejat.arinik@univ-avignon.fr

# Voting behavior patterns: France vs. Italy

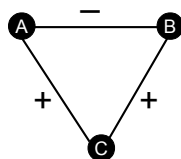
Subject	French Eurodeputies' positioning	Italian Eurodeputies' positioning
Natura 2000 and Water Framework Directive payments: foreseeing incompatible requirements in some regions	Conservatives vs. All (voting pattern 1)	S&D/ALDE vs. the Rest (voting pattern 2)
Reduction to direct payments to farmers	Conservatives vs. All (voting pattern 1)	S&D/ALDE vs. the Rest (voting pattern 2)
Inclusion of permanent grassland into cross-compliance scheme	Environmentalists vs. All (voting pattern 2)	Unanimity (voting pattern 1)
Crop rotation/diversification	Environmentalists vs. All (voting pattern 2)	Unanimity (voting pattern 1)
Measures against market disturbance: milk and sugar quota	Environmentalists vs. All (voting pattern 2)	Unanimity (voting pattern 1)
Elimination of export refunds	Environmentalists vs. All (voting pattern 2)	Unanimity (voting pattern 1)

# Structural Balance

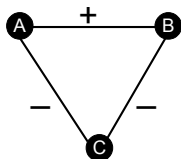
- People strive for cognitive balance in their network of likes and dislikes.



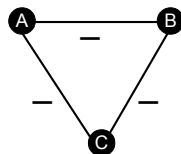
(a) Balanced



(b) Not balanced

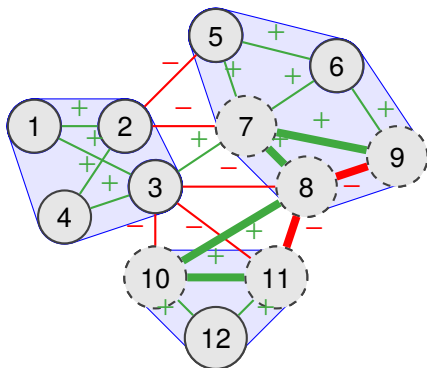


(c) Balanced



(d) Not balanced

# Structural Balance (checking of local property)



# Positioning of EP political groups (Manifestos)

Subject	GUE-NGL	G-EFA	S&D	ALDE	EPP	EFD	ECR	NI
1) Reduction of direct payments	FOR reduction (ceiling at 100 k€) [78]	FOR reduction (starting from 50 k€) [70]	FOR reduction (starting from 150 k€) [66]	-	AGAINST reduction [66]	AGAINST reduction [66]	AGAINST reduction [66]	-
2) Maintaining milk quotas	FOR quotas (with flexibility) [66]	FOR quotas (food security purposes) [66]	FOR quotas [66]	AGAINST quotas (competitiveness purposes); FOR transition period [69]	AGAINST quotas (competitiveness purposes) [66]	-	AGAINST quotas (low food price purposes) [66]	FOR quotas (fair price purposes) [79, 80]
3) Export subsidies	-	AGAINST subsidies [70, 64]	AGAINST subsidies [66]	AGAINST subsidies (with transition period) [69]	AGAINST subsidies; FOR exceptional subsidies [66]	-	-	-
4) Competitiveness	AGAINST current system (too competitive) [66]	AGAINST current system (too competitive) [66]	-	FOR competitiveness [69]	FOR competitiveness (better functioning of supply chain purposes) [66, 81]	-	FOR competitiveness (low food price purposes) [66]	AGAINST current system (fair price purposes) [79, 80]
5) Aid for rural development	-	AGAINST current scheme (not enough); FOR co-financing [66]	FOR transfer from Pillar I to Pillar II; AGAINST co-financing [60]	FOR transfer from Pillar I to Pillar II, co-financing [66]	FOR transfer from Pillar I to Pillar II, producers as market actors [66]	-	AGAINST risk management measures (not enough) [66]	AGAINST the current scheme (not enough) [79]
6) Food quality vs. quantity	FOR quality [78]	FOR quality [64, 66]	-	FOR quantity [69]	FOR quantity and quality [81]	FOR quality [82]	FOR quantity and quality [66]	FOR quality [80]
7) Enhancing environmental measures	FOR organic farming; AGAINST GMO [78, 66]	FOR crop rotation and diversification, promoting biodiversity, organic farming, permanent grasslands, AGAINST GMO, intensive agriculture [64, 70]	FOR reducing use of chemicals, promoting biodiversity, energy savings; AGAINST intensive farming [60]	FOR greener CAP, energy savings, tackling climate change through innovative solutions [69]	-	AGAINST GMO [82]	Not the priority [66]	-