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Moving fingers and the density of rational numbers: An inclusive materialist approach to digital technology in the classroom

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This study aims to examine the relationship between the body and the meaning-making process in mathematics classroom with digital technologies. We investigate a mathematical activity in which students explore the meaning of the concept 'density' of rational numbers with a multi-touch dynamic digital apparatus. A case of elementary school students is studied from the inclusive materialist perspective through microethnographic analysis. The result indicates that students' finger movements toward the digital apparatus played a critical role in developing the meaning of density throughout the activity.

Keywords: Touchpad, GeoGebra, Density, Body, Inclusive materialism.

Introduction

Contemporary digital technologies have become more 'body-friendly' (Sinclair, 2014) with the widespread adoption of haptic input devices. In a haptic technological environment - such as that with touchpad or touchscreen - users interact with technologies through more direct body movements compared to the keyboard-and-mouse environment, and hence a more powerful status is granted to the user's body. This change in the interaction asks the studies concerning the technologies in mathematics classrooms to include our body as one of the central interests (Ferrara, Faggiano, & Montone, 2017).

On this basis, this paper aims to examine the dynamic relationship between the body and the meaning-making process in mathematics classroom with digital technologies. We study a small group of grade 5 students exploring the meaning of density of rational numbers using multi-touch dynamic digital apparatus. We take the inclusive materialist perspective (de Freitas & Sinclair, 2014) to focus on the body movement in mathematical activity with digital technologies. Through a microethnographic analysis, we pay attention to the bodily movement toward the given technology and the development of meaning during the activity.

Our specific research question in this paper is as follows: *In a multi-touch dynamic digital technology environment, how does the meaning of the density of rational numbers evolve in relation to the students' body movements toward the technology?*

Theoretical framework: Inclusive materialism

Existing theoretical approaches

When theorizing the relation between the meaning-making process and technological environment in the mathematics classroom, two theoretical perspectives have been dominant in recent studies, namely the instrumental approach and the semiotic perspective (Drijvers, Kieran, & Mariotti, 2010). These two perspectives together seize the material, social and psychological nature of mathematical

activity with tools. Nevertheless, they do not leave much theoretical room to embrace the role which the body plays. The inclusive materialism, based on the new materialist ontology in which the physical and the mathematical reside on the same ontological realm, reexamine the role the material elements - including the body - play in mathematical activity. In recent years, the studies concerning the multi-touch digital technologies started to adopt this approach to investigate the relationship between the hand movement on touchscreen and mathematical thinking (e.g., de Freitas & Sinclair, 2017).

Inclusive materialism

Breaking with anthropo-centric views, the inclusive materialism argues that all human and non-human elements partake varying degrees of agency and power during a mathematical activity. Therefore, de Freitas and Sinclair (2014) redefine body so that the theory fuses the physical and psycho-social dimensions of the bodily interaction in mathematical activity. They expand the concept of the body and propose the body be "an assemblage of human and non-human components, always in the process of becoming " (p. 25). Here, the boundaries are flexible and porous, and agencies are distributed across the environment of the mathematical activity. By proposing such a comprehensive definition of the body, they suggest that we pay delicate attention to the matters and their material configuration - the physical bodies, utterance, artifacts, signs, representations, and their dynamic relationship - in mathematical activity.

What does the material configuration have to do with the meaning of a mathematical concept? The inclusive materialism takes an ontological turn by which mathematical meanings are not abstract or disembodied, but rather material and concrete. That is, the meaning of a concept is ontologically entangled with specific physical arrangements. In a mathematical activity constructing a circle with a compass, for instance, it is not the case that the compass is a disposable medium for some transcendental, determinate and a priori meaning of the concept 'circle'; instead the meaning of a circle is performed through the process where the circle and the compass are assembled. In other words, meanings emerge through mathematical activity and entail all the material specificity implicated in the activity.

In terms of research, the practical goal of the inclusive materialism lies in capturing the emergent and creative aspect of the meaning-making process. Drawing on the insights from the history of mathematical inventions, de Freitas and Sinclair (2014) deem *mobility* to be critical for such inventiveness. Moving mathematical objects or attending to how they are determined in terms of movement—rather than logically—configure the material elements in new ways. In short, movement actualizes the potential mobility of mathematical objects, creates a new material assemblage, and hence a new meaning as well. In that respect, the design of a mathematical activity should encourage learners to attend to the mobility of material elements and hence new senses that were previously imperceptible, instead of passively following the procedures which already seem possible or logical.

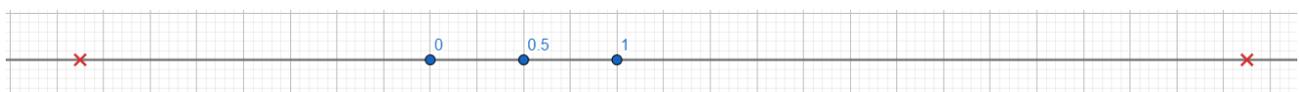


Figure 1: The initial setup of the activity

Research context and method

Participants and the activity

Participants involve a group of grade 5 students (age 10-11) in elementary schools at Seoul, South Korea. Through preliminary oral interview of a class (N=20), we selected four students in each school who had exhibited mid-level achievements in mathematics from the previous semester and, at the same time, displayed low-level understandings of the density in their responses to a questionnaire based on Vamvakoussi and Vosniadou (2007). This paper reports an episode of students A, B, C, D from one of the schools.

The activity is a game called 'point-and-name.' It was a part of a teaching experiment that the authors designed for students to explore the number line for the density of rational numbers using GeoGebra and spanned three lessons. In the beginning, on the screen was a horizontal line with three points on it, each named 0, 0.5, 1 respectively. On the background lied grid lines: In between two adjacent thick grid lines, four equidistant thin lines were placed and divided the space into five parts [Figure 1]. Students sat facing each other and were given touchpad keyboards that were connected simultaneously to a single GeoGebra environment on a shared monitor screen. Split into two teams (AB vs. CD) and taking turns, each team was asked to place a point anywhere on the line in between the two Xs on left and right; name it with a rational number in reference to the previously placed points and their names. To achieve the goal, students could move the points on the line horizontally, and zoom in and out. Once students zoom in, the gaps between the grid lines widen, and new grid lines appear filling the gap. The game ends when a team fails to place a point or give an appropriate name to their points. One of the researchers took the teacher's role only offering technical help.

Data collection and analysis

All three lessons were video recorded and transcribed. We conduct a microethnographic analysis (Streeck & Mehus, 2005) which is effective for the in-depth examination of the interplay among speech, gesture and artifacts in a mathematical activity (e.g., Nemirovsky, Kelton, & Rhodehamel, 2013).

Thereby, we pay particular attention to the following elements to understand the interaction between the learners' physical body and the digital apparatus: (a)the hand motion on touchpads, (b)the cursor movement on the screen, (c)the movement of screen itself (e.g. zooming in/out, panning), (d)the speech of students, (e)the hand gesture in the air. We must watch the hand motions on the touchpad since they are the primary means of manipulating the apparatus for students. The cursor and screen movements are also important for they embody the learner's attention and its shift. Furthermore, in a multi-touch digital technology environment, the gestures in the air are specifically important not only because they communicate meanings coupled with the speech but also they are part of a gestural continuum together with the gestures toward the haptic input device, preserving senses from one to the other (de Freitas & Sinclair, 2017).

On the cognitive side, Vamvakoussi and Vosniadou (2007) had identified various aspects of understanding of the density of rational numbers. Their result suggests that a critical cognitive action

in developing the meaning of density is to find a smaller unit than the least common unit of numbers in an interval in order to conceive a number with the smaller unit which would locate in somewhere 'in-between' the existing numbers (e.g., $\frac{3}{4}$ in the interval $(\frac{1}{2}, 1)$). Understanding that this action could be implemented in any interval of two rational numbers is considered as indicating a sophisticated development of the meaning of density.

Result and discussion

Our major findings are drawn from three notable sequences of the episode. As students carried out the activity through these sequences, the meaning of density had become more and more sophisticated. Each sequence is thematized by a significant development of the meaning of density that had come to the surface. We present each sequence through transcription or a brief depiction that highlights key events, then follow up by discussion.

Sequence 1: “You must follow the rules.”

Shortly after the beginning, students started to establish collectively a set of implicit rules of engagement (*First-rules*) and eventually all began to abide by them to play the game: **First-rule 1.** A new point must be 0.5-away from an existing point and named accordingly; **First-rule 2.** A new point must be placed on a thick grid line. The highlight of the first sequence is presented below.

- 1 C: (C drags his index finger to the left and then gently taps.) [C’s cursor moves to the left side of '0', and a point appears on the left side of the line.] [Figure 2 (a)]
- 2 D: Why are you putting it there?
- 3 C: You don’t like it?
- 4 D: No. You can’t just do this in whatever way you want.
- 5 C: Then, how?
- 6 D: You must follow the rules. See, it goes from 0 to 0.5, then to 1. What do you think will come next?
- 7 C: 1.5, it is. Let me throw this away. (C right-clicks with his middle finger) [C opens the menu on the point and deletes it.]
- 8 D: (D spread her index and middle fingers away on the touchpad, then double-taps the pad, and drags with her index finger to the right.) [The screen zooms in. The point appears on the line and is dragged right to the place where a thick grid line adjacent to '1' meets the line.] [Figure 2 (b)]
- 9 C: Now you name the point.
- 10 D: (D tabs and then types '1.5') [The name '1.5' appears above the point.] [Figure 2 (c)]

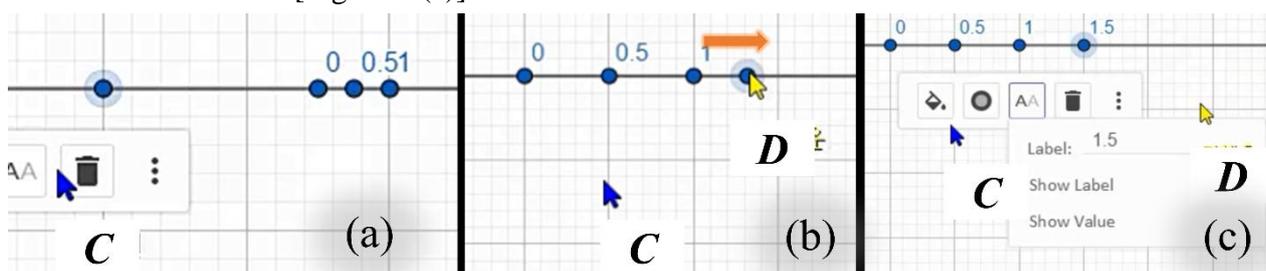
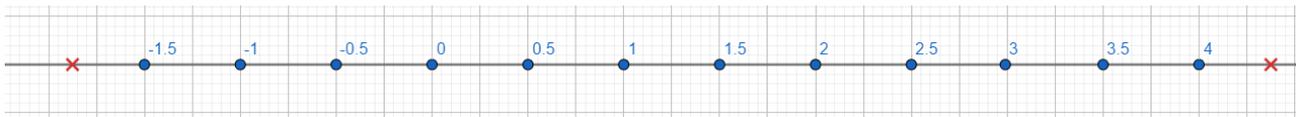


Figure 2: The establishment of First-rules

After Team CD pointed '1.5', A began to lead Team AB's turn. A zoomed in, and '0.5' and '1.5' got off the thick grid line because of that. A started counting the number of thick grid lines - including newly appeared ones - between '1' and '1.5' with his cursor. After expressing confusion about how far they should zoom in, A zoomed out, coming back to the scale at which they were. Then A placed '2' on a thick grid line as far away as the distance between '1' and '1.5'. From then on, every new point ended up on a thick grid line, 0.5-away from the point either at far-right or far-left, subsequently producing



'2.5', '3', '3.5' and so on [Figure 3].

Figure 3: The screen capture at the end of Sequence 1

Although the students did not articulate the rules explicitly, we could infer the First-rules from their discourse and actions in this sequence. Line 6 and Line 10 are good examples where they defined the way the game should be played and complied. These rules suggest that 0.5, the least common unit of numbers given in the initial setup, remained indivisible to students and they did not conceive any smaller unit by which they could play the game. The multiples of 0.5 (e.g., 0, 0.5, 1.5) are treated as if they were successive.

We must note that these rules are neither given by the teacher nor randomly established by the students. Rather, they resulted from the structure consisted of the salient perceptual features of the initial technological terrain: the three points on the line with names '0', '0.5', '1'; evenly distanced from each other; all aligned with the thick grid lines [Figure 1]. This regulation was not included in the instructions from the teacher. Students could have laid their point anywhere in between the Xs and claim its legitimacy by naming it with a proper rational-number. Instead, their attention was captivated by the given structure and, as a result, their subsequent practice began to adapt to it. Although Team AB had zoomed in and thus had a better chance to place their point in between the existing points, they were only *looking closely* at the line to measure the distance between 1 and 1.5. Instead, having their attention fixed only at the thick grid lines, they failed to obtain the measure they wanted since 1.5 was not on one of the thick grid lines. Eventually, they zoomed out so that they could place '2' one thick-grid-line away to the right.

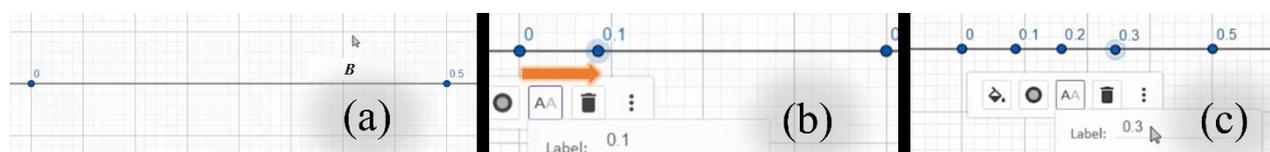
Sequence 2: “We can’t go beyond X.”

In the following sequence, to resolve the problem on hand, students devised new rules (*Second-rules*): **Second-rule 1.** A new point must be 0.1-away from an existing point and named accordingly; **Second-rule 2.** A new point must be placed on a grid line, either thick or thin. The highlights of this sequence are described below.

Once Team CD placed '4', the last point in Figure 3, students encountered an issue: There is no available place for new points. According to the First-rules, they were only supposed to point on the thick grid lines between the two Xs. Zooming in at the interval (3.5, 4), frustrated, A commented,

"We can't go beyond X." Instead of giving up, however, Team AB then decided to seek a new place for '0.1'. B zoomed in and started counting thick grid lines between '0' and '0.5', of which she counted eight [Figure 4 (a)].

However, students therein encountered the second issue: No thick grid line corresponded to the proper location of '0.1'. While B was having a hard time finding a thick grid line corresponding to '0.1', A started to notice the use of thin grid lines and suggested B to make use of them. Not long after, B agreed on A. There, they began to count, finding out the interval (0, 0.5) comprised 20 thin grid lines and thus 0.1 corresponded to four of them. At last, a new point appeared near 0 and got dragged slowly to the fourth thin grid line and so did the name '0.1', shortly [Figure 4 (b)]. From then on, students unanimously followed Team AB's method with little verbal exchange, counting four thin grid lines to the right and pointed '0.2', then '0.3' [Figure 4 (c)]. The activity had to stop there since the



lesson time was over.

Figure 4: The establishment of Second-rules

After Team AB had devised a solution to place '0.1' in violation of the First-rules, Team CD simply followed their solution for '0.2' with no dispute at all, and Team AB repeated the action for '0.3' in the same way. Even though we were not able to observe further after '0.3', it is reasonable to assume they would have continued to play in the same manner judging based on their previous performance.

This new set of rules suggests the students became able to conceive a smaller unit (e.g., 0.1). Moreover, it shows that they could come up with the numbers which can be placed in between two once-seemingly-successive numbers. At this point, 0.5 was not the indivisible least common unit, and the multiples of 0.5 were not successive for students any longer.

Throughout this sequence, the body movements were at the center. Especially, the act of zooming in and out, rendered by spreading and pinching fingers on the touchpad, eventually evoked the change in students' perceptual habit and made them recognize the potential of thin grid lines – which was imperceptible earlier.

Another noteworthy thing is that the same perceptual change did not occur in the first sequence even though they zoomed in/out in precisely the same manner. The fundamental difference here lies in the degree of X's agency-in-play in the two situations. Once the students reached the barricades, unlike the first sequence, X became a major player in this student-technology-concept assemblage, which dismissed the students' propensity to go further outward according to the First-rules. Instead, it turned their attention to the intervals in between the points. It was not until the formation of this particular configuration that spreading fingers catalyzed such a dramatic shift in practice. Only then, students were *stretching* the number line and space rather than merely *looking closely* at them.

Sequence 3: "We keep stretching."

After the activity was over, the teacher conducted an ill-structured group interview with students on-site. Through students' discourse and gestures, it became apparent that the meaning of density had evolved further, and it was indeed inseparable from the technological environment of the activity.

- 11 T: Let's say we already have placed like, so many points and, say, y'all are really smart people. Then, can we place another point at this time?
- 12 All: Yes.
- 13 B: By stretching more and more (*palm facing forward and all fingers open, B spreads her thumb and index and close them in repeatedly*) [Figure 5 (a)].
- 14 T: Then, until when?
- 15 A, B: Forever.
- 16 B: We do **zero point zero, zero, zero, zero, zero, zero, one [0.0000001]**
- 17 A: **Zero point zero, zero, zero, zero, zero, zero, ... zero, one [0.00000001]**¹ (...)
- 18 C: The further you stretch (*palm facing downward and only index and middle finger open, C spreads them away and close in quickly and repeatedly*) [Figure 5 (b)], the more lines you will have (*palm facing forward, C repeats to spread and squeeze all of his fingers swiftly*) [Figure 5 (c)].
- 19 T: Then, let's say, I would place my points at all the empty place on the line there, and now we don't see any empty place. Can we still lay another point?
- 20 A: We can.
- 21 B: Yes. We keep stretching (*palm facing downward, B spreads her index and middle finger away and bring them in repeatedly*) [Figure 5 (d)].

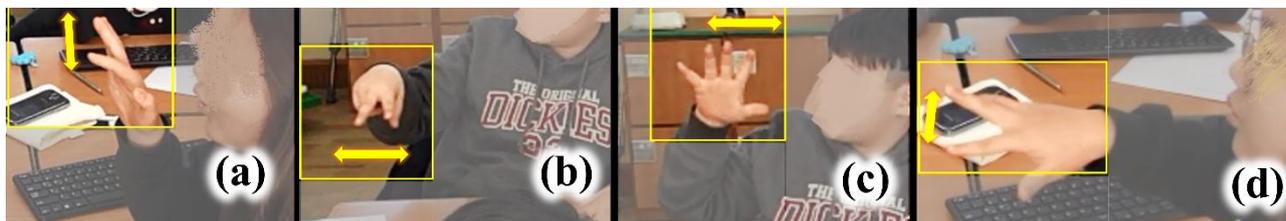


Figure 5: The students' hand gestures in the air

The students realized that no matter how many points are on the line and how dense they appear to be, they could always find a place for another point in between them. B was enumerating zeros in 0.0000001 out loud (Line 16) to imply that the game could go on by finding a number with a sufficiently small unit in response to the teacher's question (Line 11). Moreover, while enumerating zeros out loud simultaneously with B, A noticed that he could find an even tinier unit if he continued to add zeros before finishing with one. This made A take a brief halt when B finished and then quickly added one more zero and one (Line 17). Also, Line 13, 18 and 21 indicate that students became able to conceive a room to put such numbers by virtually stretching the line and space.

¹ This bolded and underlined part in the transcript indicates the moment when speeches of A and B were overlapping. Note that B deliberately continued to add a zero even after A stopped.

As we have observed from the episode, the body movement is a crucial factor in the invention of new meanings. Students' hand gestures on and off the touchpads demonstrate the catalyzing role of the mobility in the meaning-making process or, in other words, becoming the assemblage of fingers-touchpads-number line-space-'density'. No one possessed the emerged meaning from the beginning. No one was able to produce it by only repeating what seemed possible. The meaning did not exist until the finger movements actualized the latent material configurations and allowed the new senses in unanticipated ways. It is this emergent nature what makes the meaning genuinely new and inventive.

Concluding remarks

We have scrutinized a case of mathematical activity to examine the relationship between the body movement and the meaning development in a multi-touch digital technology environment. Through the inclusive materialist lens, we could observe how the finger movements toward the digital apparatus played a crucial role in developing the meaning of the density of rational numbers. As Sinclair and de Freitas (2014) note, movements of the hand bring about new material reconfigurations and hence new hand gestures, which participate in the process of communication as well as the invention of meaning. We hope this study will contribute both to the embodied cognition theory and the studies on the integration of digital technologies in mathematics education by encouraging to incorporate the body and the material as central foci of investigation.

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