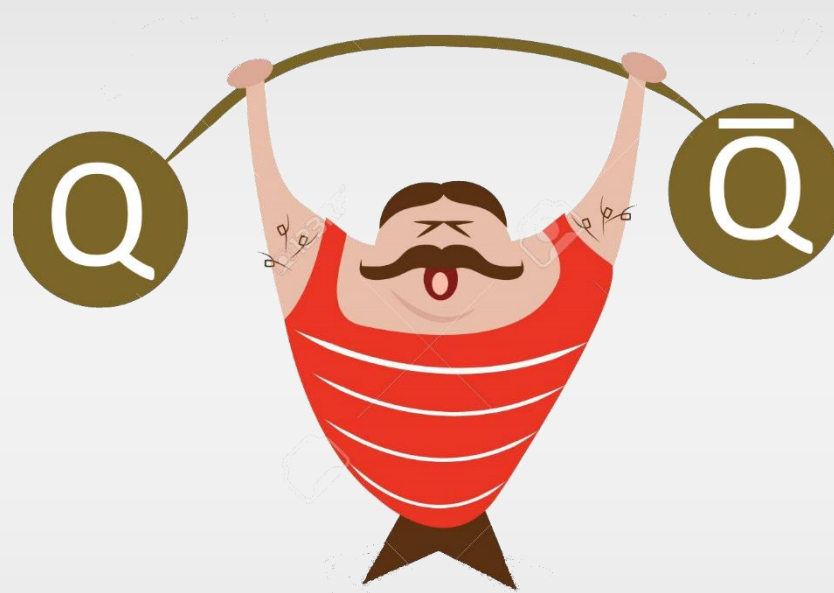


Day 3: Quarkonium equilibrium limits

The Nantes Schrödinger-Langevin approach



Quarkonia Task Force

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EMMI Rapid Reaction Task Force – GSI Darmstadt

18/12/2019

Schrödinger-Langevin approach

Main ideas:

- Langevin-like Schrödinger equation to mock open quantum systems
- Real time dynamics of correlations, dissociation, recombination & transitions

$$i\hbar \frac{\partial \Psi_{Q\bar{Q}}(\mathbf{r}, t)}{\partial t} = \left(\underbrace{\hat{H}_{\text{MF}}(\mathbf{r})}_{\text{Mean Field}} - \underbrace{\mathbf{F}_{\mathbf{R}}(t) \cdot \mathbf{r}}_{\text{Fluctuations}} + \underbrace{A(S(\mathbf{r}, t) - \langle S(\mathbf{r}, t) \rangle_{\mathbf{r}})}_{\text{Dissipation}} \right) \Psi_{Q\bar{Q}}(\mathbf{r}, t)$$

Mean Field:

T-dependent screened potential from IQCD

Fluctuations

Stochastic force scaled such as $T_{Q\bar{Q}} = T_{QGP}$ at equilibrium

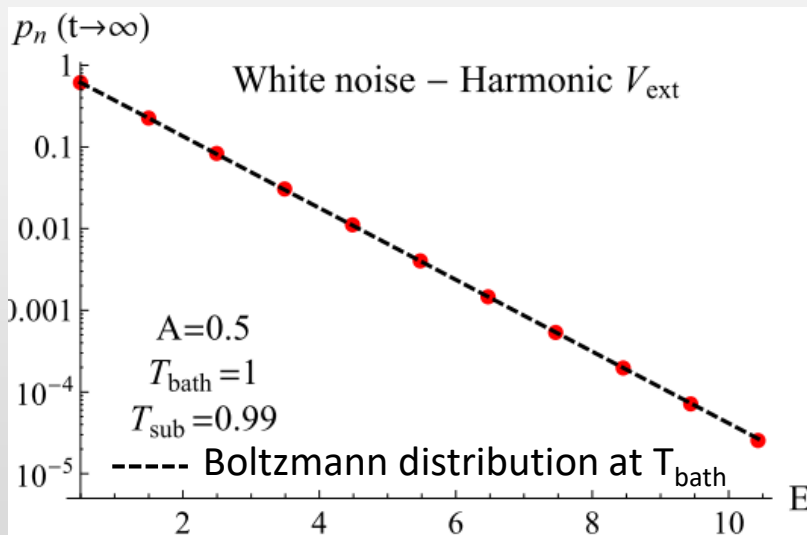
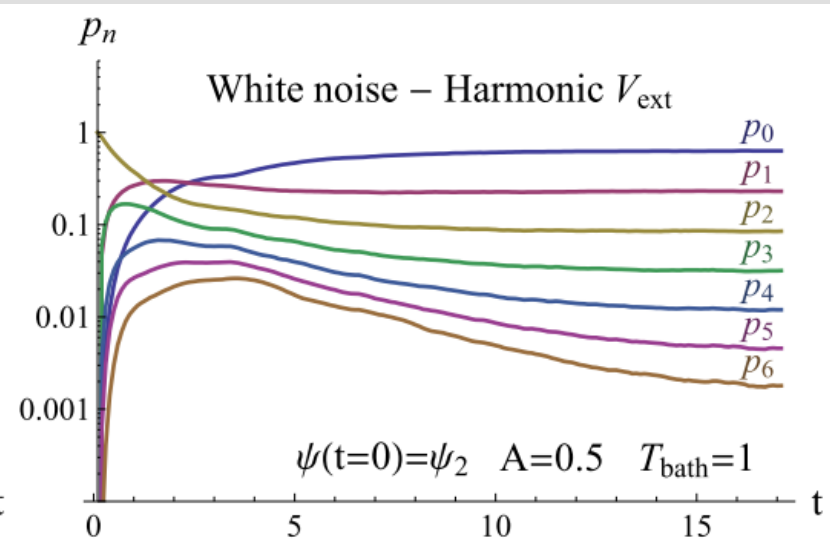
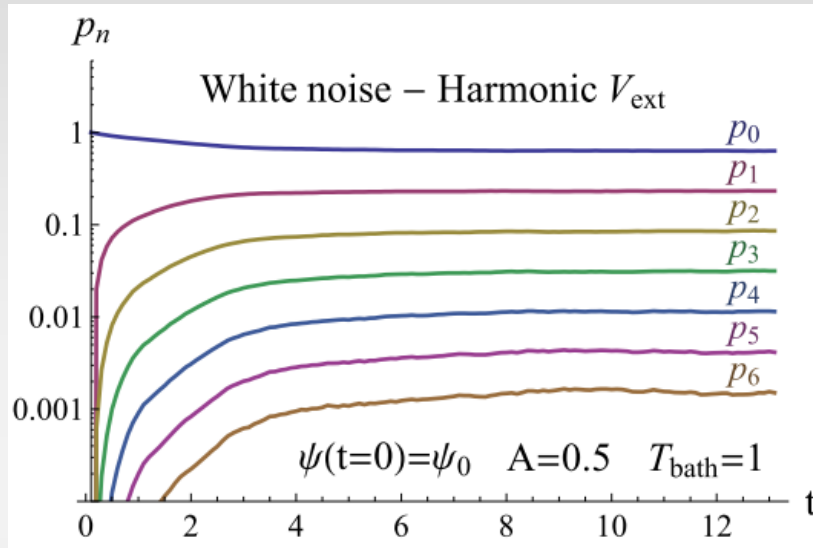
Dissipation

$S(\mathbf{r}, t) = \arg(\Psi_{Q\bar{Q}}(\mathbf{r}, t))$
- non-linear, real and ohmic
- brings to the lowest state
- A(T): Drag

- Most quantum features ok – Allows quantum initial/final states
- Mixed state from statistics – Easy to implement
- Leads to local « thermal » distributions: Boltzmann behaviour for at least the low lying states
- A priori not related to a quantum master equation

Equilibrium with SL equation

Harmonic potential $\frac{1}{2} x^2$



Analytically: correct thermal equilibrium reached if

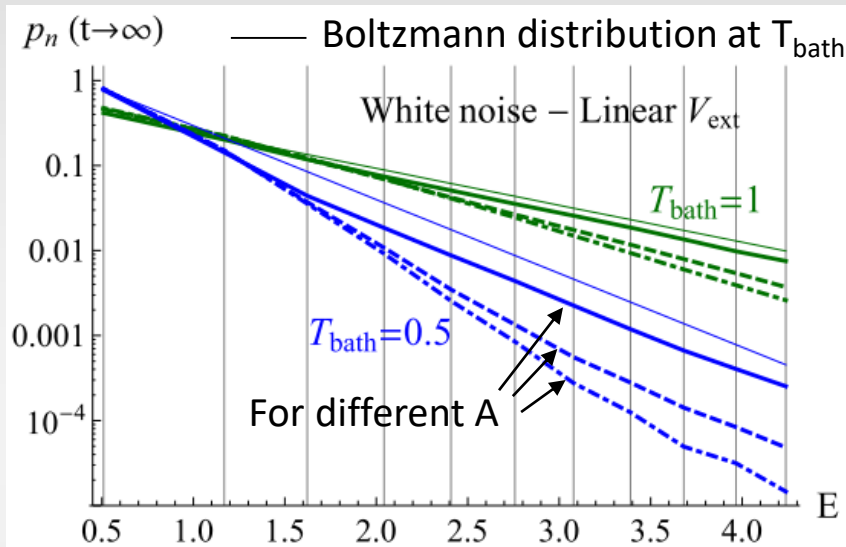
$$B = 2mA E_0 \left[\coth \left(\frac{E_0}{kT_{\text{bath}}} \right) - 1 \right]$$

« quantum fluctuation-dissipation relation for white noise »

with $\langle F_R(t) F_R(t + \tau) \rangle = B \delta(\tau)$

Equilibrium with SL equation

Other potentials: linear example $\frac{1}{2}|x|$



No Boltzmann behaviour for higher states

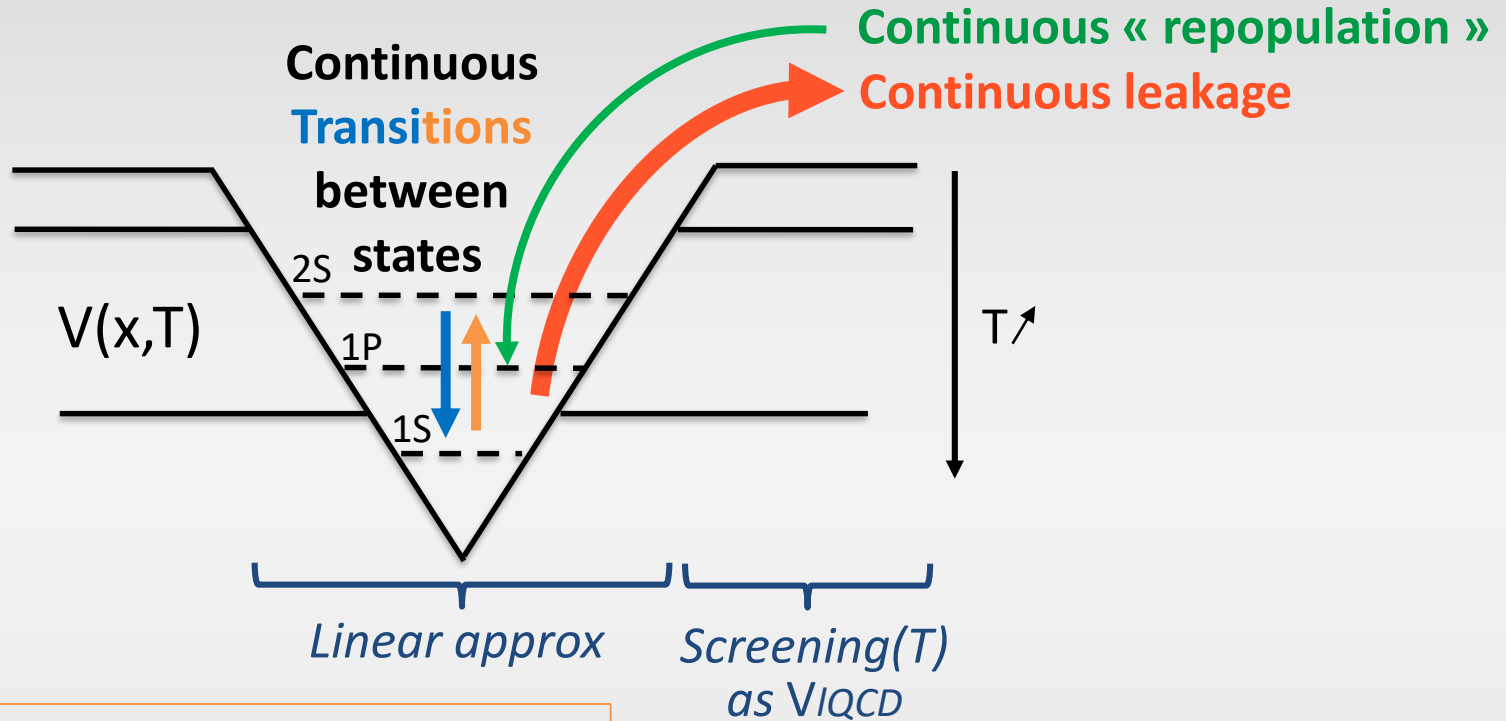
Disagreements between
subsystem T_{sub} and bath T_{bath}

=> Correct Boltzmann distribution obtained for the lower states
if noise is rescaled such as to get $T_{\text{sub}}=T_{\text{bath}}$

SLE equation and equilibrium: ok in phenomenology if only the lower states are considered (=> ok for quarkonia)

Dynamics of $Q\bar{Q}$ with SL equation

simplified 1D singlet model => insights on the dynamics



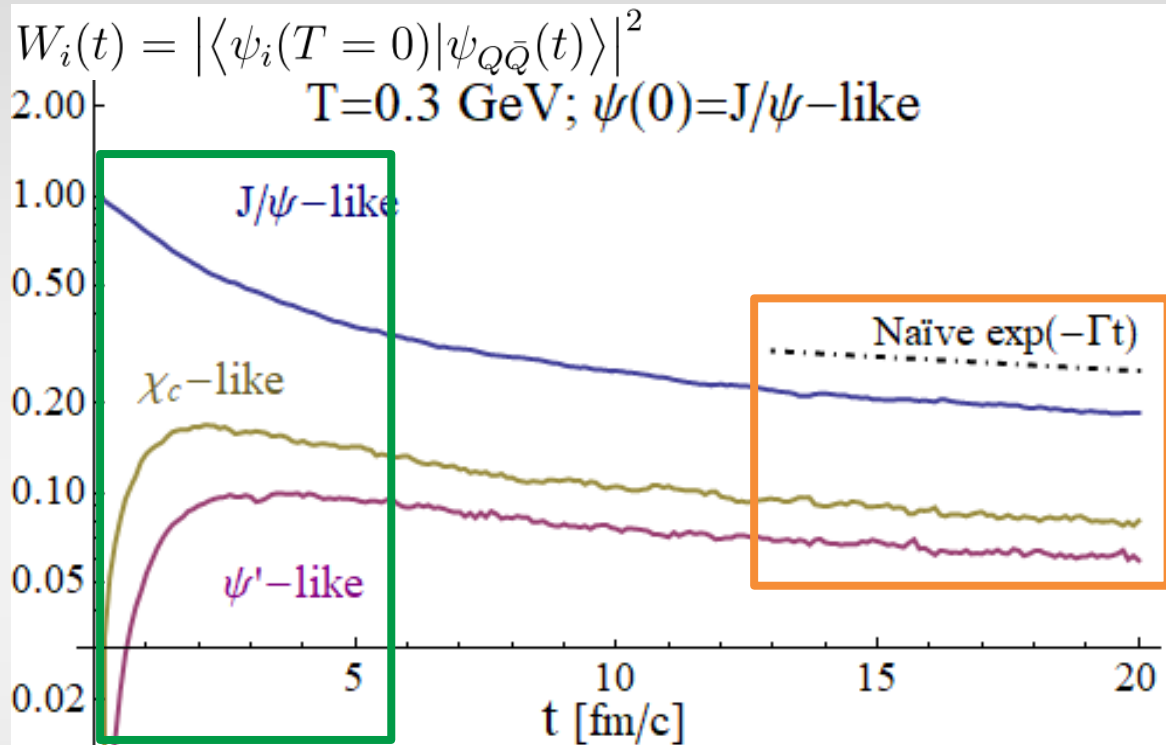
Stochastic forces =>

- Feed up of higher states
- Continuous leakage of bound component to continuum
- Reverse effect: repopulation from unbound component diffusion

Dissipation =>

- Feed down of lower states
- Deceleration of unbound component

Typical evolution at constant T



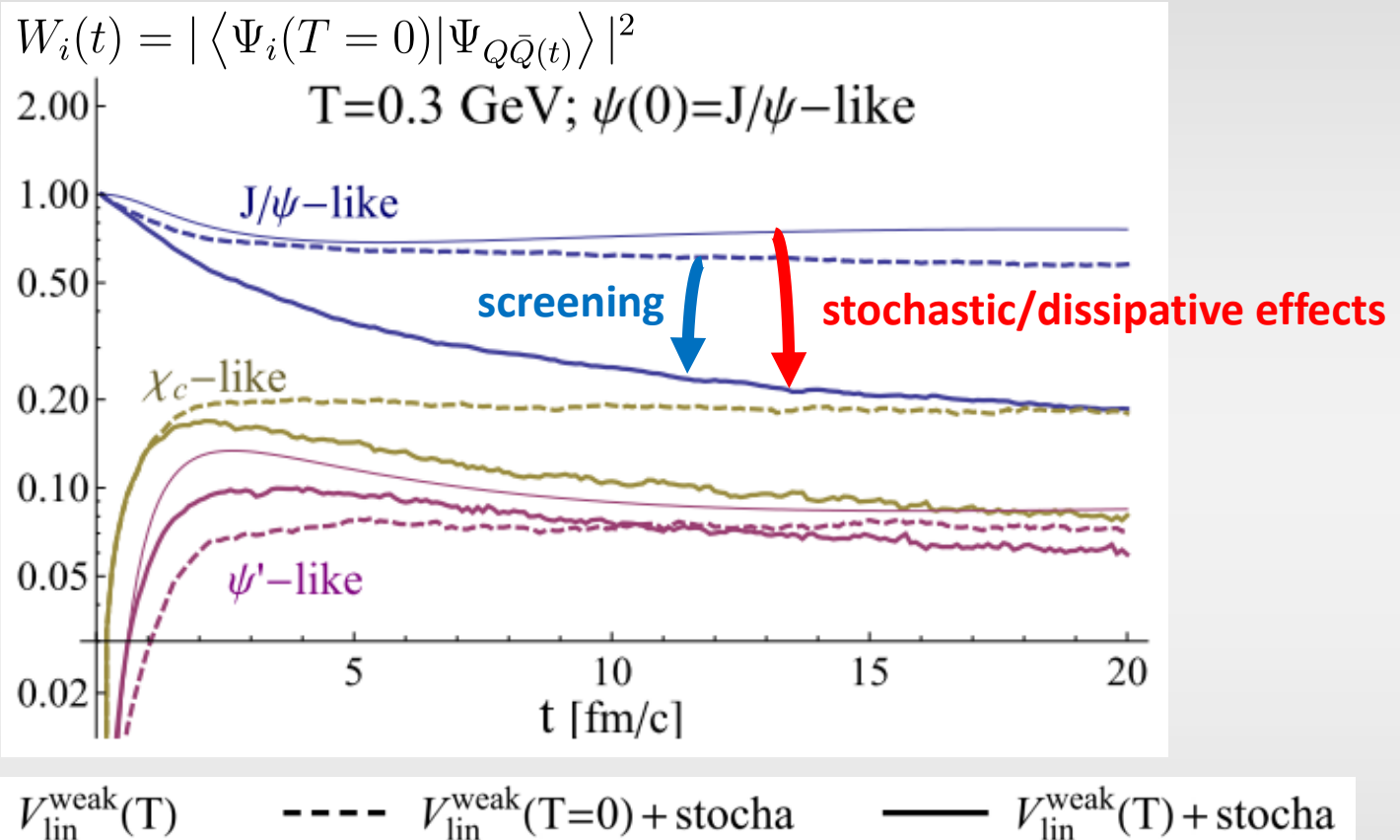
Decay
 of the global $Q\bar{Q}$
 system with a
 common rate

Long transient phase:
 reequilibration of the population

Continuous leakage out of the potential well
 => no stable equilibrium but only local equilibriums at each $t \gg \tau_{\text{relax}}$

Typical evolution at constant T

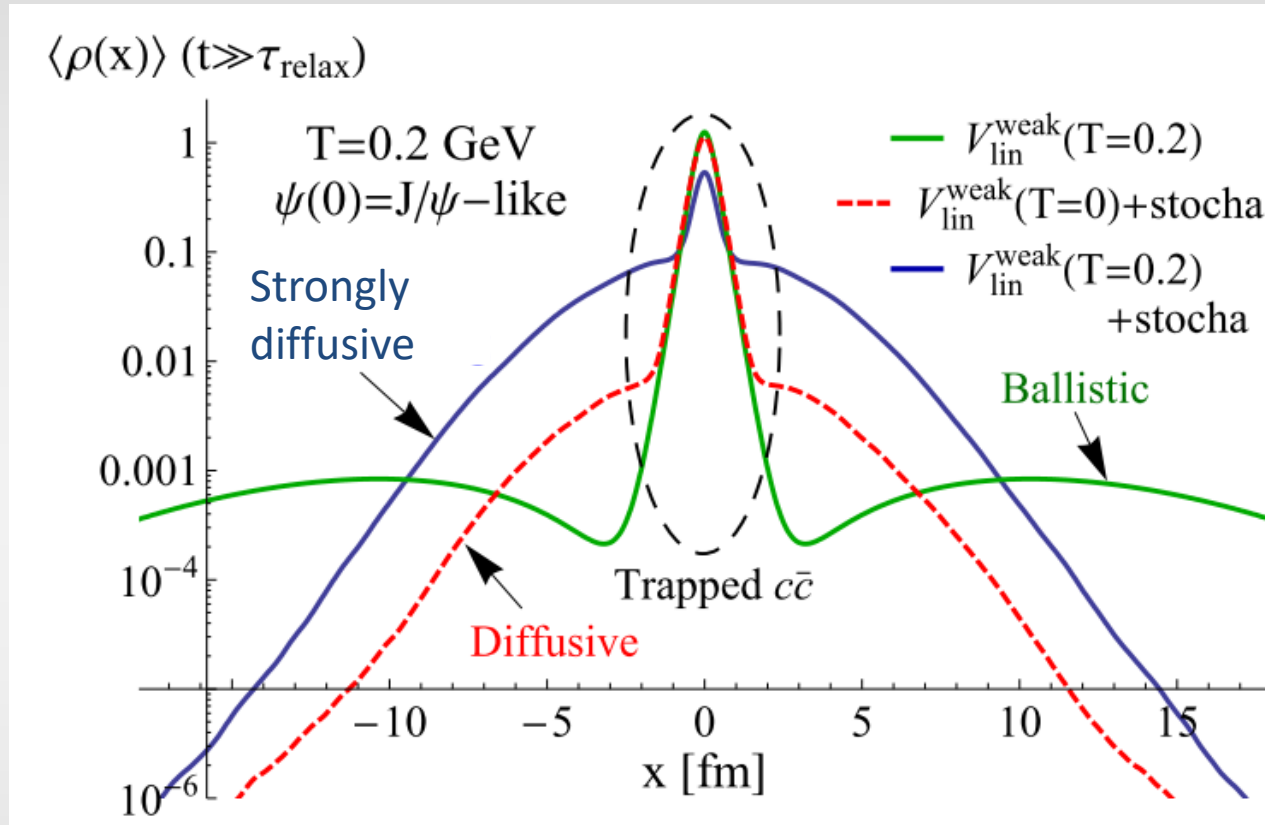
Reminder: here populations = projection on vacuum states



Both the screening and the stochastic/dissipative effects are important

Local equilibria in stationary QGP

Densities at some large time

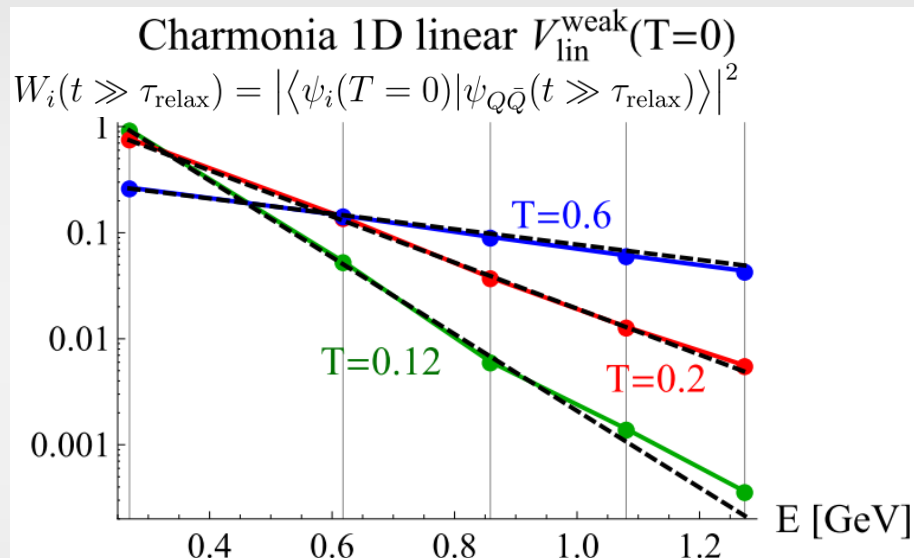


- Part of the QQ subsystem remains trapped inside the potential “well” and populates the eigenstate weights
- Different regimes depending if screening and/or thermal forces are included

Local equilibriums in stationary QGP

Populations at some large time

No screening



----- Boltzmann distribution at T

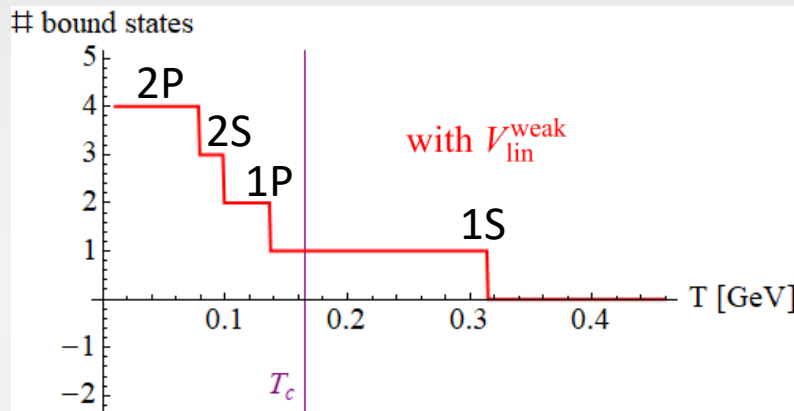
- Correct thermal equilibrium for the relative population of the bound states $[E_i \leq V_{\text{lin}}(\mathbf{x} \rightarrow \infty, T=0)]$ despite the presence of the saturation (string breaking)
- Effect of time : global shift of the distribution toward smaller values

Local equilibria in stationary QGP

Reminder: here

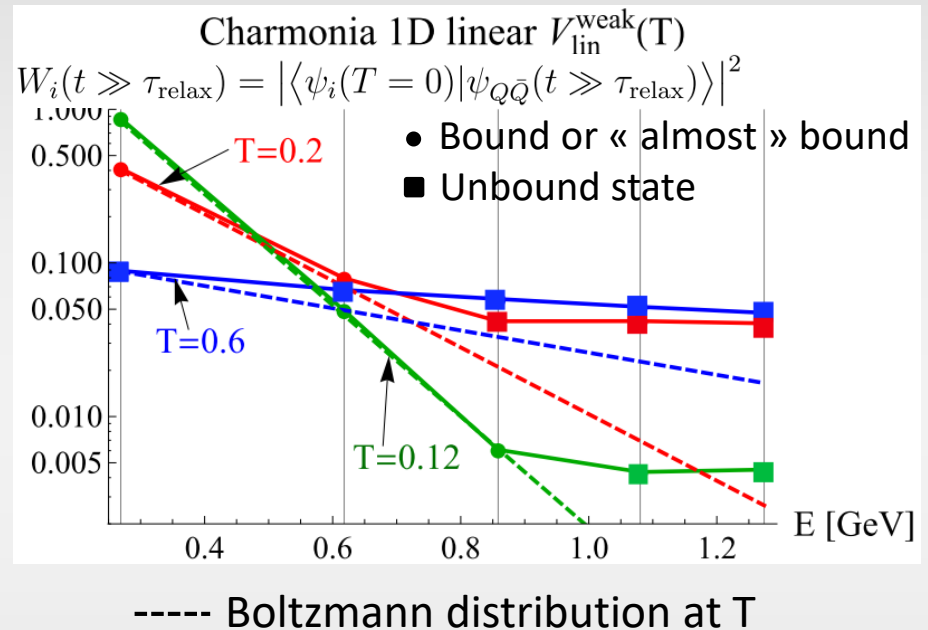
populations = projection on vacuum states

bound states with T



Populations at some large time

With screening



- Correct thermal equilibrium for the relative population of the bound states or “almost bound” [$E_i \leq$ or $\sim V_{\text{lin}}(x \rightarrow \infty, T)$]
- For the unbound states [$E_i > V_{\text{lin}}(x \rightarrow \infty, T)$]: flatish distributions => reflect the density of the free $Q\bar{Q}$ component in the vicinity of the potential well that overlap with the unbound states

Thank you

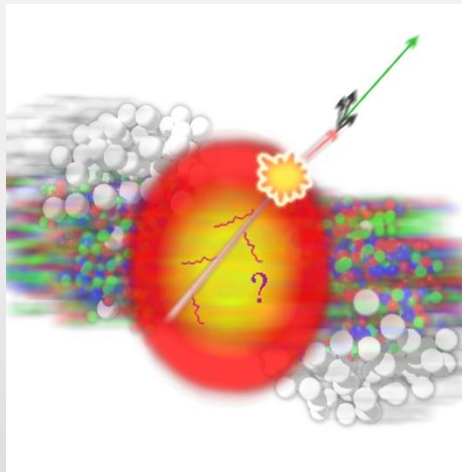
Back-up

Hard probes

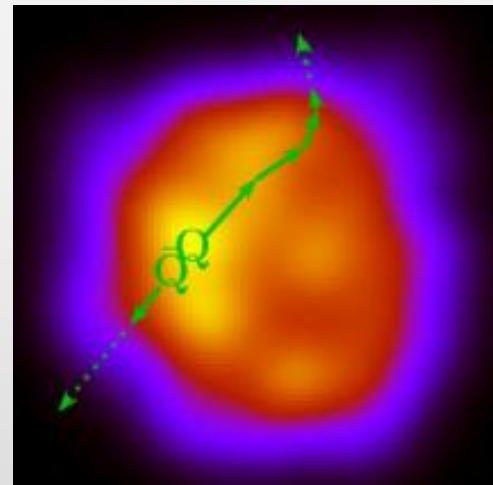
To study the medium properties beyond the freeze out «horizon»...

- ✓ Nature ? Weakly or strongly interacting ?
 - ✓ Density and temperature ?
- ✓ Transport properties (viscosity, spatial diffusion...) ?

... one can analyse the « tomography » of the medium realised by the hard probes (high p_T or massive partons)



Jet quenching



Open heavy flavours and quarkonia propagations

Hard probes

Why hard probes are interesting ?

- ✓ Produced in pQCD processes, only at the very beginning of the collision
- ✓ Do not flow hydrodynamically but propagate/interact through other processes that are sensitive to the medium properties

They can propagate inside the QGP all along its evolution to probe it

And quarkonia specifically ?

- ✓ Known $Q\bar{Q}$ initial production (CNM ?)
- ✓ Simple 2 particles systems
- ✓ Binding described by potentials
- ✓ Some bound states may survive above T_c
- ✓ Weakly affected by the final hadronic phase

Can be mastered to probe the QGP temperature and deconfinement

- ✓ However: the state formation times are subject to debate
=> states form before, in-medium or at the freeze out ?

Common models of quarkonia suppression in QGP

Sequential suppression (Matsui and Satz)

- Based on states T_{diss}
- If $T_{\text{QGP}} > T_{\text{diss}}$ the state is dissociated forever
- If $T_{\text{QGP}} < T_{\text{diss}}$ the state evolves adiabatically

=> Quarkonia as early QGP thermometer

Statistical hadronisation (Braun-Munzinger, Stachel...)

- All $Q\bar{Q}$ pairs are dissociated
- Statistical recombination at freeze-out

=> Quarkonia as thermometer of T_c

Transport models (Zhao, Rapp, Zhuang...)

- Sequential suppression like but
- Quarkonia \leftrightarrow $Q+\bar{Q}$ possible during the evolution

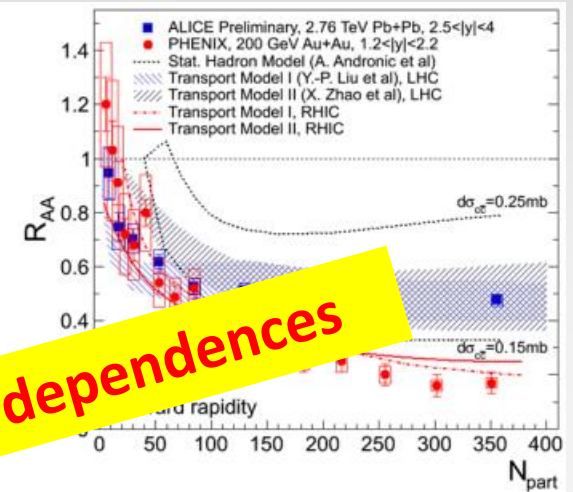
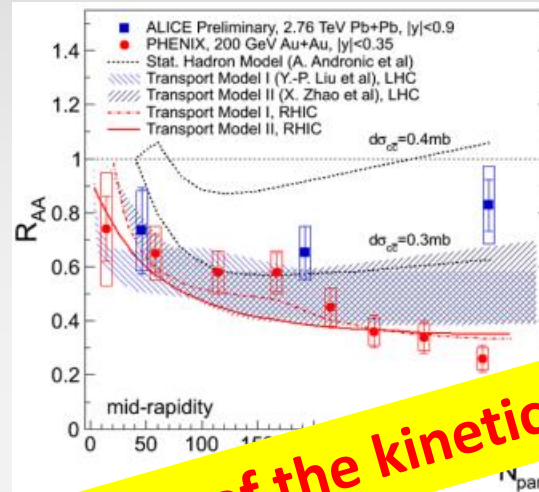
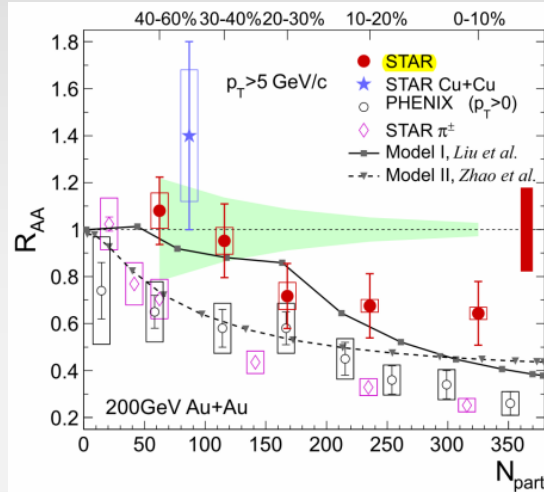
=> Quarkonia as continuous QGP thermometer

Description of the data

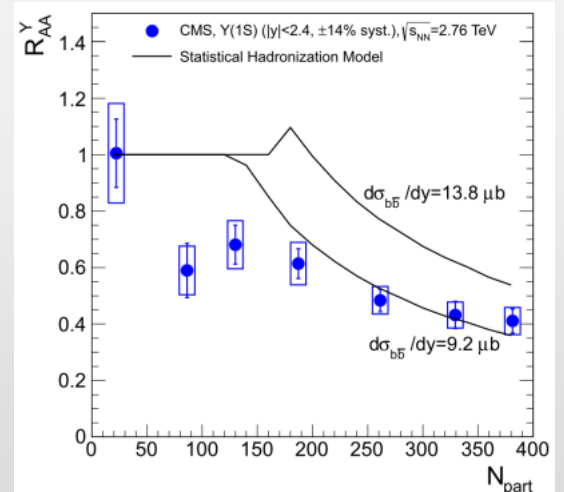
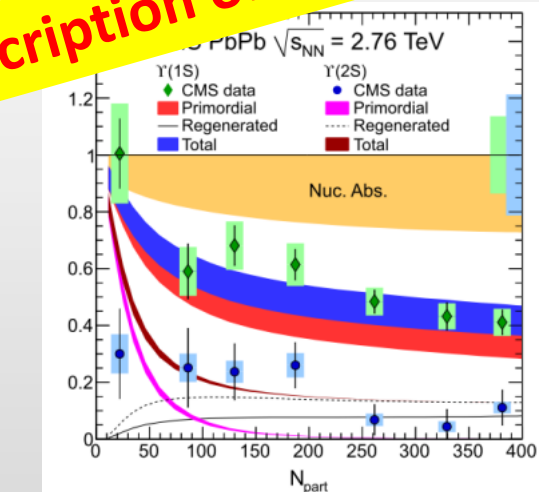
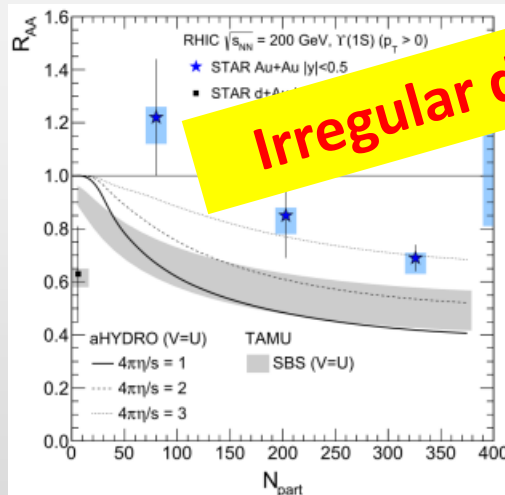
RHIC

LHC run 1

J/ψ



BOTTOMONIA

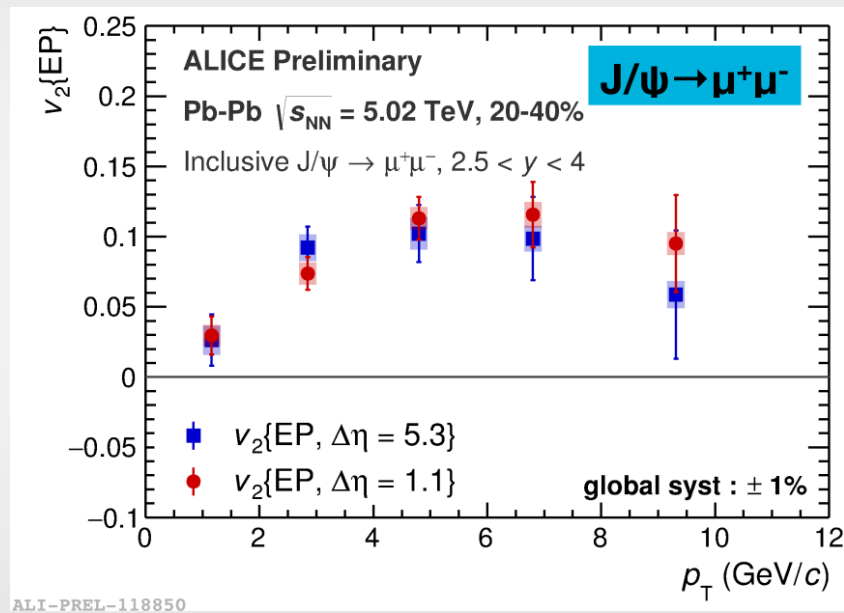


Irregular description of the kinetic dependences

Looking at recent data

- ✓ Hints for some recombination into low p_T J/ψ

Significant v_2

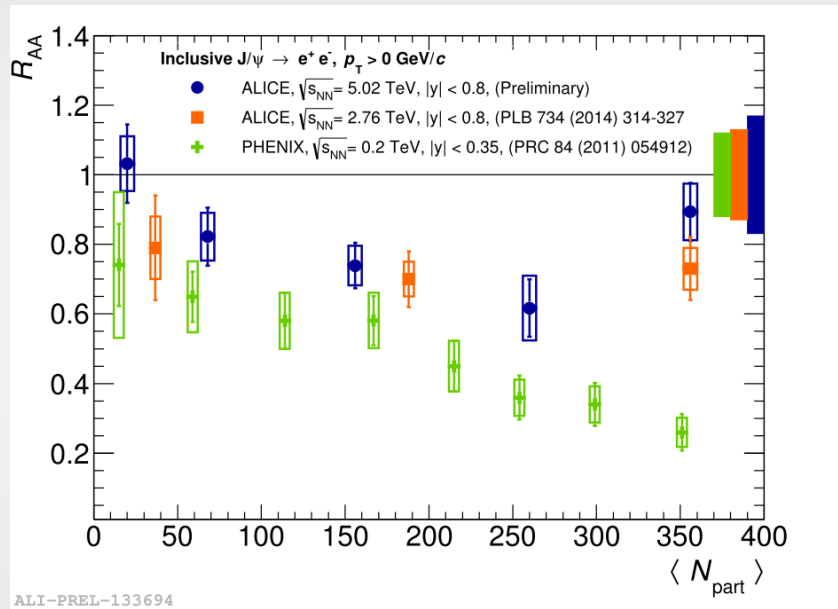


(Alternative idea: partial thermalisation of color octet states ?)

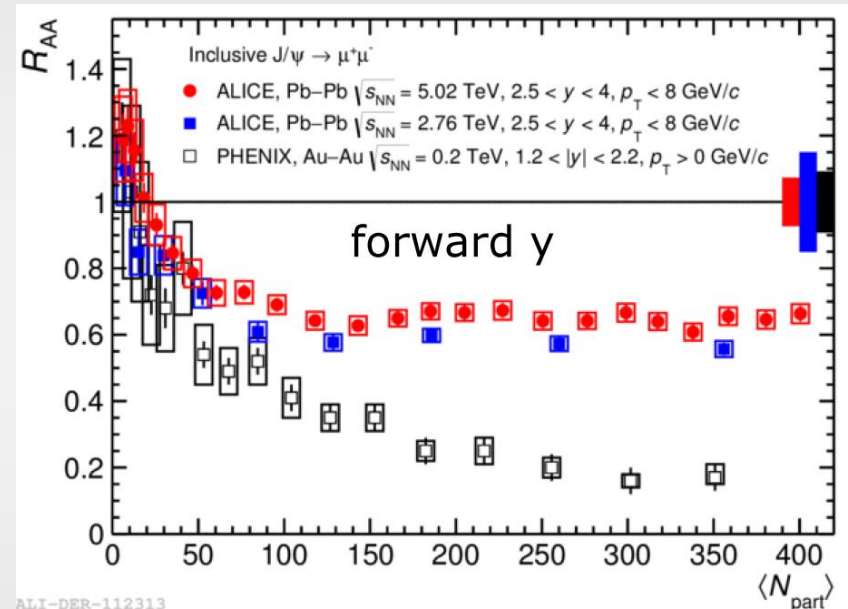
Looking at recent data

✓ Hints for some recombination into low p_T J/ψ

At mid-y: ordering with s_{NN}



At forward-y: ordering with s_{NN}



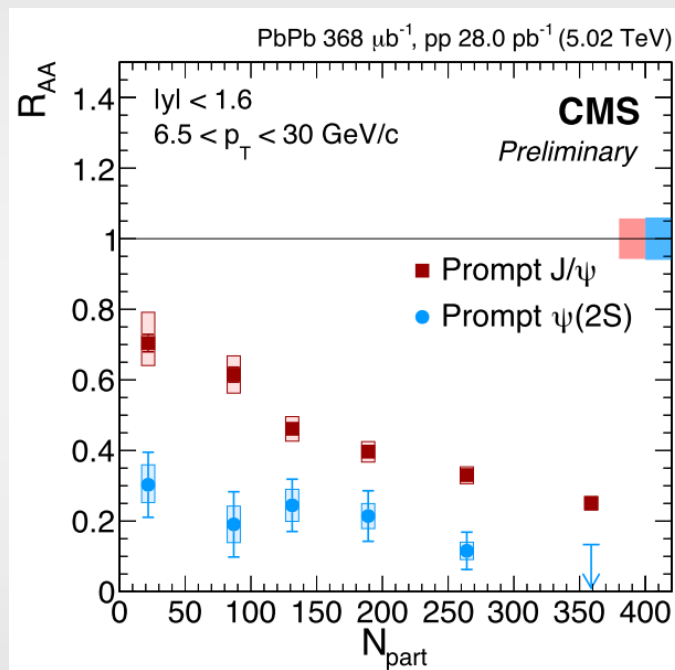
while not observed for the bottomonia (more suppression as s_{NN} increases)

-> Considering some recombination of uncorrelated charm/anticharm quarks is most probably necessary to any model aiming to describe the data

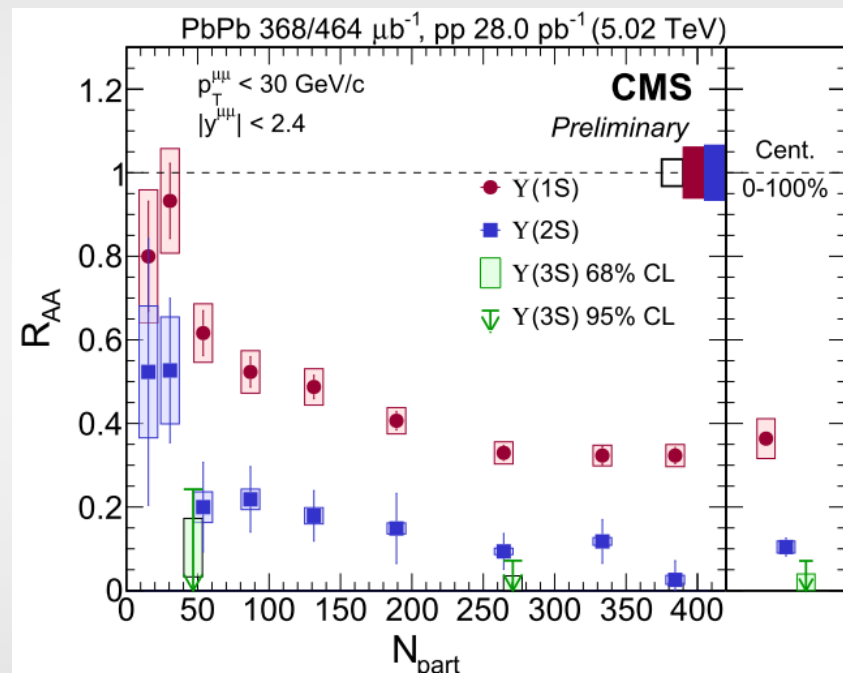
Looking at recent data

- ✓ Relative sequential-like suppression between the quarkonium states

Charmonia



Bottomonia

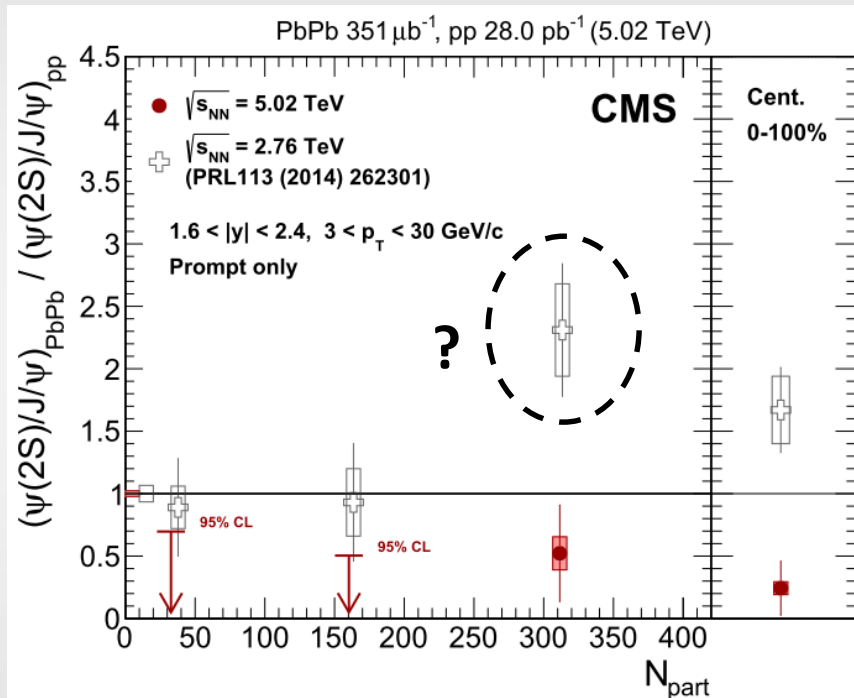


But: smooth evolution with centrality (not really « steps-like » behaviour)

Looking at recent data

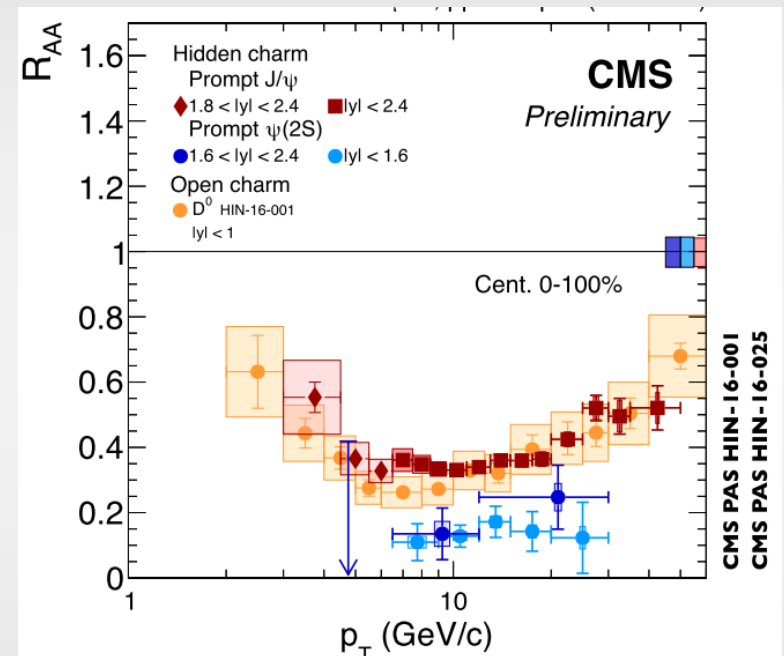
But a bit puzzling...

Ratio $\psi(2S) / J/\psi$ at forward-y



LHC run 1: « unexpected » ratio >1
 no more observed at LHC run 2

Comparison J/ψ and D^0



Similar R_{AA} even at higher p_{T} :
 - cannot be recombination
 - troubling that sequential-like (J/ψ) and
 Eloss/diffusion+fragmentation (D^0) dynamics
 give the same behaviour...

Common model assumptions

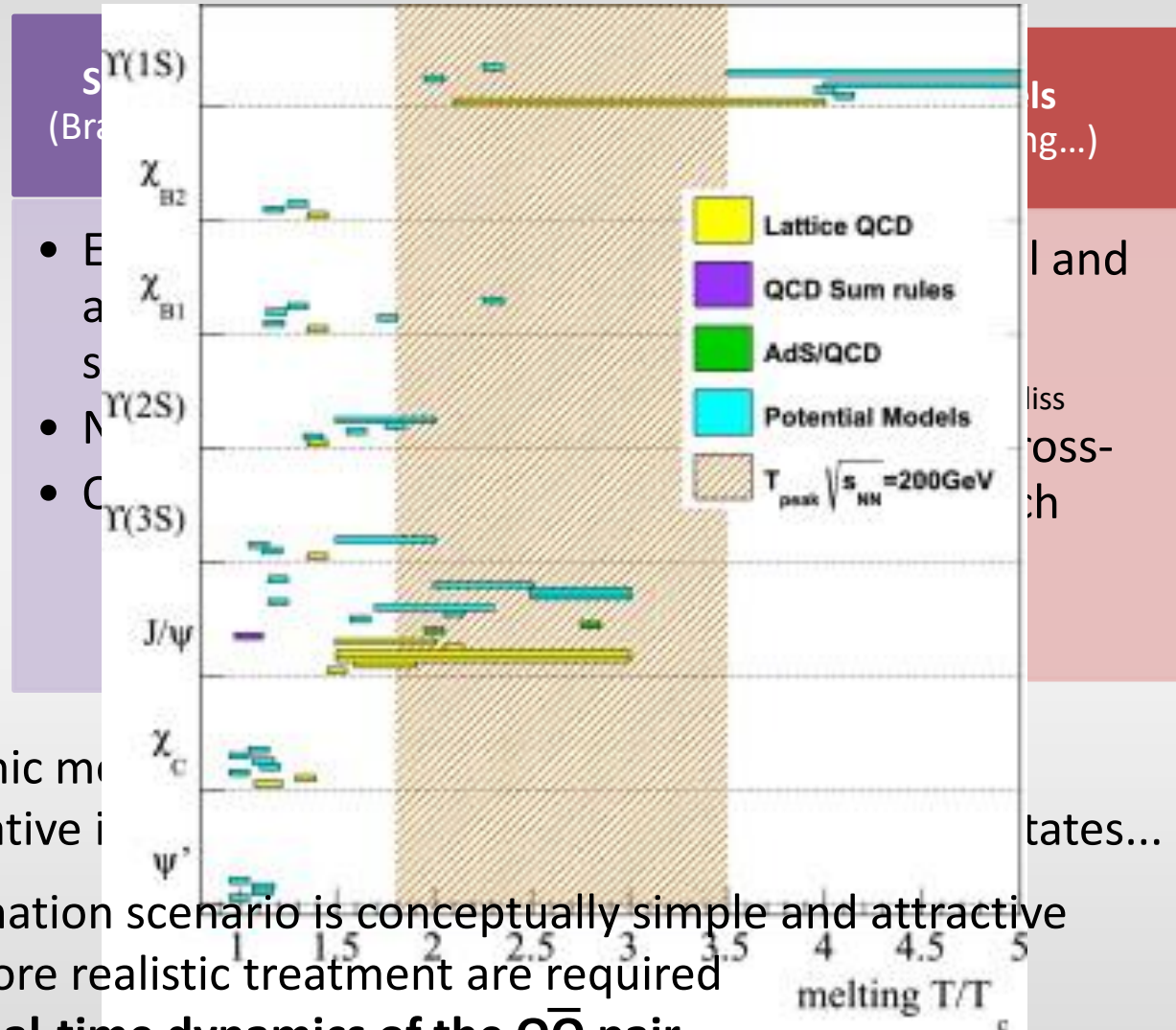
Sequential suppression (Matsui and Satz)

- Everything happens at the BEGINING in a quasi-stationnary medium
- Questionable T_{diss}
- Adiabatic evolution
- No recombinations

Problem: very dynamic medium
no effects from non-dissociative i

A screening + recombination scenario is conceptually simple and attractive
but more realistic treatment are required

**-> real-time dynamics of the $Q\bar{Q}$ pair
(and not only of its bound states)**



Our motto

Consider:

color screening, thermal effects and QGP dynamics

INNER DYNAMICS OF EACH $Q\bar{Q}$ PAIR

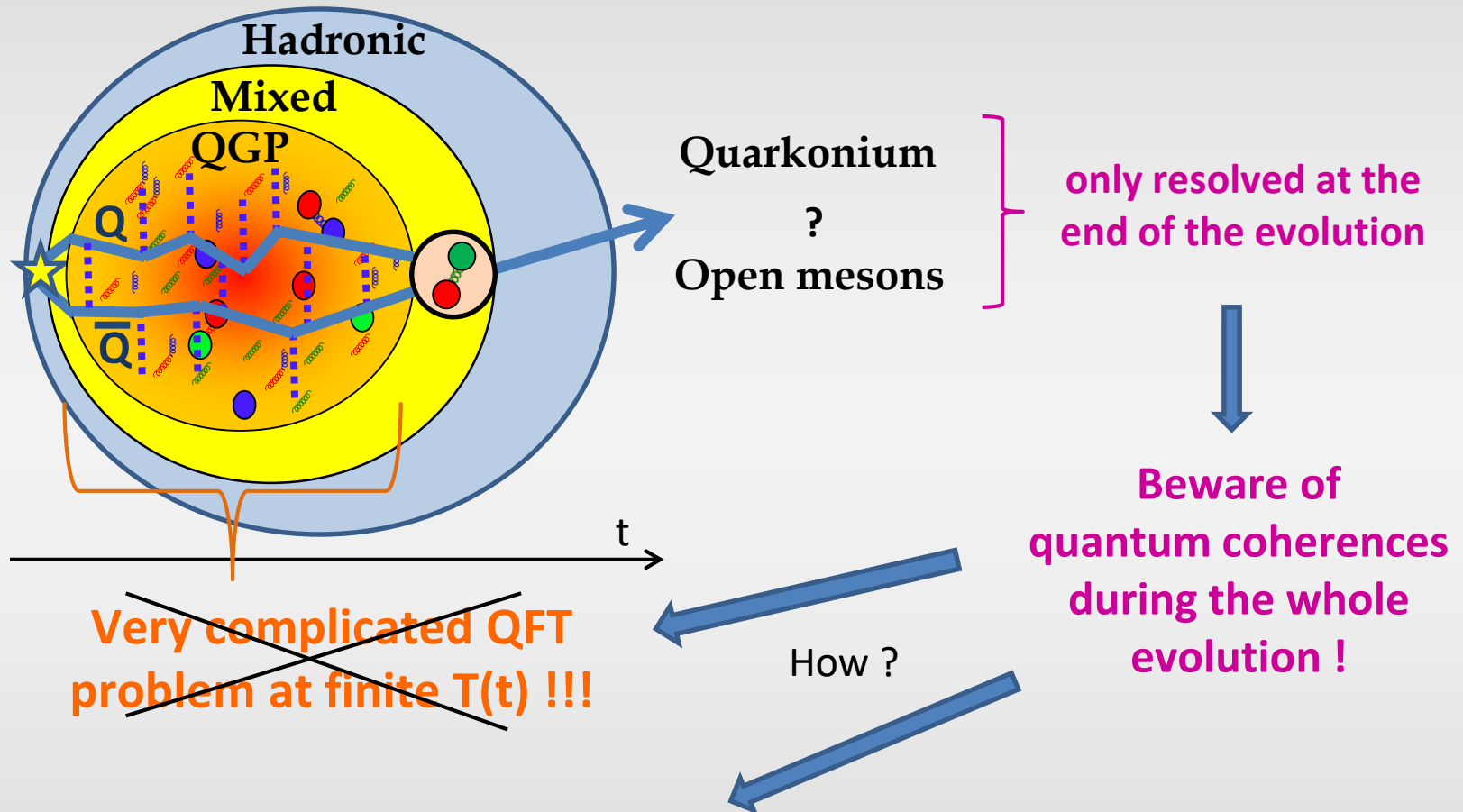
A dynamical and continuous picture of the dissociation, recombination, transitions to other bound states and energy exchanges with the QGP.

+

$Q\bar{Q}$ PAIRS EVOLUTION IN QGP

Realistic t-dependent background:
Monte-Carlo event generator with initial fluctuations

Inner dynamics ? -> back to concepts



Treatment within the open quantum system framework !

Other frameworks: cross sections, imaginary potentials from QFT at finite T , EFT and IQCD...

Inner dynamics: open quantum systems

QGP = thermal bath

The common open quantum system approach

- **Idea:** density matrix and {quarkonia + bath} => bath integrated out
 - ⇒ non unitary evolution + decoherence effects
 - ⇒ At equilibrium: Boltzmann distributions $\propto \exp\left(\frac{-E_n}{k_B T}\right)$

But defining the bath/interaction is complex and application entangled

Borghini and Gombeaud*

simple model of bath and Einstein rate equation

But: unable to thermalise the inner dynamics

Akamatsu, Rothkopf et al.**

closed-time path integral formalism + LO thermal QCD

⇒ complex potential / master equations

⇒ stochastic potential equation in the recoilless limit

But: rising energy problem.

See also Young and Dusling (2013), Blaizot (2016), De Boni (2017)...

Will a pQCD + open quantum system formalism describe the data ?

* N. Borghini et al., Eur. Phys. J. C 72 (2012) 2000;

** Y. Akamatsu Phys.Rev. D87 (2013) 045016; Phys. Rev. D 91, 056002 (2015); Kajimoto, Akamatsu, Asakawa & Rothkopf (2017) 1705.03365

Inner dynamics: open quantum systems

Langevin-like approaches

- **Idea:** Effective equations (possibly not from first QCD principles) with few parameters to unravel/mock the open quantum approach while keeping most of the quantum features

Heavy quarks are Brownian particles ($M_Q \gg T$) + **Drag $A(T)$**
=> need for Langevin-like equation

($A(T)$ from single heavy quark observables or IQCD calculations)

Semi-classical *

Inspired by
Young and Shuryak **

Schrödinger-Langevin
equation

Include fluctuation/dissipation mechanisms to mimic the dense QGP-Q \bar{Q} collisions and real in-medium potentials for the screening

Inner dynamics: semi-classical model

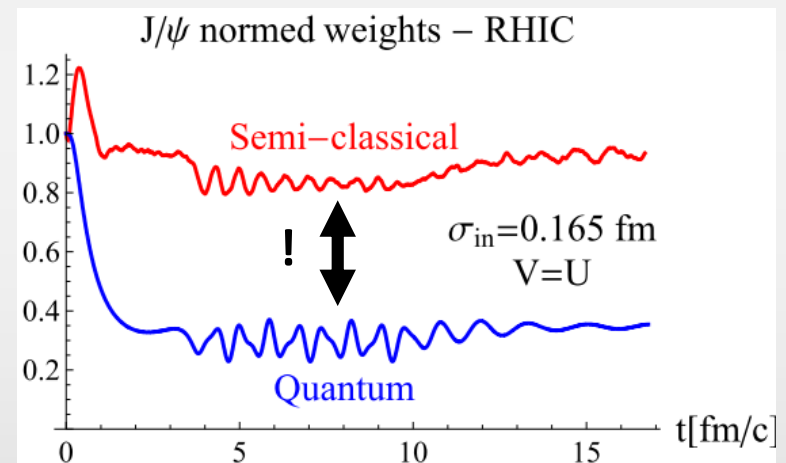
In few words: Evolution of Wigner distributions with a classical Langevin dynamic

=> Interesting suppression patterns:
smooth evolutions, more suppression at LHC than at RHIC...

BUT

1) Without fluctuation-dissipation:
discrepancies with pure quantum results

2) Uncorrect thermalisation
+ violation of Heisenberg principle at low T



=> Need for a fluctuation-dissipation mechanism
compatible with quantum mechanics !!

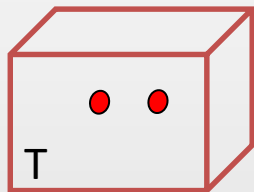
Inner dynamics: Schrödinger-Langevin (SL) equation

Derived from the Heisenberg-Langevin equation*, in Bohmian mechanics** ...

$$i\hbar \frac{\partial \Psi_{Q\bar{Q}}(\mathbf{r}, t)}{\partial t} = \left(\hat{H}_{\text{MF}}(\mathbf{r}) - \mathbf{F}_{\mathbf{R}}(t) \cdot \mathbf{r} + A(S(\mathbf{r}, t) - \langle S(\mathbf{r}, t) \rangle_{\mathbf{r}}) \right) \Psi_{Q\bar{Q}}(\mathbf{r}, t)$$

Hamiltonian: Mean Field: T-dependent color screened potential
Generally taken from lattice-QCD

Static IQCD calculations (maximum heat exchange with the medium):



$\left\{ \begin{array}{l} F : \text{free energy} \\ S : \text{entropy} \end{array} \right.$



$U = F + TS$: internal energy
(no heat exchange)

- “Weak potential” $F < V_{\text{weak}} < U \Rightarrow$ some heat exchange
- “Strong potential” $V = U \Rightarrow$ adiabatic evolution

* Kostin The J. of Chem. Phys. 57(9):3589–3590, (1972) ** Garashchuk et al. J. of Chem. Phys. 138, 054107 (2013)

Mócsy & Petreczky Phys.Rev.D77:014501,2008 ; Kaczmarek & Zantow arXiv:hep-lat/0512031v1

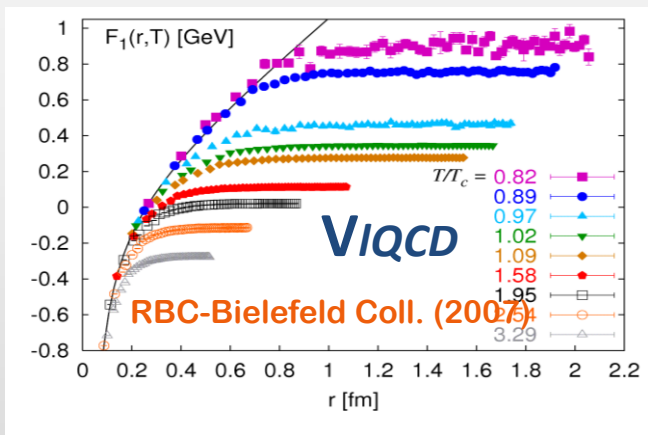
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Hamiltonian: Mean Field: T-dependent color screened potential
Generally taken from lattice-QCD. Only singlet for now.

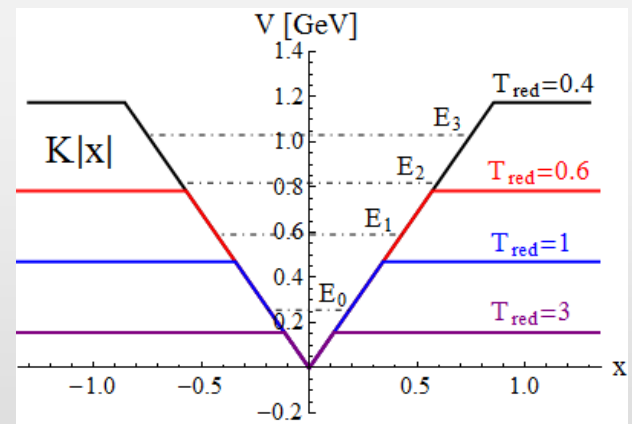
3D not easy to implement => 1D simplified model
not aim to reproduce the data but rather gives insights on the dynamics.



1D



simplification



Linear approx

Screening(T)
as VIQCD₂₈

Parameters (K, Vmax) chosen to reproduce
quarkonium spectrum + $B\bar{B}$ or $D\bar{D}$ threshold

Inner dynamics: SL equation

Derived from the Heisenberg-Langevin equation, in Bohmian mechanics ...

$$i\hbar \frac{\partial \Psi_{Q\bar{Q}}(\mathbf{r}, t)}{\partial t} = \left(\hat{H}_{\text{MF}}(\mathbf{r}) - \mathbf{F}_{\mathbf{R}}(t) \cdot \mathbf{r} + A(S(\mathbf{r}, t) - \langle S(\mathbf{r}, t) \rangle_{\mathbf{r}}) \right) \Psi_{Q\bar{Q}}(\mathbf{r}, t)$$

Fluctuations: thermal excitation

Taken as a « classical » white stochastic force/noise scaled such as to obtain $T_{Q\bar{Q}} = T_{QGP}$ at equilibrium

The noise operator is assumed here to be a commuting c-number whereas it is a non-commuting q-number within the Heisenberg-Langevin framework.

I. R. Senitzky, Phys. Rev. 119, 670 (1960); 124, 642 (1961).

G. W. Ford, M. Kac, and P. Mazur, J. Math. Phys. 6, 504 (1965).

R. Katz and P.B. Gossiaux, Annals Phys. 368 (2016) 267-295

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Dissipation: thermal de-excitation

$$S(\mathbf{r}, t) = \arg(\Psi_{Q\bar{Q}}(\mathbf{r}, t))$$

- ✓ non-linearly dependent on $\Psi_{Q\bar{Q}}$
 - ✓ real and ohmic
- ✓ brings the system to the lowest state
- ✓ with $A(T) \propto T^2$ the Drag coefficient from a microscopic model (pQCD - HTL) by Gossiaux and Aichelin

Properties of the SL equation

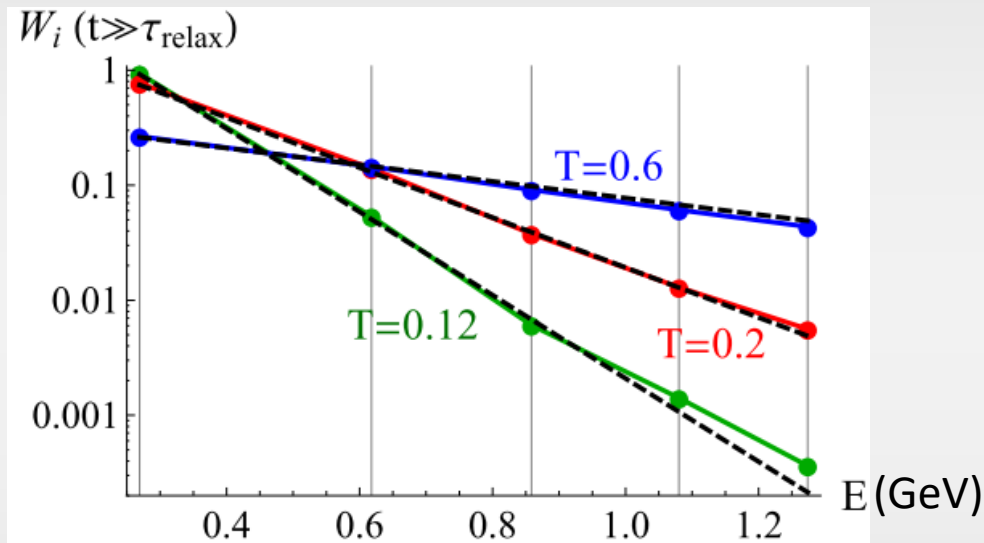
- 2 parameters: **A** (Drag) and **T** (temperature)
- Unitarity and Heisenberg principle satisfied at any T
- Non linear => Violation of the superposition principle (=> decoherence)
- A priori not related to a quantum master equation: the decoherence effects are probably not rigorous. We might be missing some physics.
- Mixed state observables from statistics:

$$\left\langle \langle \psi(t) | \hat{O} | \psi(t) \rangle \right\rangle_{\text{stat}} = \lim_{n_{\text{stat}} \rightarrow \infty} \frac{1}{n_{\text{stat}}} \sum_{r=1}^{n_{\text{stat}}} \langle \psi^{(r)}(t) | \hat{O} | \psi^{(r)}(t) \rangle$$

- Easy to implement numerically (especially in Monte-Carlo generator)

Properties of the SL equation

- Leads to local « thermal » distributions: Boltzmann behaviour for at least the low lying states



$$\text{Populations} \propto \exp\left(\frac{-E_n}{k_B T}\right)$$

(weak coupling limit: no shift and broadening of the energy levels assumed)

**=> Fluctuation-dissipation mechanism
compatible with quantum mechanics and effective !!**

Initial $Q\bar{Q}$ pair wavefunction ?

The $Q\bar{Q}$ pairs are produced at the very beginning
 BUT state formation times are subject to debate
 => we test the two extrem behaviours:

- the $Q\bar{Q}$ pair is fully decoupled into eigenstates:

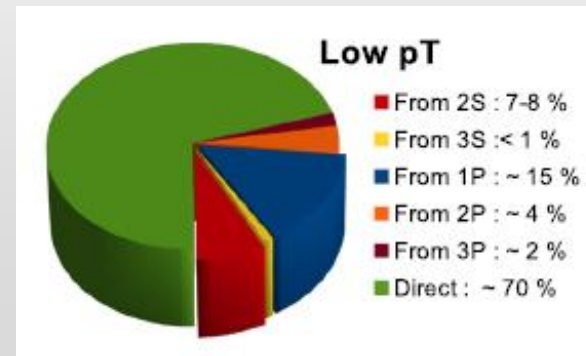
$$\psi_{Q\bar{Q}}(t=0) = \psi_i(T=0)$$

or

- the $Q\bar{Q}$ pair is not decoupled:

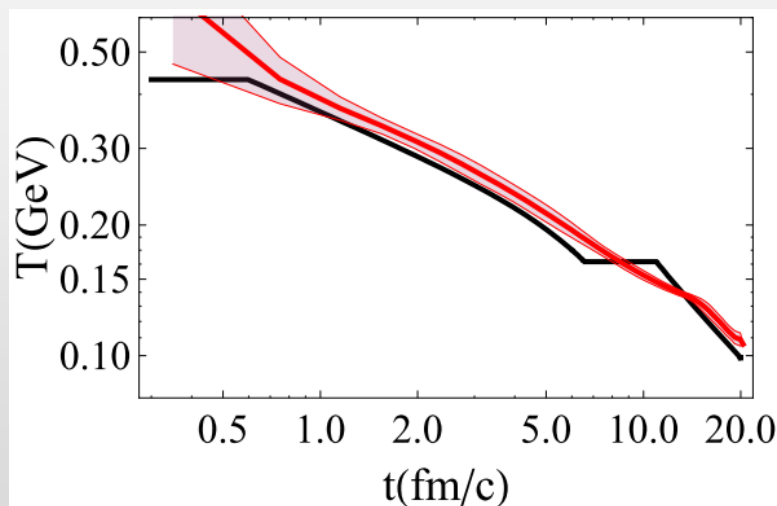
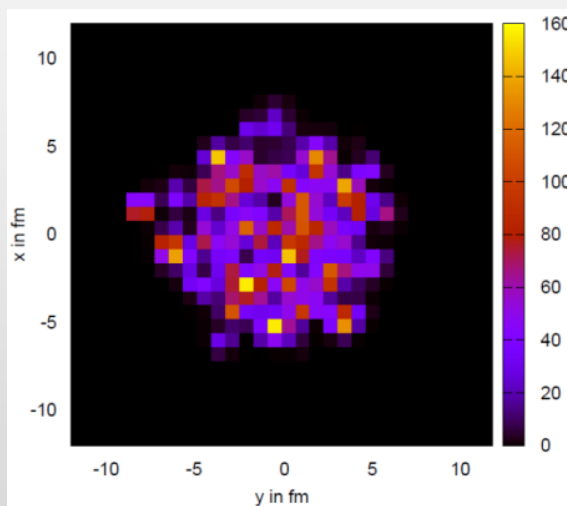
$\psi_{Q\bar{Q}}(t=0)$ = “a mixture of Gaussian S and P components”
 tuned to obtain correct feed-downs and production ratios.

e.g.: contribution to $Y(1S)$ from feed downs:



Evolution of the $Q\bar{Q}$ pairs on EPOS2 background

- Very good model for heavy ion collisions with initial fluctuations and ideal 3D hydrodynamics
- $Q\bar{Q}$ pairs initial positions: given by Glauber model
- No Cold Nuclear Matter effects (no shadowing and no hadronic scatterings)
- $Q\bar{Q}$ pair center-of-masse motion: along straight lines with no E_{loss} (assumed to be color singlet)
- Focus on bottomonia for now (CNM and statistical recombination small)



Observables

« weight » (population) W_i :

$$W_i(t) = |\langle \Psi_i(T=0) | \Psi_{Q\bar{Q}(t)} \rangle|^2$$

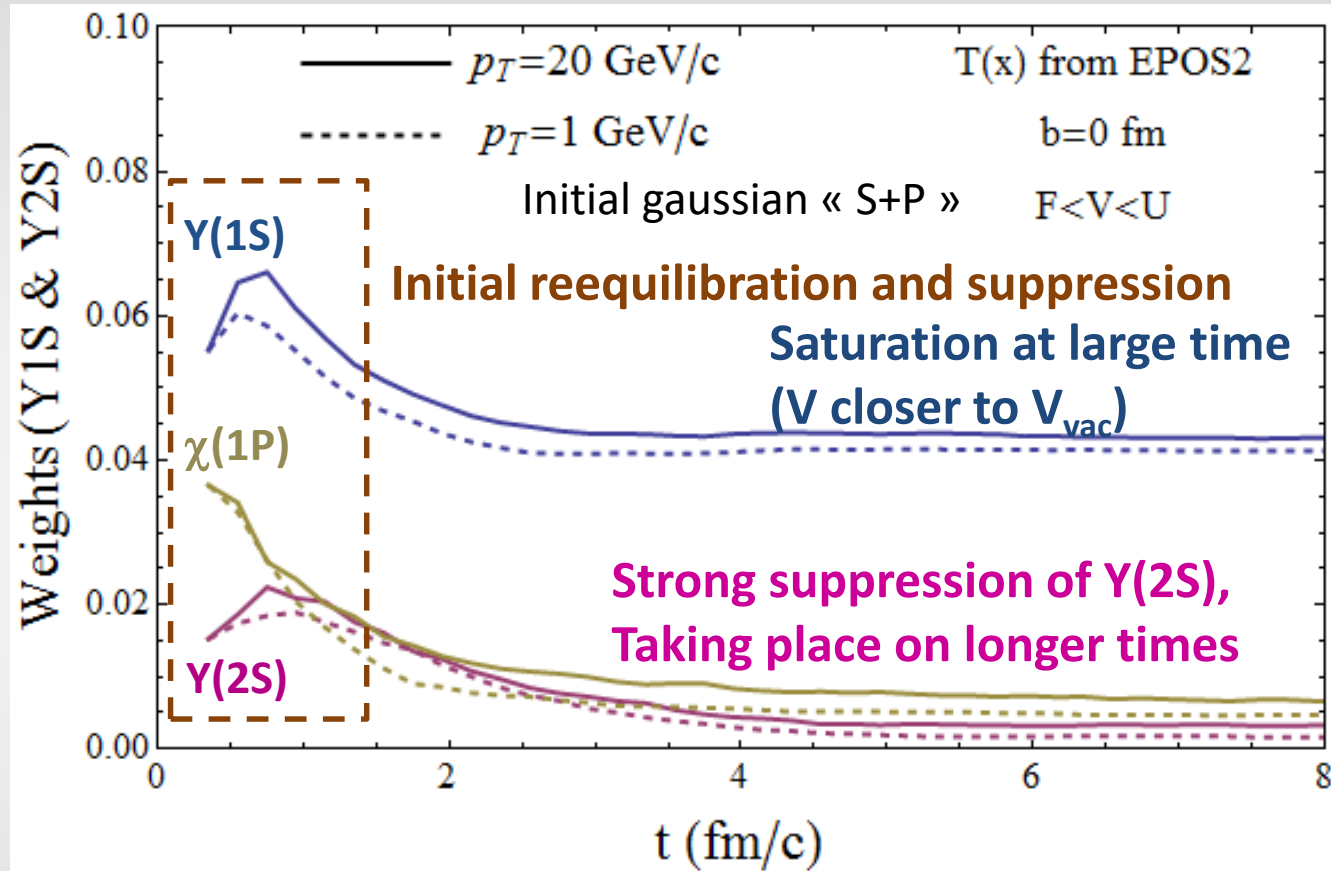
Normed weights S_i :

$$S_i(t) = W_i(t) / W_i(t=0)$$

The only « physical » values are at the freeze out.

$S_i(t)$ at freeze out convoluted with p_T - y spectra in pp collisions $\Rightarrow R_{AA}$

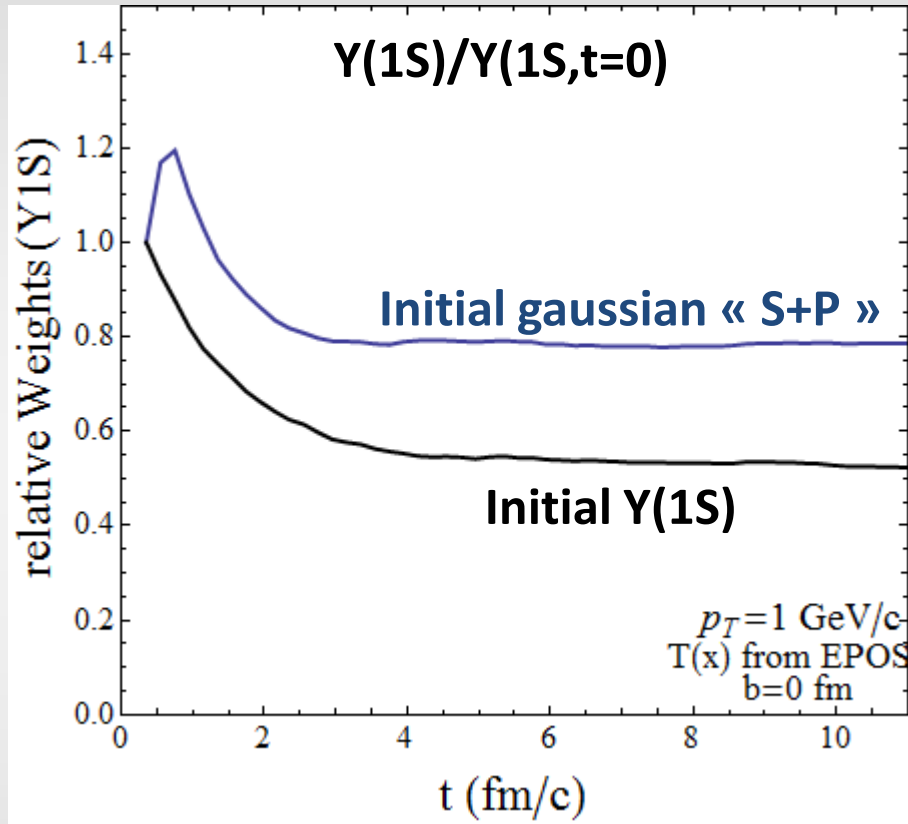
Example of evolution



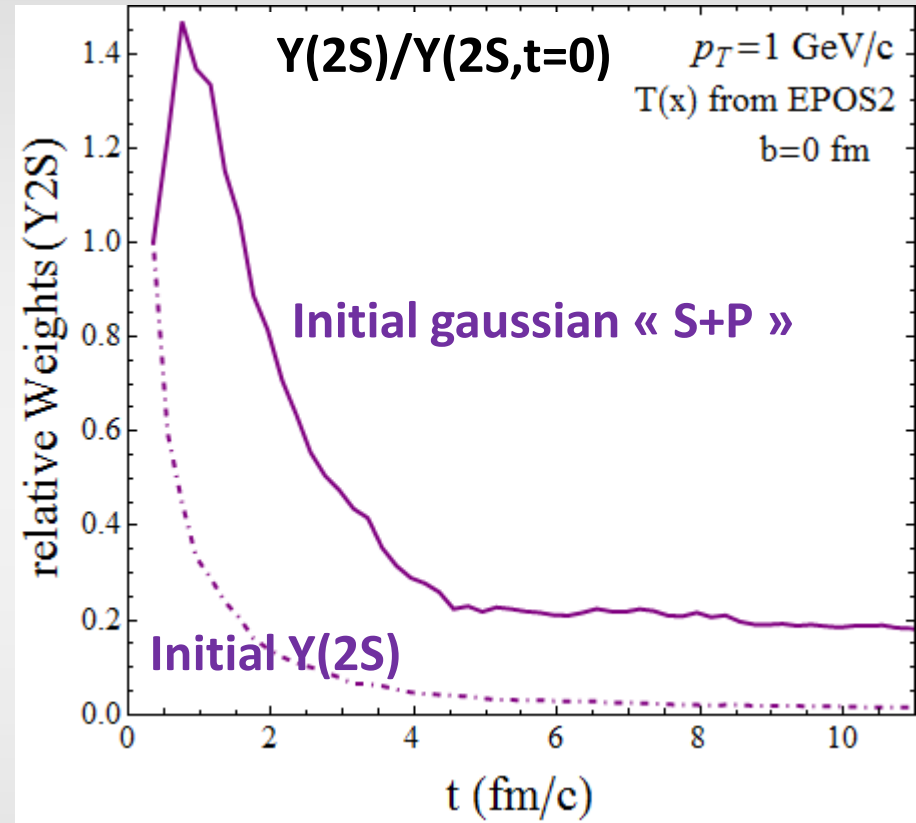
Observations

- Smooth evolutions (especially for higher excited states)
- No strong p_T dependence
- Important transitions between bound states
- Not everything is about thermal decay widths

Influence of initial state



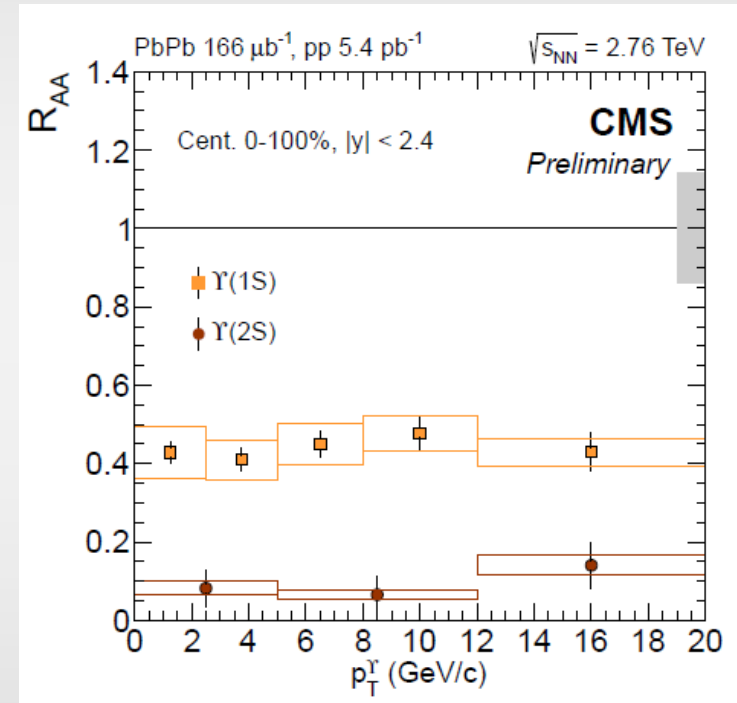
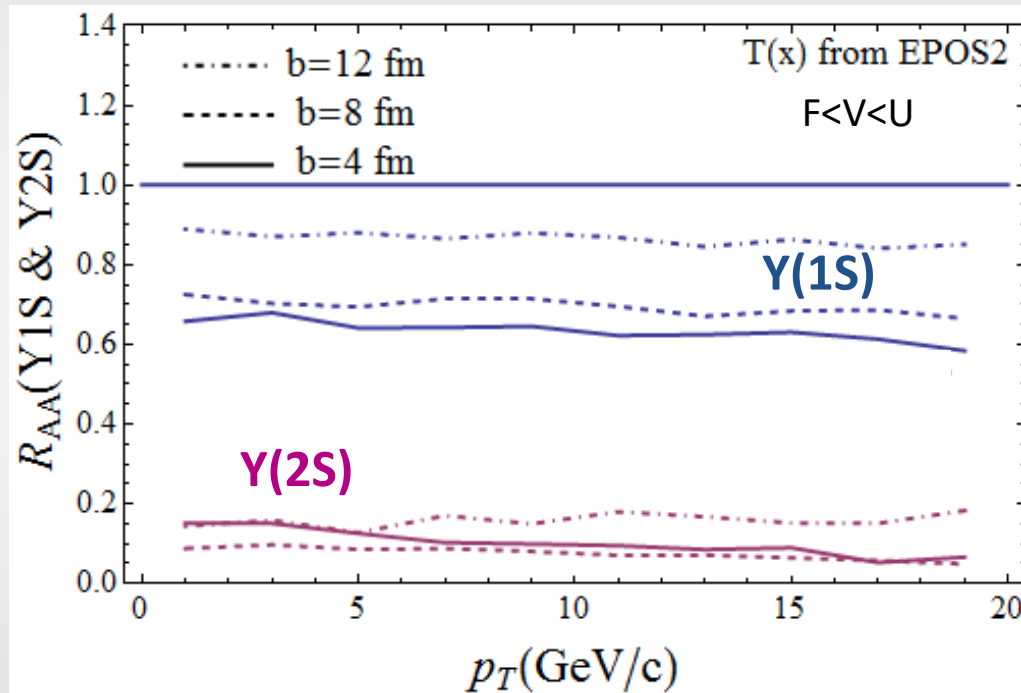
**1P component feeds the Y(1S)
at small times**



**Y(2S) found at the end of QGP
evolution are mostly the ones
regenerated from the 1S & 1P**

LHC (2.76 TeV): $R_{AA}(p_T)$

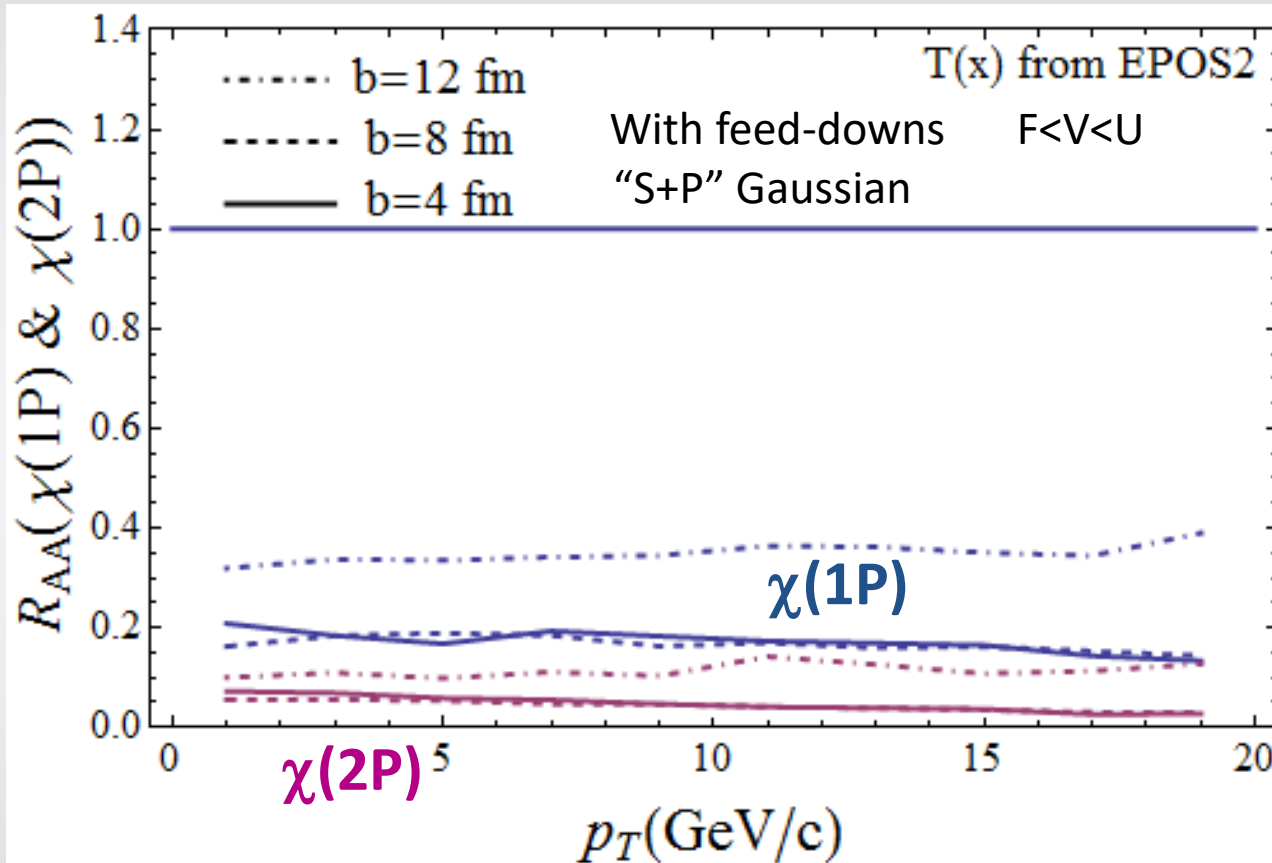
With initial Gaussian “S+P” and feed-downs



Flatish !

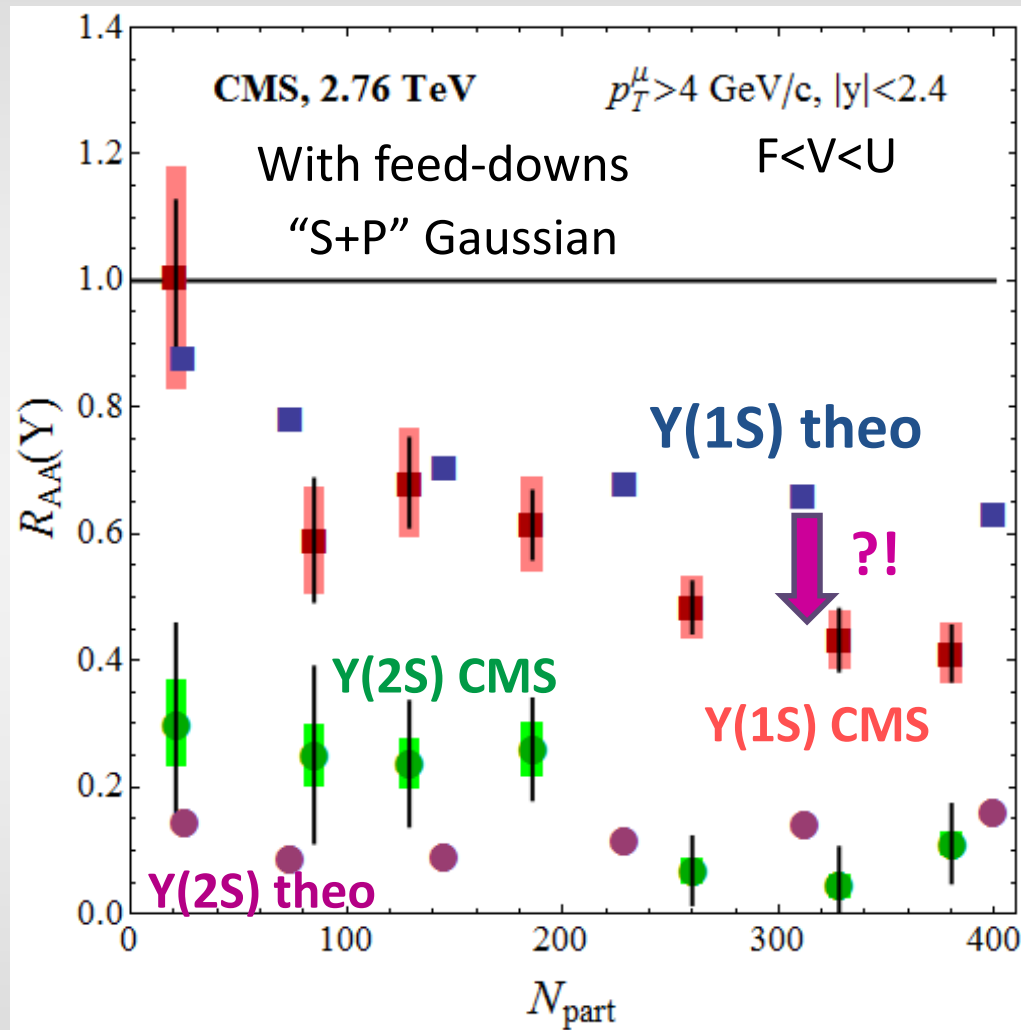
(helped by approximations: drag A and feed downs independent of momentum)

LHC (2.76 TeV): $R_{AA}(p_T)$



Same idea for P states

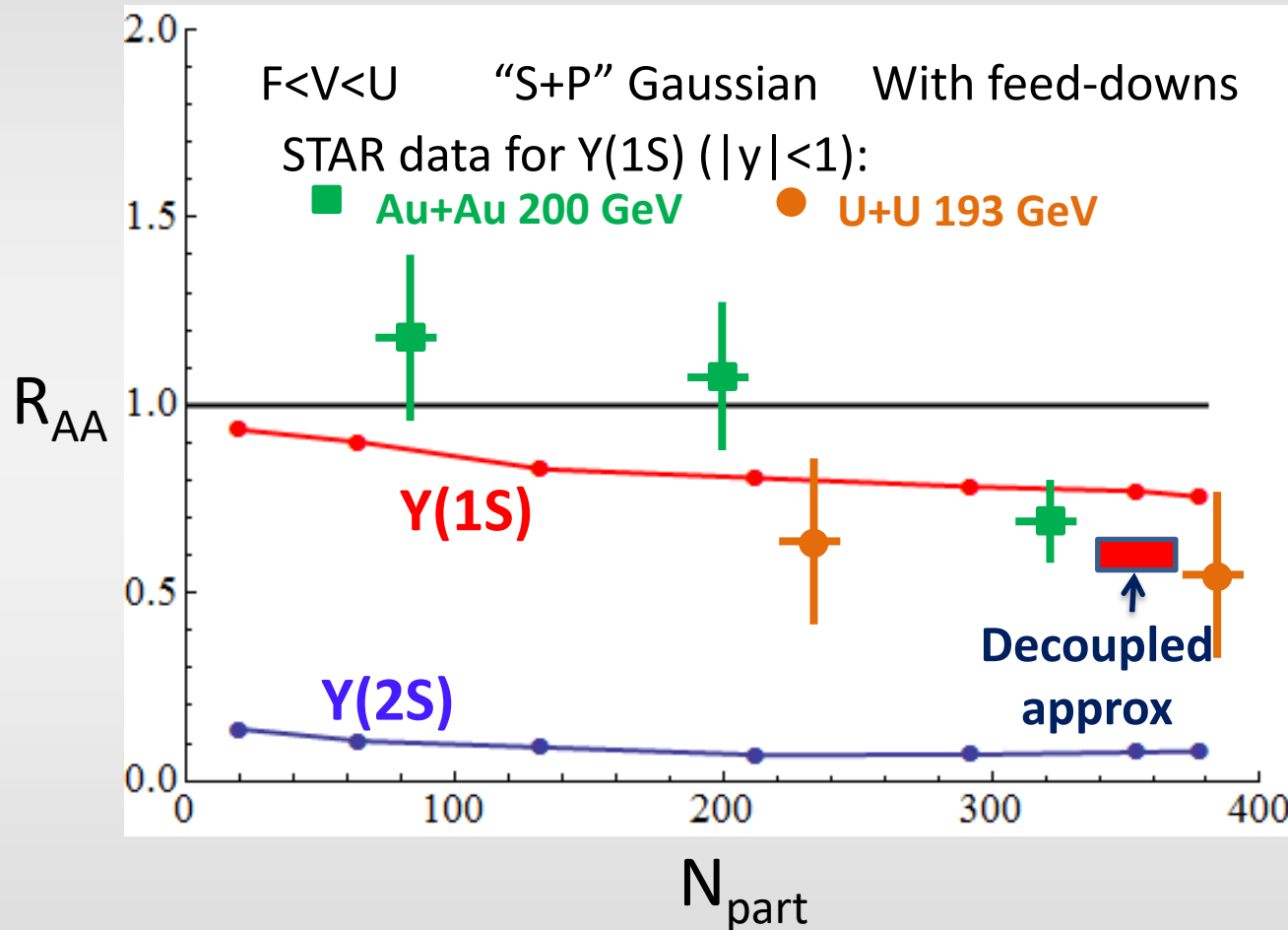
LHC (2.76 TeV): $R_{AA}(N_{part})$



lack of suppression in most central events (CNM ?)

RHIC (200 GeV): $R_{AA}(N_{part})$

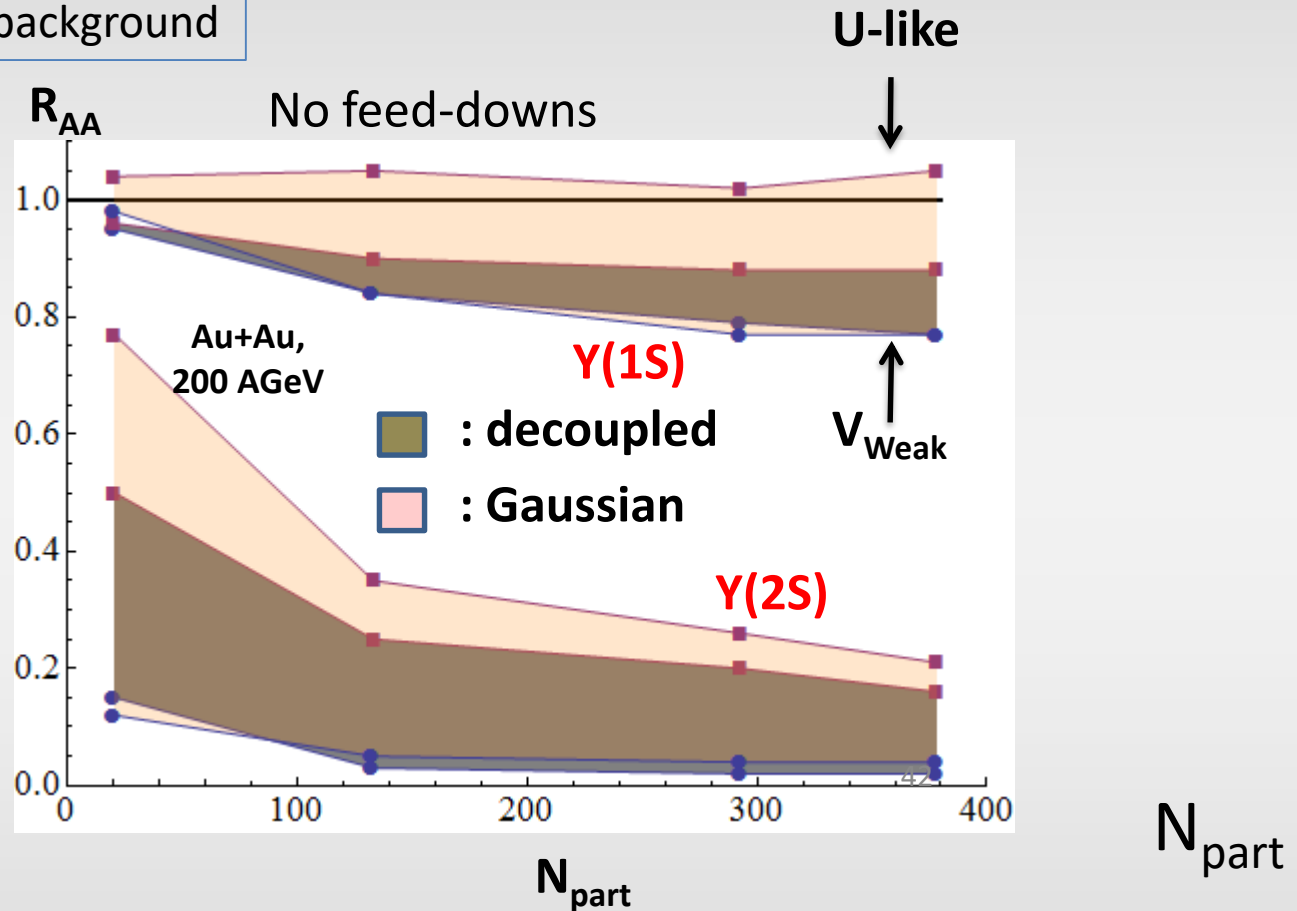
With Kolb-Heinz QGP background



Roughly good agreement with the data (with $F < V_{weak} < U$)

RHIC (200 GeV): $R_{AA}(N_{part})$

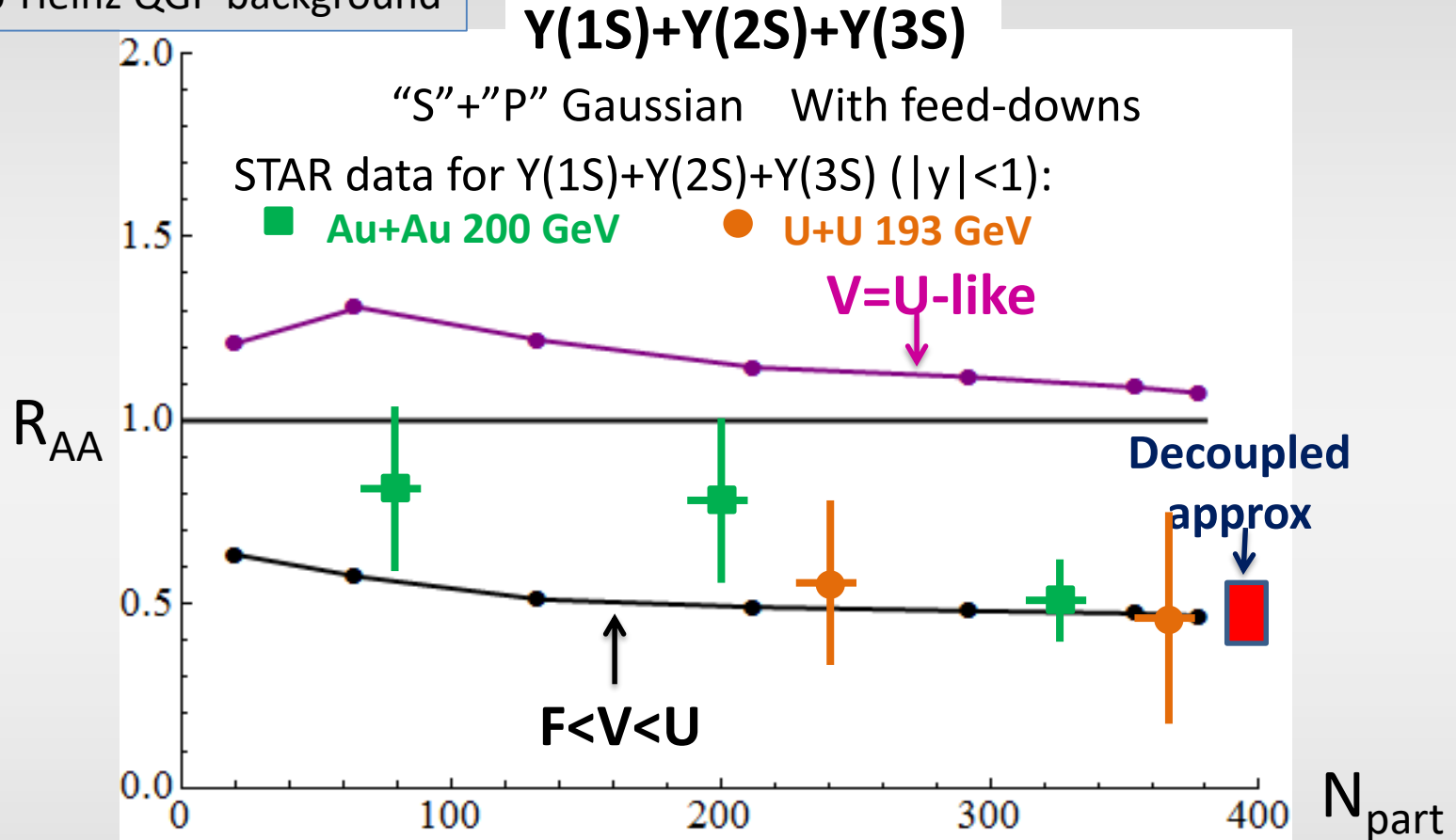
With Kolb-Heinz QGP background



With potential U : stronger dependence on initial state

RHIC (200 GeV): $R_{AA}(N_{part})$

With Kolb-Heinz QGP background



With potential U : less good results as compared to data